To do ...

- HW2 PL due Tues
  - Prairie Learn guide on website
- Quiz 1 (Tues – Fri)
  - CBTF instructions on website
- HW 3 ME due Thurs
- WA 2 due Sun

Good luck!!
*note: As you can see, using the triangle method

\[ \vec{r}_a + \vec{r} = \vec{r}_b \]

but we know \( \vec{r}_a \) and \( \vec{r}_b \), not \( \vec{r} \) so we solve and get

\[ \vec{r} = \vec{r}_b - \vec{r}_a \]
Note: if you switch the order, you get a different vector! For example

\[ \vec{r}_1 = \vec{r}_c - \vec{r}_a \]

vs.

\[ \vec{r}_1 = \vec{r}_a - \vec{r}_c \]
not the same!
Rapid Refresh ...

- i
- $<$3
- c
- l
- i
- c
- k
- e
- r
1. Vector algebra, as we are going to use it, is based on a __________ coordinate system.
   - A) Euclidean
   - B) Left-handed
   - C) Greek
   - D) Right-handed
   - E) Egyptian

2. The symbols $\alpha$, $\beta$, and $\gamma$ designate the __________ of a 3-D Cartesian vector.
   - A) Unit vectors
   - B) Coordinate direction angles
   - C) Greek societies
   - D) $X, Y$ and $Z$ components
iQ>clicker

3. If you know only $\mathbf{u}_A$, you can determine the ________ of $\mathbf{A}$ uniquely.

A) magnitude        B) angles ($\alpha$, $\beta$ and $\gamma$)
C) components ($A_x$, $A_y$, & $A_z$)        D) All of the above.

4. What is not true about an unit vector, e.g., $\mathbf{u}_A$?

A) It is dimensionless.

B) Its magnitude is one.

C) It always points in the direction of positive X-axis.

D) It always points in the direction of vector $\mathbf{A}$. 
Force vector directed along a line

The force vector \( \vec{F} \) acting along the rope can be defined by the unit vector \( \vec{u} \) (defined the direction of the rope) and the magnitude of the force.

\[
\vec{F} = F \vec{u}
\]

The unit vector \( \vec{u} \) is specified by the position vector:

\[
\vec{r}_B = (x_B - x_A) \hat{i} + (y_B - y_A) \hat{j} + (z_B - z_A) \hat{k}
\]

\[
\vec{r}_{AB} = \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|}
\]

The man pulls on the cord with a force of 70 lb. Represent the force \( \vec{F} \) as a Cartesian vector:

\[
\vec{F}_{AB} = (12 \hat{i} - 8 \hat{j} - 24 \hat{k}) \text{ ft}
\]

magnitude (length of the rope)

\[
|\vec{F}_{AB}| = \sqrt{12^2 + 8^2 + 24^2} = 28 \text{ ft}
\]

the unit vector is

\[
\vec{u}_{AB} = \frac{\vec{F}_{AB}}{|\vec{F}_{AB}|} = \frac{12}{28} \hat{i} - \frac{8}{28} \hat{j} - \frac{24}{28} \hat{k}
\]

\[
\vec{F}_{AB} = F \vec{u}_{AB} = \left[70 \left(\frac{12}{28}\right) \hat{i} - 70 \left(\frac{8}{28}\right) \hat{j} - 70 \left(\frac{24}{28}\right) \hat{k}\right] \text{ N}
\]

Quick check:

\[
\vec{F} = F \vec{u} = F \frac{\vec{r}}{|\vec{r}|} = F \frac{x \hat{i} + y \hat{j} + z \hat{k}}{|\vec{r}|} = F \frac{x^2}{|\vec{r}|^2} \hat{i} + F \frac{y^2}{|\vec{r}|^2} \hat{j} + F \frac{z^2}{|\vec{r}|^2} \hat{k}
\]

\[
|\vec{F}| = \sqrt{F^2 \frac{x^2}{|\vec{r}|^2} + F^2 \frac{y^2}{|\vec{r}|^2} + F^2 \frac{z^2}{|\vec{r}|^2}} = \frac{F}{|\vec{r}|} \sqrt{x^2 + y^2 + z^2} = \frac{F}{|\vec{r}|} |\vec{r}| = F
\]
Force vector directed along a line

Don’t look up!
Dot (or scalar) product

The dot product of vectors $\mathbf{A}$ and $\mathbf{B}$ is defined as such:

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$

$$\cos \theta = \left( \frac{\mathbf{A} \cdot \mathbf{B}}{AB} \right)$$

* Find angle between two vectors
* Find components of a vector parallel/perp to a line

Cartesian vector formulation:

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

Note that:

$$i \cdot j = 0 \quad i \cdot i = 1$$

$$\theta = 90 \degree \quad \cos(90\degree) = 0 \quad \theta = 0 \quad \cos 0 = 1$$

$$\mathbf{A} \cdot \mathbf{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x B_x \hat{i} \cdot \hat{i} + A_x B_y \hat{i} \cdot \hat{j} + A_x B_z \hat{i} \cdot \hat{k} + \cdots$$

$$\hat{i} \cdot \hat{i} = 1 \quad \hat{i} \cdot \hat{j} = 0 \quad \hat{i} \cdot \hat{k} = 0$$

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

Very important!

* $\mathbf{A} \cdot \mathbf{B}$ is a scalar
* The units are $\mathbf{A} \cdot \mathbf{B}$

i.e. $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{B}$
Example

Given: The force acting on the hook at point A.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.

Plan:
1. find \( \vec{r}_{AO} \)
2. find \( \theta \)
3. find \( F_{AO} \) with dot product

1. \( \vec{r}_{AO} = (x_o - x_A)\hat{i} + (y_o - y_A)\hat{j} + (z_o - z_A)\hat{k} \)

\[ \vec{r}_{AO} = (0 - 1)\hat{i} + (0 - 2)\hat{j} + (0 - 2)\hat{k} = -\hat{i} + 2\hat{j} - 2\hat{k} \text{ m} \]

\[ |\vec{r}_{AO}| = \sqrt{1^2 + 2^2 + 2^2} = 3 \text{ m} \]

2. \[ \theta = \cos^{-1} \left( \frac{\vec{F} \cdot \vec{r}_{AO}}{|\vec{F}| |\vec{r}_{AO}|} \right) = \cos^{-1} \left( \frac{(-6\hat{i} + 9\hat{j} + 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - 2\hat{k})}{1 \cdot 3} \right) \]

\[ \theta = \cos^{-1} \left( \frac{18 \text{ KN.m}}{3} \right) \]

3. \( \vec{F}_{AO} = \vec{F} \cos \theta \)

or

\[ \vec{F}_{AO} = \vec{F} \cdot \hat{u}_{AO} = \vec{F} \cdot \frac{\vec{r}_{AO}}{|\vec{r}_{AO}|} = \frac{F_{x}r_{x} + F_{y}r_{y} + F_{z}r_{z}}{|\vec{r}_{AO}|} \]
Is the force due to $F_{AC}$ acting along the axis of strut AO:

(A) > 60

(B) < 60

(C) = 60

(D) I’m too tired to answer because I was up all night trying to get access to Compass/Mastering Engineering/PrairieLearn/other
Example

Determine the projected component of the force vector $\vec{F}_{AC}$ along the axis of strut AO. Express your result as a Cartesian vector.

**Given:** $|\vec{F}_{AB}|$, $|\vec{F}_{AC}|$, $x$, $y$, $z$ of $A$, $B$, $C$

**Plan:**
- Use dot product
- Find unit vectors $\vec{U}_{AC}$ and $\vec{U}_{AO}$
- Write force vector $\vec{F}_{AC}$ as Cartesian vector
- Take dot product of $\vec{F}_{AC}$ and $\vec{U}_{AO}$

\[
\vec{U}_{AC} = \frac{(x_c - x_a)\hat{i} + (y_c - y_a)\hat{j} + (z_c - z_a)\hat{k}}{|\vec{U}_{AC}|}
\]

\[
\vec{U}_{AO} = \frac{(5\cos(60) - 0)\hat{i} + (0 - 6)\hat{j} + (5\sin(60) - 2)\hat{k}}{|\vec{U}_{AO}|}
\]

\[
\vec{U}_{AO} = 0.362\hat{i} - 0.869\hat{j} + 0.338\hat{k}
\]

\[
\vec{U}_{AC} = \frac{(x_c - x_a)\hat{i} + (y_c - y_a)\hat{j} + (z_c - z_a)\hat{k}}{|\vec{U}_{AC}|}
\]

\[
\vec{U}_{AO} = \frac{(0 - 0)\hat{i} + (0 - 6)\hat{j} + (0 - 2)\hat{k}}{|\vec{U}_{AO}|}
\]

\[
\vec{U}_{AO} = 0.949\hat{i} - 0.316\hat{k}
\]

\[
\vec{F}_{AC} = \vec{F}_{AC} \cdot \vec{U}_{AC} = 60 (0.362\hat{i} - 0.869\hat{j} + 0.338\hat{k}) |\vec{F}_{AC}|
\]

\[
\vec{F}_{AC} \cdot \vec{U}_{AO} = \vec{F}_{AC} \cdot \vec{U}_{AC} - \vec{U}_{AO} = 43.057 |\vec{F}_{AC}|
\]

*Cartesian vector*
\[ (T_{AC})_{AO} = T_{AC} \cdot U_{AO} = T_{AC} U_{AO} \]

* unit of lb because
unit vector \( \vec{U}_{AO} \) dimensionless

\[
(F_{AC})_{AO} = (F_{AC})_{AO} \vec{U}_{AO} = 43 \left( \begin{array}{c} 0 \hat{i} - 0.9487 \hat{j} - 0.3162 \hat{k} \end{array} \right) \times (\text{cartesian}) \vec{vector}
\]
Cross (or vector) product

The cross product of vectors $\mathbf{A}$ and $\mathbf{B}$ yields the vector $\mathbf{C}$, which is written

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

The magnitude of vector $\mathbf{C}$ is given by:

$$|\mathbf{C}| = |\mathbf{A}| |\mathbf{B}| \sin \theta$$

The vector $\mathbf{C}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ (specified by the right-hand rule). Hence,

$$\mathbf{C} = \mathbf{A} B \sin \theta \mathbf{u_c}$$

What is $\mathbf{B} \times \mathbf{A} = ?$

$\mathbf{A} \times -\mathbf{C} = ?$
Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$

Considering the cross product in Cartesian coordinates

$$\mathbf{A} \times \mathbf{B} = (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \times (B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k})$$

$$\mathbf{A} \times \mathbf{B} = A_x B_y (\mathbf{i} \times \mathbf{i}) + A_x B_z (\mathbf{i} \times \mathbf{j}) + A_y B_z (\mathbf{i} \times \mathbf{k}) + \cdots$$

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \mathbf{i} - (A_x B_z - A_z B_x) \mathbf{j} + (A_x B_y - A_y B_x) \mathbf{k}$$

* VECTOR quantity!
Cross (or vector) product

Also, the cross product can be written as a determinant.

\[
A \times B = \begin{vmatrix}
i & j & k \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix}
\]

Each component can be determined using 2 \times 2 determinants.

\[
\vec{A} \times \vec{B} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix} - \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix} + \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix}
\]

\[
\vec{A} \times \vec{B} = (A_yB_z - A_zB_y) \vec{i} - (A_xB_z - A_zB_x) \vec{j} + (A_xB_y - A_yB_x) \vec{k}
\]
Chap 2 - recap

- Scalars — magnitude (positive or negative)
- Vectors — magnitude, direction
- Dot product — 1. determine angle b/n vectors 2. components of vector in a given direction
- Cross product — 1. normal direction to a surface 2. area of parallelogram 3. calculate moment of a force

Rectangular components of a vector
Cartesian vector representation
Cartesian vector using direction cosines
Cartesian vector using unit vector
Cartesian position vectors

\[ \vec{A} = A_x \hat{i} + A_y \hat{j} \]

\[ \vec{A} = A \hat{u} \]

\[ \vec{r} = \vec{r}_B - \vec{r}_A \]

\[ \vec{A} = A \cos(\omega) \hat{i} + A \cos(\phi) \hat{j} \]