To do ...

- Quiz 2 this week (ends on Sat)

- WA 4 due Sun

- HW 8 PL due Tues

- HW 9 ME due Thurs
What is the equivalent system?

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note that \( \vec{F}_R \perp \vec{M}_R \)
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\[
\bar{x} = \frac{\vec{M}_R}{\vec{F}_R}
\]
Replace the force system acting on the post by a resultant force and resultant moment about point A, and specify where its line of action intersects the post AB measured from point A.

- Sum forces
- Find and sum moments

\[ \Sigma F_x : \frac{4}{5} \times 250 - 300 - 500 \cos(30) = -533 \text{ N} \]

\[ \Sigma F_y : 500 \sin(30) - \frac{3}{5} \times 250 = 100 \text{ N} \]

\[ \left| \vec{F}_r \right| = \sqrt{F_x^2 + F_y^2} = 54.12 \text{ N} \]

\[ \theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = 10.6^\circ \]

\[ \vec{M}_R = \Sigma M = (1 \times 300) + (2 \times 500 \cos(30)) - (0.2 \times 500 \sin(30)) \]

\[ - \left( 0.5 \times \frac{3}{5} \times 250 \right) - (3 \times \frac{4}{5} \times 250) = \bar{x} \vec{F}_{R_x} \]

\[ \bar{x} = 0.827 \text{ m} \]

Only x-component creates
Reduction of a simple distributed load

\[ w \]

 VS. force applied at single point.
Reduction of a simple distributed load

In structural analysis, we often are presented with a distributed load \( W(x) \) (force/unit length) and we need to find the equivalent loading \( F \).

Example of such forces are winds, fluids, or the weight of items on the body’s surface:

\[
W(x) = p(x) \cdot b = \frac{N}{m^2} \cdot m = \frac{N}{m} = \frac{\text{force}}{\text{length}}
\]

\[
W(x) = \frac{dF}{dx}
\]

Replace coplanar parallel force system with a single equivalent resultant force

\[
M_i = \int dF(x) = \int_0^L w(x)dx = \text{Area under the curve.}
\]

\[
M_e = \sum x dF(x) = \sum_0^L x w(x)dx = \text{Resultant moment.}
\]

Where is the resultant force located?

\[
\bar{x} = \frac{M_e}{F_e}
\]
line of action passes through centroid.

\[ M_f = x \cdot 1 \cdot \ell \]

\[ \int_0^L x \omega(x) \, dx = \bar{x} \int_0^L \omega(x) \, dx \]

\[ \bar{x} = \frac{\int_0^L x \omega(x) \, dx}{\int_0^L \omega(x) \, dx} = \text{geometric center, centroid} \]
**Triangular loading**

\[ \omega(x) = \frac{w_0}{L} x \]

1. Find \( \overrightarrow{F}_R = \int_0^L w(x) dx \)
2. Find \( \overrightarrow{M}_F = \int_0^L x w(x) dx \)
3. Find \( \overline{x} = \frac{\overrightarrow{M}_F}{\overrightarrow{F}_R} \)

\[ \text{Area} = \frac{1}{2} b h = \frac{1}{2} L w_0 = \overrightarrow{F}_R \]

\[ \overrightarrow{F}_R = \frac{1}{2} w_0 L \times \text{(Area of triangle)} \]

\[ \overrightarrow{M}_R = \int_0^L x w(x) dx = \int_0^L \frac{w_0}{L} x^2 dx = \frac{w_0}{L} \int_0^L x^2 dx = \frac{w_0}{L} \left[ \frac{x^3}{3} \right]_0^L \]

\[ \overrightarrow{M}_R = \frac{1}{3} w_0 L^2 \]

\[ \overrightarrow{N}_R = \overline{x} \overrightarrow{F}_R \quad \Rightarrow \quad \overline{x} = \frac{\overrightarrow{M}_R}{\overrightarrow{F}_R} = \frac{\frac{1}{3} w_0 L^2}{\frac{1}{2} w_0 L} = \frac{2}{3} L \]
The diagrams illustrate the relationship between the variables and the area of a triangle. The left diagram has a label \( \overrightarrow{F_E} \cdot (\text{Area of triangle}) \) and the right diagram has a label \( \overrightarrow{F_E} \cdot (\text{Area of triangle}) \).

The text on the diagrams states:

Left diagram: \( \chi = \frac{2}{3}L \)

Right diagram: \( \chi = \frac{1}{3}L \)
Rectangular loading \( w(x) = w_0 \)

1. Find \( F_R = \int w(x) \, dx \)
2. Find \( M_R = \int x \, dF = \int x \, w(x) \, dx \)
3. Find \( \bar{x} = M_R / F_R \)

\[ \omega(x) = mx + b_x = 0 + w_0 = \omega(x) = w_0 \]

\[ F_R = \int_0^L w_0 \, dx = w_0 \left[ x \right]_0^L = w_0 L \quad \text{*(Area of Rectangle!)} \]

\[ M_R = \int_0^L x \, dF = \int_0^L w_0 x \, dx = w_0 \left[ \frac{x^2}{2} \right]_0^L = \frac{1}{2} w_0 L^2 \]

\[ \bar{x} = \frac{M_R}{F_R} = \frac{1}{2} \frac{w_0 L^2}{w_0 L} = \frac{L}{2} \quad \text{Always!} \]

\( \bar{x} = \frac{L}{2} \)

\( \frac{F_R}{(\text{Area of Rectangle} - w_0 L)} \)
Determine the magnitude and location of the equivalent resultant of this load.

- **DRAW FBD**
- **Split into simple loads.**

\[ \vec{F}_{R_1} = \frac{1}{2} (9 \times 100 - 50) = 225 \text{ lb} \]

\[ \vec{F}_{R_2} = (9 \times 50) = 450 \text{ lb} \]

\[ \vec{F}_R = \vec{F}_{R_1} + \vec{F}_{R_2} = 675 \text{ lb} \]

*Sum moments.*

\[ (3 \times 225) + (4.5 \times 450) = \bar{x} \times (675) \]

\[ \bar{x} = 4 \text{ ft} \]

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\[ \vec{F}_R = 675 \text{ lb} \]
\[ \vec{F}_R = \int_A 12(1 + 2x^2)dx = \ldots \]

\[ \vec{M}_R = \int_A 12x(1 + 2x^2)dx = \ldots \]

\[ \bar{x} = \frac{\vec{M}_R}{\vec{F}_R} \]