To do ...

- Extra credit due Mon (today!)
- Quiz 2 this week (Tues-Sat)
- HW 6PL due Tues
- HW 7ME due Thurs
- WA 4 due Sun
Moment of a force about a specified axis

\[
\begin{align*}
\mathbf{r} &= [0, 0, r] \\
\mathbf{r}_m &= [-x, y, 0] \\
\mathbf{M}_0 &= \mathbf{r}_m \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -x & y & 0 \\ 0 & 0 & r \end{vmatrix} = [y_F, x_F, 0] \\
M_y &= \mathbf{u}_j \cdot \mathbf{M}_0 = \mathbf{u}_j \cdot (\mathbf{r}_m \times \mathbf{F}) = [0, 1, 0] \cdot [y_F, x_F, 0] = x_F \\
\text{scalar magnitude.}
\end{align*}
\]
A force is applied to the tool as shown. Find the magnitude of the moment of this force about the x axis of the value.

given: \( \vec{F} \), \( \alpha, \beta, \gamma \), \( \vec{u}_x \)

\[
\vec{r}_{OA} = \vec{r}_A - \vec{r}_o = \begin{bmatrix} 0, 0.3, 0.25 \end{bmatrix}_M
\]

\[
\vec{F} = 200 \begin{bmatrix} \cos(120) \cos(60) \cos(45) \end{bmatrix} = \begin{bmatrix} -100, 100, 141.4 \end{bmatrix}_N
\]

\[
M_x = \vec{u}_x \cdot (\vec{r}_{OA} \times \vec{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix} = 1 \left( 0.3(141.4) - 0.25(100) \right) \text{ N\cdotm}
\]

\[
M_x = 17.4 \text{ N\cdotm \ (CCW) BECAUSE POSITIVE.}
\]
Rapid Refresh ...

- i
- <3
- c
- l
- i
- c
- k
- e
- r
When determining the moment of a force about a specified axis, the axis must be along ____________.

A) the x axis  
B) the y axis  
C) the z axis  
D) any line in 3-D space  
E) any line in the x-y plane
iQ>clicker

For finding the moment of the force $F$ about the x-axis, the position vector in the triple scalar product should be $\overrightarrow{r}$.

A) $r_{AC}$  
B) $r_{BA}$  
C) $r_{AB}$  
D) $r_{BC}$

\[
\vec{M}_A = \vec{r}_{AB} \times \vec{F}
\]

\[
\vec{M}_x = \vec{u}_x \cdot (\vec{r}_{AB} \times \vec{F})
\]
iQ>clicker

With the force $\mathbf{P}$, a person is creating a moment $\mathbf{M}_A$ using this flex-handle socket wrench. Does all of $\mathbf{M}_A$ act to turn the socket?

A) YES

B) NO
Couples ...
Moment of a couple

Q: What happens?

\[ \sum F \neq 0 \]

\[ \sum M \neq 0 \]

* Translation and rotation

* Rotation only

\[ \sum F = 0 \]

\[ \sum M = 0 \]
Moment of a couple

A couple is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance \( d \).

Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction.

The moment produced by a couple is called couple moment.

**Requirements:**

* At least two forces
* Lines of action are parallel
* Same magnitude: \( |\vec{F}_1| = |\vec{F}_2| \)
* Opposite direction: \( \vec{F}_1 = -\vec{F}_2 \)
* Forces separated by perpendicular distance \( d \)

Since \( \vec{F}_1 = -\vec{F}_2 \)

the \( \Sigma \vec{F} = \vec{F}_1 - \vec{F}_2 = 0 \) \( \Rightarrow \) produces no net force, no translation!

But due to the separation there is pure rotation!

Moment of a couple is a free vector!
Direction:

- Use right-hand rule convention.
- CW → positive
- CCW → negative

Magnitude of a couple moment:

\[ M = dF \]

(vector analysis)

(Scalar)

\[ M = \overrightarrow{r} \times \overrightarrow{F} \]

Any position vector from line of action of \( \overrightarrow{F} \) to line of action of \( \overrightarrow{F} \)!

Lo only direction and magnitude are important.

Not confident to unique line of action.
Consider the equivalent couples

A torque or moment of 12 N·m is required to rotate the wheel. Why does one of the two grips of the wheel above require less force to rotate the wheel?

\[
\begin{align*}
F_1 &= 30 \text{ N} \\
\alpha_1 &= 0.4 \text{ m} \\
\vec{M}_1 &= (0.4)(30) = -12 \text{ N·m}
\end{align*}
\]

\[
\begin{align*}
F_2 &= 40 \text{ N} \\
\alpha_2 &= 0.3 \text{ m} \\
\vec{M}_2 &= (0.3)(40) = -12 \text{ N·m}
\end{align*}
\]

Both \(\vec{M}_1\) and \(\vec{M}_2\) are clockwise (cw). Note that \(F_2 > F_1\) because \(\alpha_1 > \alpha_2\).
Two couples act on the beam with the geometry shown. Find the magnitude of \( F \) so that the resultant couple moment is 1.5 kN·m clockwise.

- Identify couples, \( F \leq 1.0 \)

\[ \sum M_F = (0.3\ m)(2\ kN) - (0.9)F = -1.5 \ \text{kN·m} \]

\[ 0.6 \ \text{kN·m} + 1.5 \ \text{kN·m} = 0.9 \ \text{kN·m} \]

\[ F = 2.33 \ \text{kN} \]
Two couples act on the beam with the geometry shown and \( d = 4 \) ft. Find the resultant couple:

- identify couples.
- find perpendicular distance or resolve forces into \( x \) \& \( y \) components.

First consider forces on the top beam.

Resolve into \( x \) \& \( y \) components.

Q: Which components would lead to rotation of the body?

\( \bullet \) \( F_x \) would not cause rotation,

\( \bullet \) \( F_y \) components would rotate in CW direction.
\[ \vec{M}_1 = -dF_y = -(3)(50 \cos(50)) \text{ ft-lb} \]

Now the second beam.

- $F_y$ does not rotate body, only $F_x$ would in CCW direction.

\[ \vec{M}_2 = dF_x = (4 \text{ ft} \times \frac{4}{5} 80) \text{ ft-lb} \]

\[ \vec{M} = \vec{M}_1 + \vec{M}_2 = 126 \text{ ft-lb CCW} \]
Determine the couple moment acting on the pipe.

**Approach 1:**

\[ \vec{N} = \vec{r}_{oa} \times \vec{F}_a + \vec{r}_{ob} \times \vec{F}_b \]

\[ \vec{M} = \begin{bmatrix} 1 & 0 & k \\ 0 & 8 & 0 \\ 0 & 0 & -25 \end{bmatrix} + \begin{bmatrix} 1 & 0 & k \\ 6\cos\left(\frac{\pi}{6}\right) & 8 & -6\sin\left(\frac{\pi}{6}\right) \\ 0 & 0 & 25 \end{bmatrix} = \ldots = \]

\[ \vec{M} = \begin{bmatrix} 0, & -25\left(6\cos\left(\frac{\pi}{6}\right)\right), & 0 \end{bmatrix} \text{ lb}\cdot\text{in} \]
Determine the couple moment acting on the pipe

**Approach 2:**

Use couple moment since

\[ F_1 = F_2 \]

\[ \vec{F}_1 = -\vec{F}_2 \]

\[ \vec{r}_{AB} = \vec{r}_{OB} - \vec{r}_{OA} = [6 \cos(30), 0, -6 \sin(30)] \text{ in} \]

\[ \vec{r}_A = [0, 8, 0] \text{ in} \]

\[ \vec{r}_B = [6 \cos(30), 8, -6 \sin(30)] \]

\[ \vec{M} = \vec{r}_{AB} \times \vec{F}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 \cos(30) & 0 & -6 \sin(30) \\ 0 & 0 & 25 \end{vmatrix} = \cdots = \]

\[ \vec{M} = [0, -25 \cdot 6 \cos(30), 0] \text{ lb \cdot in} \]

*Same as previous,*
Moving a force on its line of action (L.O.A)

Moving a force from A to B, when both points are on the vector’s line of action, does not change the external effect.

\[ \rightarrow \text{translation/rotation of body.} \]

\[ \rightarrow \text{depends on where and how force is applied to body} \]

\[ F \text{ is a Sliding Vector} \]

\[ \rightarrow \text{Line of Action is defined, produces the same external effect on a body.} \]