Announcements

- Echo360 is available to supplement lecture notes.

- Upcoming deadlines:
  - Friday (10/5)
    - Written Assignment
  - Tuesday (10/9)
    - PL HW
Objectives

- 2D rigid body equilibrium examples
- 3D rigid body support reactions
The uniform rod $AB$ has a mass of 40 kg. Determine the force in the cable when the rod is in the position shown. There is a smooth collar at $A$.

1) ID support vars $\rightarrow$ forces/moments they provide

2) FBD

3) EoE

$\sum F_x = A - T = 0$

$\sum F_y = -W + B = 0$

$\sum M_B = (1.5\cos 60^\circ m)W - (3\sin 60^\circ m)A = 0$

$A = \frac{(1.5\cos 60^\circ)W}{3\sin 60^\circ}$

$B = W$

$T = A = \frac{\cos 60^\circ}{2\sin 60^\circ} W$
The man has a weight $W$ and stands at the center of a plank with negligible weight. If the planes at $A$ and $B$ are smooth, determine the tension in the cord in terms of $W$ and $\theta$.

3.) EoE

$$\sum F_x = -B \sin \theta + T \cos \theta = 0$$

$$\sum F_y = A - W + B \cos \theta + T \sin \theta = 0$$

$$\sum M_B = + W \left( \frac{L}{2} \cos \phi \right) - A \left( L \cos \phi \right) = 0$$
Equilibrium of a rigid body

Now we add the z-axis to the coordinate system!

What are the possible movements for a 3-D body?
Equilibrium of a rigid body

Now we add the z-axis to the coordinate system!

6 Equations of Equilibriums:

\[ \Sigma F_x = 0 \quad \Sigma M_{ox} = 0 \]
\[ \Sigma F_y = 0 \quad \Sigma M_{oy} = 0 \]
\[ \Sigma F_z = 0 \quad \Sigma M_{oz} = 0 \]
<table>
<thead>
<tr>
<th>Types of Connection</th>
<th>Reaction</th>
<th>Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) cable</td>
<td><img src="cable.png" alt="" /></td>
<td>![cable.png]</td>
</tr>
<tr>
<td>(2) smooth surface support</td>
<td><img src="smooth.png" alt="" /></td>
<td>![smooth.png]</td>
</tr>
<tr>
<td>(3) roller</td>
<td><img src="roller.png" alt="" /></td>
<td>![roller.png]</td>
</tr>
</tbody>
</table>
### TABLE 5-2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

<table>
<thead>
<tr>
<th>Types of Connection</th>
<th>Reaction</th>
<th>Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4) ball and socket</td>
<td>$F_x$, $F_y$, $F_z$, $M_x$, $M_y$, $M_z$</td>
<td>7</td>
</tr>
<tr>
<td>(5) single journal bearing</td>
<td>$F_x$, $F_y$, $M_x$, $M_y$</td>
<td>4</td>
</tr>
<tr>
<td>Types of Connection</td>
<td>Reaction</td>
<td>Number of Unknowns</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>----------</td>
<td>-------------------</td>
</tr>
<tr>
<td>(6) single journal bearing with square shaft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7) single thrust bearing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8) single smooth pin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Types of Connection</td>
<td>Reaction</td>
<td>Number of Unknowns</td>
</tr>
<tr>
<td>---------------------</td>
<td>----------</td>
<td>-------------------</td>
</tr>
<tr>
<td>(9) single hinge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10) fixed support</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Calculate the reaction forces at the support.

1) 1D Ren: \( D \) is a fixed support.

\[
\begin{align*}
\Sigma F_x &= 0 = F_x \cos \theta + D_x \\
\Sigma F_y &= 0 = -F_x \sin \theta + D_y \\
\Sigma F_z &= 0 = -F_1 + D_z
\end{align*}
\]

(assuming negligible mass)

\[
\Rightarrow D_x = -F_x \cos \theta \\
D_y = F_x \sin \theta \\
D_z = F_1
\]

2) \( F_6D \)

\[
\begin{align*}
\Sigma M_D &= \Sigma M_{x0} = \sum \vec{M}_D = \vec{F}_1 \times \vec{F}_1 + \vec{F}_2 \times \vec{F}_2 + \vec{M}_D = 0 \\
\Sigma M_{y0} &= 0 \\
\Sigma M_{z0} &= 0
\end{align*}
\]

\[
\begin{align*}
\vec{F}_1 &= (5\hat{j} + 1.2\hat{k}) \text{ in} \\
\vec{F}_2 &= (3.25\hat{i} + 6\hat{j}) \text{ in}
\end{align*}
\]

\[
\vec{M}_1 = \begin{bmatrix} 0 & 5 & 1.2 \\ 0 & 0 & -6000 \end{bmatrix} = -30000 \hat{l} \text{ lb \cdot in}
\]

\[
\vec{M}_2 = \begin{bmatrix} 3.25 & 0 & 0 \\ 500 & 600 & 0 \end{bmatrix} \begin{bmatrix} -16.25 \sin \theta \\ -3000 \cos \theta + M_{Dz} \end{bmatrix} \hat{k}
\]

\[
\Rightarrow \Sigma M_0 = (30000 + M_{Dz}) \hat{z} + (M_{Dy}) \hat{j} + (-16.25 \sin \theta - 3000 \cos \theta + M_{Dz}) \hat{k}
\]

\[
\Rightarrow M_{Dx} = 3000 \text{ lb \cdot in} \\
\Rightarrow M_{Dy} = 0 \text{ lb \cdot in} \\
\Rightarrow M_{Dz} = 16.25 \sin \theta + 3000 \cos \theta
\]