Announcements

- CBTF Quiz 5 this week
- Written exam viewing: Tuesday, Nov 26, 9am-4pm, 224 MEB

❑ Upcoming deadlines:

- Tuesday (11/27)
  - PL HW
- Friday (11/30)
  - Written Assignment

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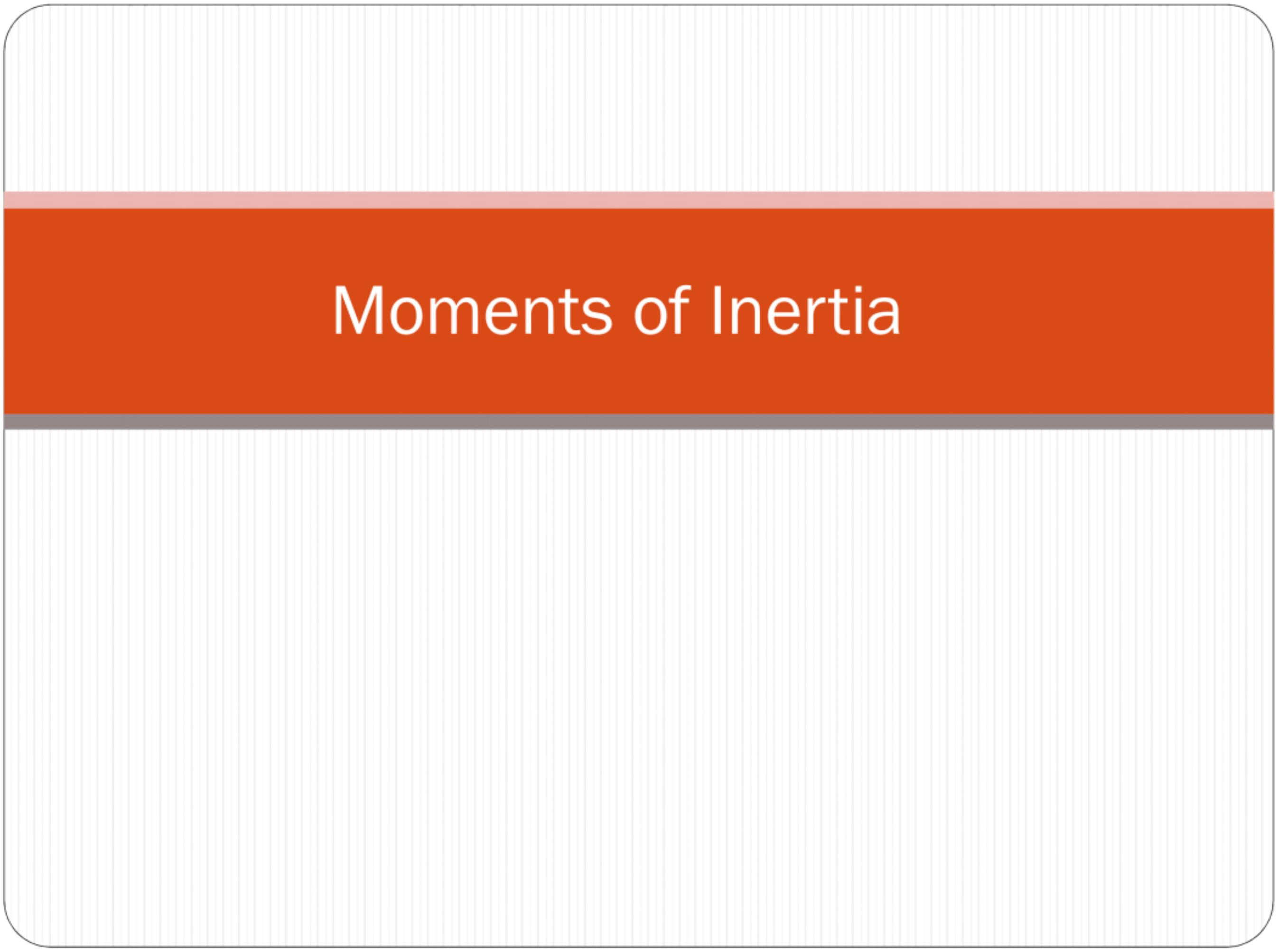


Applications

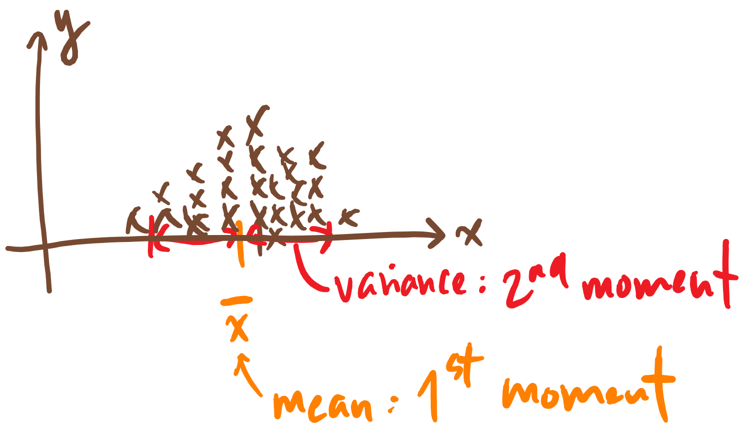
Many structural members like beams and columns have cross sectional shapes like an I, H, C, etc..

Why do they usually not have solid rectangular, square, or circular cross sectional areas?

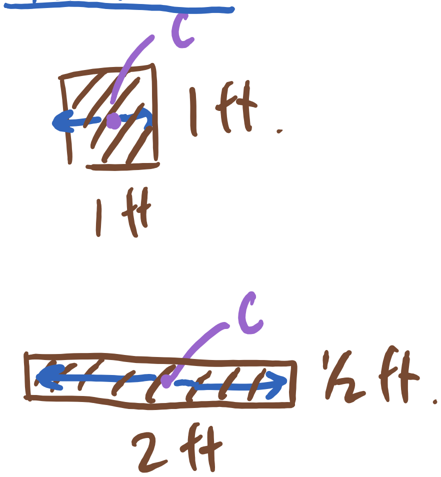
What primary property of these members influences design decisions?

Terminology: the term **moment** in this module refers to the mathematical sense of different "measures" of an area or volume.

- The *zeroth* moment is the total mass.
- The *first* moment (a single power of position) gave us the centroid.
- The *second* moment will allow us to describe the "width."
- An analogy that may help: in *probability* the first moment gives you the mean (the center of the distribution), and the second is the standard deviation (the width of the distribution).



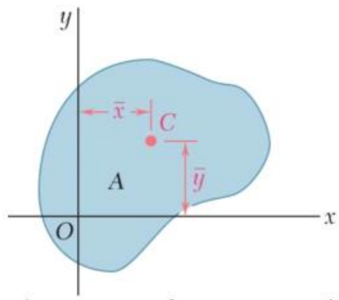
For areas



→ same 1st moment.
= same total area.
→ different 2nd moment.

Recap: First moment of an area (centroid of an area)

- The first moment of the area A with respect to the x-axis is given by $Q_x = \int_A y\, dA$
- The first moment of the area A with respect to the y-axis is given by $Q_y = \int_A x\, dA$
- The centroid of the area A is defined as the point C of coordinates and , which satisfies the relation

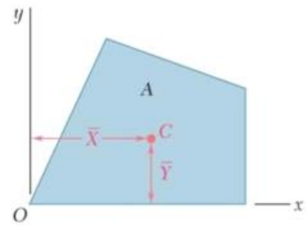


$$\int_A x\, dA = A\,\bar{x}$$

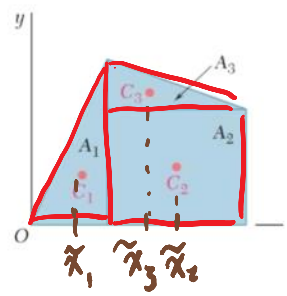
$$\int_A y\, dA = A\,\bar{y}$$

$$\bar{x} = \frac{\int x\, dA}{A}$$

$$\bar{y} = \frac{\int y\, dA}{A}$$



- In the case of a composite area, we divide the area A into parts



$$A_{total}\,\bar{X} = \sum_i A_i\,\bar{x}_i \qquad A_{total}\,\bar{Y} = \sum_i A_i\,\bar{y}_i$$

$$\bar{X} = \frac{\Sigma A_i \tilde{x}_i}{A} \qquad \bar{Y} = \frac{\Sigma A_i \tilde{y}}{A}$$

$$= \frac{A_1\tilde{x}_1 + A_2\tilde{x}_2 + A_3\tilde{x}_3}{A_1 + A_2 + A_3}$$

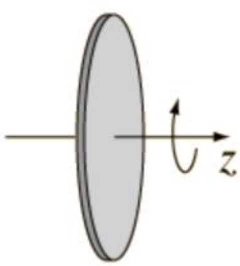
Mass Moment of Inertia

- **Mass moment of inertia** is the mass property of a rigid body that determines the torque needed for a desired angular acceleration about an axis of rotation.

$\alpha = [rad/s^2]$

- A larger mass moment of inertia around a given axis requires more torque to increase the rotation, or to stop the rotation, of a body about that axis
- Mass moment of inertia depends on the shape and density of the body and is different around different axes of rotation.

Torque-acceleration relation:



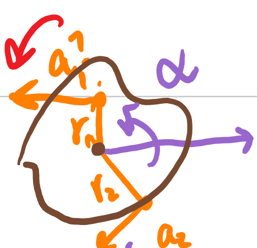
$F = ma$

For TAM 211:

$\Sigma M_o = 0$

For TAM 212:

$\Sigma M_o \neq 0 \rightarrow \vec{r} \times \vec{F} = \vec{M} = \vec{r} \times (m\vec{a})$



$= \underline{\vec{r} \times (mr\vec{\alpha})}$

$\vec{F} = m\vec{a}$

$\vec{M} = I\vec{\alpha}$

mass moment of inertia

$I = mr^2$

α: angular acceleration.

$a = r\alpha$

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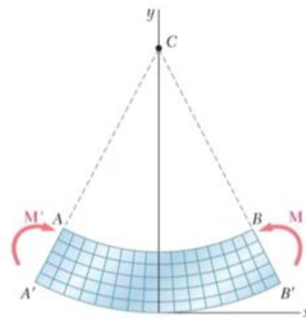
Second moment of area

Moment of inertia is the property of a deformable body that determines the moment needed to obtain a desired curvature about an axis.
Moment of inertia depends on the shape of the body and may be different around different axes of rotation.

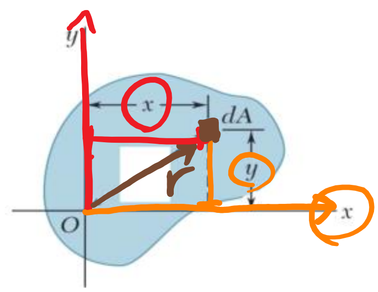


- The moment of inertia of the area A with respect to the x-axis is given by

$$I_x = \int y^2 \, dA$$

- The moment of inertia of the area A with respect to the y-axis is given by

$$I_y = \int x^2 \, dA$$



- The moment of inertia of the area A with respect to the origin is given by (Polar MoI)

$$I_o = \int r^2 \, dA, \quad r^2 = x^2 + y^2$$

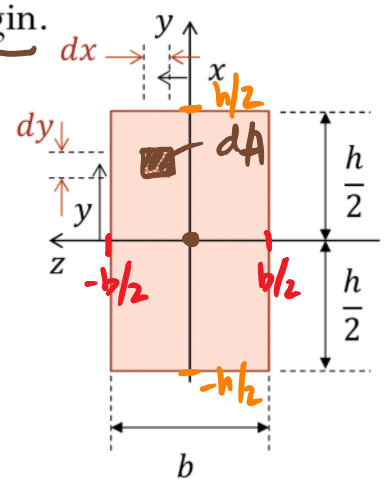
$$= \int (x^2 + y^2) \, dA$$

$$= \underbrace{\int x^2 \, dA} + \underbrace{\int y^2 \, dA}$$

$$I_o = I_y + I_x$$

- For mass moment of inertia: $I_x = \int y^2 \, dm$, $I_y = \int x^2 \, dm$

Moment of inertia of a rectangular area about the origin.



Find: $I_o = I_x + I_y$

① $I_x = \int y^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}} y^2\, dx\, dy = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 x \Big|_{-\frac{b}{2}}^{\frac{b}{2}} dy$

$= \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 b\, dy = b \frac{y^3}{3}\Big|_{-\frac{h}{2}}^{\frac{h}{2}} = b\left[\frac{(h/2)^3}{3} - \frac{(-h/2)^3}{3}\right]$

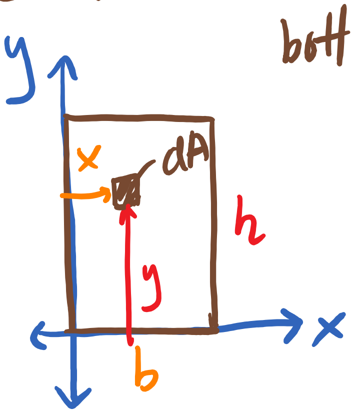
$I_x = \frac{bh^3}{12}$ ← moment of inertia for area about the x-axis through the origin for a rectangle with dimensions $b \times h$.

$dA = dx\, dy$

② $I_y = \int x^2 dA = \int_{-h/2}^{h/2}\int_{-b/2}^{b/2} x^2\, dx\, dy = \frac{hb^3}{12}$

$I_o = I_x + I_y = \frac{bh^3}{12} + \frac{hb^3}{12}$

Example 2: Find I_x where the origin is located at the bottom left of the rectangle.



$I_x = \int y^2 dA = \int_0^h\int_0^b y^2\, dx\, dy$

$= \int_0^h y^2 x \Big]_0^b dy = b\int_0^h y^2 dy = \frac{by^3}{3}\Big|_0^h = \frac{bh^3}{3}$

$I_y = \int x^2 dA = \int_0^h\int_0^b x^2\, dx\, dy = \int_0^h \frac{x^3}{3}\Big|_0^b dy$

$\int^h \ldots \quad b^3 y \Big|^h \quad b^3 h$

$$= \int_0^h \frac{b^3}{3}\,dy = \frac{b^3 y}{3}\Big|_0^h = \frac{b^3 h}{3}$$

→ MoI is the smallest at its centroid.

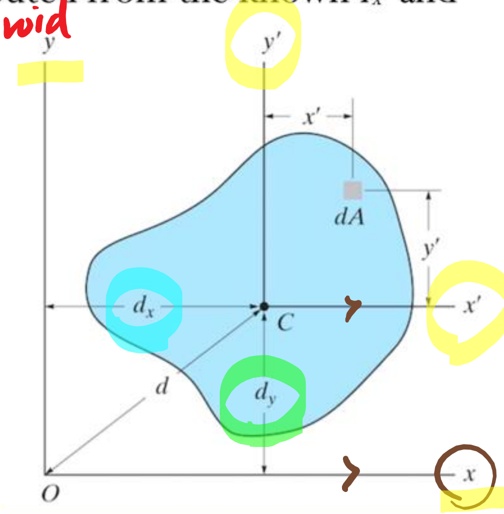
Parallel axis theorem

- Often, the **moment of inertia** of an area is known for an axis passing through the **centroid**; e.g., x' and y':
- The moments around other axes can be computed from the known $I_{x'}$ and $I_{y'}$:

HAS TO start at the centroid

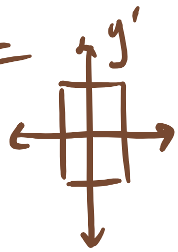
$$I_x = I_{x'} + (d_y)^2 A$$

$$I_y = I_{y'} + (d_x)^2 A$$



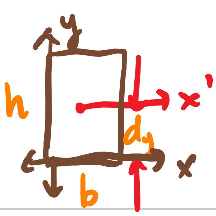
Note: the integral over y' gives zero *when done through the centroid axis.*

Ex 1



$$I_{x'} = \frac{bh^3}{12}$$

Ex 2



$$d_y = \frac{h}{2}$$

$$I_x = I_{x'} + (d_y)^2 A$$

$$= \frac{bh^3}{12} + \left(\frac{h}{2}\right)^2 (bh)$$

$$= \frac{bh^3}{12} + \frac{bh^3}{4}$$

$$= \frac{bh^3}{12} + \frac{3bh^3}{12} = \frac{4bh^3}{12}$$

$$= \frac{bh^3}{3}$$ ✓ same as previous page using $I_x = \int y^2 dA$.