To do ...

- Quiz 1 (*Last day today!*)
  - CBTF instructions on website
- WA 2 due *Sun*
- CATME due *Mon*
- NO lecture on *Mon*
- HW 4PL due *Tues*
- HW 5ME due *Thurs*
Idealizations

Pulleys are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side. Springs are (usually) regarded as linearly elastic; then the tension is proportional to the change in length $s$.

- Normal force at point of contact perpendicular to surface!
- Tension is the same
- Pulley - massless, frictionless
- Cable - rigid, massless
- Spring force opposes external force
- $s > 0$ - stretch
- $s < 0$ - compress
Free body diagram

- System of objects
- Choose subsystem/object to analyze
- 3 unknowns!

**Draw FBD of A**

**Draw FBD of C**

**Scalar eqns. of equilibrium:**

\[ \sum F_x: \quad T_2 \cos \theta - T_1 = 0 \]

\[ \sum F_y: \quad T_2 \sin \theta - T_3 = 0 \]

**Eqs. of EM:**

\[ \sum F_x: \quad \text{none} \]

\[ \sum F_y: \quad T_3 - mg = 0 \]

\[ \therefore \quad T_3 = mg \]

Now use \( T_3 \) and solve for \( T_2 \), then \( T_1 \).
\[ T_2 = \frac{T_2}{\sin \theta} \Rightarrow T_2 = \frac{mg}{\sin \theta} \]

\[ T_1 = T_2 \cos \theta \Rightarrow T_1 = mg \frac{\cos \theta}{\sin \theta} \]

*Q: What happens to \( T_1 \) and \( T_2 \) as \( \theta \rightarrow 0^\circ \) and \( \theta \rightarrow 90^\circ \)?

What does this mean physically?
Equilibrium of a system of particles

Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton’s first law

\[ \sum \mathbf{F} = \mathbf{0} \]

on selected multiple free-body diagrams of particles or groups of particles.

The five ropes can each take 1500 N without breaking. How heavy can \( W \) be without breaking any?

- \( Q: \) what does this mean?
- \( \text{given: wax tension in any cable} \)
- \( \text{unknown: 6 unknowns! } T_1, T_2, T_3, T_4, T_5 \text{ and } W \)

\[ \text{plan: break up problem into parts, draw FBD and solve eqns.} \]

\[ \sum T_y = 0: \]

- \( 2T_1 - T_2 = 0 \)
- \( 2T_1 = T_2 \)
- \( T_2 = T_3 \)
- \( 2T_1 - T_3 = 0 \)
- \( 2T_4 - T_1 = 0 \)
- \( 2T_5 - T_5 = 0 \)

\[ T_4 = \frac{1}{2} T_i \]

\[ T_5 = 2T_4 = 2 \cdot \frac{1}{2} T_i = T_1 \]

The critical tension is supported in \( T_2 \) and \( T_3 \) giving

\[ T_2 = T_3 = T_{\text{crit}}. \]

\[ T_1 = \frac{1}{2} T_2 = \frac{1}{2} T_3 = \frac{1}{2} T_{\text{crit}} \]

\[ T_4 = \frac{1}{2} T_i = \frac{1}{2} \frac{1}{2} T_2 = \frac{1}{4} T_{\text{crit}} \]

\[ T_5 = T_1 = \frac{1}{2} T_2 = \frac{1}{2} T_{\text{crit}}. \]

Now, the FBD of the weight in order to write the EoE is
Summing the forces:
\[ \sum F_y: \quad T_2 + T_4 + T_5 - W = 0 \]

\[ T_{cr1} + \frac{1}{4} T_{cr1} + \frac{1}{2} T_{cr1} = W \]

\[ W = \frac{3}{4} T_{cr1} = \frac{3}{4} (1500 \text{N}) = 2.63 \text{ kN} \]

Q: is \( W \) in rotational equilibrium?

Which way will it spin?

Since \( T_2 > T_5 \)

\[ \text{Rotate Clockwise.} \]
Rapid Refresh ...

- i
- <3
- c
- l
- i
- c
- k
- e
- r
iQ>clicker

Screwy Arms Rob Lowe is ready to do his lat pull downs. His trainer told him to load the machine with a whopping 25 lbs, but he is afraid that all of that weight will break the cable and thusly he will smash his pretty boy face with the bar. Assuming equal sized cables, which machine should Screwy Arms use to lessen his fears.

A)

B)
Determine the force exerted by the hand so that the box with weight $W$ can be suspended in the position shown.

a) $W \sin \theta$

b) $W \tan \theta$

c) $W / \tan \theta$

d) $W / \sin \theta$

e) $W / \cos \theta$
If the box has weight 1962 N, determine the force that has to be applied at B to make the system in equilibrium in the configuration below, where $\theta = 40^0$

\[ \sum F_x: \quad F_B - F_C \cos \theta = 0 \]

\[ \sum F_y: \quad F_C \sin \theta - W = 0 \]

2 unknowns, 2 equations!
Solve for $F_C$ then $F_B$!

Using $\sum F_y = 0$ we get:

\[ F_C = \frac{W}{\sin \theta} \]

then use and $F_B$ using $\sum F_x = 0$

\[ F_B - F_C \cos \theta = 0 \]

\[ F_B = \frac{W}{\sin \theta} \cos \theta = 0 \]

$F_B$ depends on $W$ and $\theta$!
clear; clc;

t0=40; % angle [degree]
W=1962; % weight of box [Newton]
g=9.81; % acceleration due to gravitational attraction [m/s^2]
m=W/g; % mass of box [kg]

FB=m*g*((cosd(t0))/(sind(t0))) % force at B [Newton]

t=1:0.1:90; % theta 1 -> 90 degree
w=1:10:10000; % weight 1 -> 10 kN

figure(1)
c1f
subplot(1,2,1)
plot(t, (m*g*((cosd(t))/(sind(t))))/1000,'LineWidth',3)
hold on
scatter(t0, FB/1000,100,'s','filled')
xlabel('Theta (deg)')
ylabel('Force (kN)')
axis([0 90 0 90])
axis square

subplot(1,2,2)
plot(w, (w*((cosd(t0))/(sind(t0))))/1000,'LineWidth',3)
xlabel('Weight (N)')
ylabel('Force (kN)')
axis([0 10000 0 10])
axis square
A 4 kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass of block B needed to hold it in the equilibrium position shown. The given parameters are:
\[ x_1 = 0.4 \text{m}, a = 2.5 \text{m}^{-1}, \theta = 60^\circ. \]

Draw FBD of sphere.

\[ \sum F_x: \quad T \cos \theta - N \sin \theta_1 = 0 \]
\[ \sum F_y: \quad T \sin \theta + N \cos \theta_1 - W = 0 \]

2 equations, unknowns?

\[ T = mg \]

\[ m_B = \frac{T}{g} \] (2)
Solve for $T$ using $\Sigma F_x = 0$ gives:

$$T = N \frac{\sin \theta}{\cos \theta} \quad (1)$$

Solve for $N$ using $\Sigma F_y = 0$ gives:

$$T \sin \theta + N \cos \theta \theta_1 - W = 0$$
$$N \frac{\sin \theta_1 \sin \theta + \cos \theta \cos \theta_1 - W}{\cos \theta} = M_a g$$

$$N = \frac{M_a g \cos \theta}{\sin \theta \sin \theta + \cos \theta \cos \theta_1} = \ldots = 19.7 \text{ N}$$

What about $\theta_1$? Using the equation of the curve $y = ax^2$:

$$\tan \theta_1 = \frac{dy}{dx} = \frac{d}{dx} (ax^2) = 2ax \quad \therefore \quad \theta_1 = \tan^{-1} (2ax) = 63.4^\circ$$

* Approximation only for $\frac{T}{M_a} \ll 1$
Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400N.

Given: constraint of max tension

Unknown: 6 unknowns!

Plan: find which cord will have the greatest force for given mass.

Starting with D:

\[ \Sigma F_x: \quad F_E \cos(30^\circ) - F_{DC} = 0 \]
\[ F_E = \frac{Mg}{\sin(30^\circ)} = 19.62 \text{ m} \]
\[ F_{DC} = F_E \cos(30^\circ) = 16.99 \text{ m} \]

Now moving to C:

\[ \Sigma F_x: \quad F_B \cos(45^\circ) + \frac{3}{5} F_A - F_{RC} = 0 \]
\[ \Sigma F_y: \quad \frac{4}{5} F_A - F_B \sin(45^\circ) = 0 \]

Taking \( \Sigma F_y \) gives
\[ F_A = \frac{5}{4} F_B \sin(45^\circ) \]

Put in to \( \Sigma F_x \):
\[ F_B \cos(45^\circ) + \frac{3}{4} F_B \sin(45^\circ) - F_{DC} = 0 \]
\[ F_B = \frac{F_{DC}}{\cos(45) + \frac{3}{4}\sin(45)} = 13.73 \text{ m} \]

\[ F_A = \frac{5}{4} F_B \sin(45) = 12.14 \text{ m} \]

* therefore cord DE is subject to the greatest force, then the maximum allowable mass is

\[ F_m = 400 = 19.62 \text{ m} \]

\[ m = \frac{400 \text{ N}}{19.62} = 20.4 \text{ kg} \]
Determine the mass of each of the two identical cylinders if they cause a sag of $s = 0.5 \text{ m}$ when suspended from the rings at A and B. Note that $s$ is zero when the cylinders are removed.

The spring force is given by $F = k(l - l_0)$.

1. \[1.5 \sqrt{2^2 + 1.5^2} = 2.5 \text{ m}\]

2. \[2 \sqrt{2^2 + 2^2} = 2.828 \text{ m}\]

\[m = \frac{1}{g}(F_y \sin \theta) = \frac{1}{g}(k(l - l_0) \sin \theta) = \frac{1}{g}(100 \times (2.828 - 2.5) \sin(45)) = 2.37 \text{ kg}\]
A “scale” is constructed with a 4-ft-long cord and the 10-lb block D. The cord is fixed to a pin at A and passes over two small frictionless pulleys. Determine the weight of the suspended block B if the system is in equilibrium when $s = 1.5\text{ft}$.

**Main Idea:**
- Coplanar system
- Pulleys
- Symmetry @ point B

\[ \sum F_x: \quad T_1 \cos \theta_1 - T_2 \cos \theta_2 = 0 \]
\[ T_1 = T_2 \quad \therefore \theta_1 = \theta_2 \]

\[ \sum F_y: \quad T_1 \sin \theta_1 + T_2 \sin \theta_2 - W = 0 \]

\[ T = W_B \]

\[ s = 4\text{ft} - 1.5\text{ft} \]
\[ = 2.5\text{ft} \]
\[ \Sigma F_y: \quad T_1 \sin \theta_1 + T_2 \sin \theta_2 - W = 0 \]

\[ W = 2T \sin \theta \]

\[ \theta = \arcsin \left( \frac{0.5}{1.25} \right) \Rightarrow \theta = 66.4^\circ \]

\[ W = 2(16 \text{ lb}) \sin(66.4^\circ) = \boxed{18.3 \text{ lb}} \]
Determine the unstretched length of spring AC if a force $P = 80$ lb causes the angle $\theta = 60^\circ$ for equilibrium. Cord AB is 2 ft long. Use the spring stiffness $k = 50$ lb/ft.

**Main IDEA:**

**COPLANAR:**

**CABLE / SPRING FORCE**

**find** $l_0$

\[
l_1 = 2\cos(60^\circ) = 1 \text{ ft}
\]

\[
l_2 = 2\sin(60^\circ) = 1.73 \text{ ft}
\]

\[
l_3 = 4 - l_1 = 3 \text{ ft}
\]

\[
l_f = \sqrt{l_1^2 + (4 - l_1)^2} = 3.96 \text{ ft}
\]

\[
\theta_1 = \tan^{-1}\left(\frac{l_2}{4 - l_1}\right) = 30^\circ
\]

\[
f_s = k(l_l - l_0) = \frac{l_1}{l_0 - \frac{f_s}{k}}
\]

\[
\sum F_x: \quad f_k \cos(\theta_1) - f_B \cos(60^\circ) = 0 \quad \Rightarrow \quad f_B = f_s \frac{\cos \theta_1}{\cos(60^\circ)}
\]

\[
\sum F_y: \quad f_B \sin(60^\circ) + f_s \sin \theta, -P = 0
\]

\[
\overline{f_s} \cos \theta_1 \cdot \sin(60^\circ) + f_s \sin \theta_1 - P = 0
\]
\[ F_3 \cos \theta \cdot \sin(60) + F_5 \sin \theta \cdot \tan \theta = P = 0 \]

\[ F_3 = \frac{P \cos(60)}{\cos \theta \cdot \sin(60) + \sin \theta \cdot \cos(60)} \]

\[ F_5 = \frac{P \cos(60)}{\cos \theta \cdot \sin(60) + \sin \theta \cdot \cos(60)} \]

\[ \psi = \frac{P \cos(60)}{\cos \theta \cdot \sin(60) + \sin \theta \cdot \cos(60)} \]

\[ \lambda_0 = \lambda_f - \frac{P \cos(60)}{K \cdot \cos \theta \cdot \sin(60) + \sin \theta \cdot \cos(60)} \]

\[ \lambda_0 = 3.464 \text{ ft} - \frac{80 \text{ lb} \cos(60)}{50 \text{ ft} \tan(60)} = 2.664 \text{ ft} \]
3D force systems

Find the tension developed in each cable:

- **Given:** magnitude and direction of applied force
- **Unknown:** $T_1$, $T_2$, $T_3$
- **Plan:**
  - Draw FBD of $A$
  - Find direction and magnitude of forces
  - Use equation of equilibrium
    \[ \sum F = (\sum F_x) \hat{i} + (\sum F_y) \hat{j} + (\sum F_z) \hat{k} \]

- **FBD**

- **Determine direction vectors and use**
  \[ \frac{\vec{F}}{F} = \vec{F} \hat{u} \]

- **$u_1 = [1, 0, 0]$**
- **$u_2 = [-\frac{3}{5}, \frac{4}{5}, 0]$**
- **$u_3 = [\frac{-3}{5}, \frac{4}{5}]$**

\[ \begin{array}{c|c|c|c}
\frac{\vec{F}}{F} & T & 0 & 0 \\
\end{array} \]
\[
\begin{array}{|c|c|c|c|}
\hline
\vec{T}_1 & \vec{T}_2 & \vec{T}_3 & \vec{F} \\
\hline
\frac{4}{5} \vec{T}_2 & -\frac{3}{5} \vec{T}_2 & 0 & 0 \\
\hline
\vec{F}_x = 0 & -\frac{3}{5} \vec{T}_3 & 0 & 4 \vec{T}_3 \\
\hline
\vec{F}_y = 0 & 0 & -\frac{3}{5} \vec{T}_3 & 0 \\
\hline
\vec{F}_z = 0 & 0 & 0 & -900 \\
\hline
\end{array}
\]

\[\Sigma F_x: \quad \vec{T}_1 - \frac{3}{5} \vec{T}_2 = 0\]

\[\Sigma F_y: \quad -\frac{3}{5} \vec{T}_3 + \frac{4}{5} \vec{T}_2 = 0\]

\[\Sigma F_z: \quad \frac{4}{5} \vec{T}_3 - 900 = 0 \quad \Rightarrow \quad \vec{T}_3 = \frac{5}{4} (900) N\]

\[\vec{T}_3 = 1125 N\]

Now using \(\Sigma F_y\),

\[\vec{T}_2 = \frac{3}{4} \vec{T}_3 = 844 N\]

Finally,

\[\vec{T}_1 = \frac{3}{5} \vec{T}_2 = \frac{3}{5} (844) = 506 N\]
Particle P is in equilibrium with five (5) forces acting on it in 3-D space. How many scalar equations of equilibrium can be written for point P?

A) 2
B) 3
C) 4
D) 5
E) 6