Announcements

- Remember to register for Quiz 2

❑ Upcoming deadlines:

- Friday (9/21 - Today!)
  - Written Assignment
- Tuesday (9/25)
  - PL HW
- Friday (9/28)
  - Written Assignment

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Objective

- Distributed Loading

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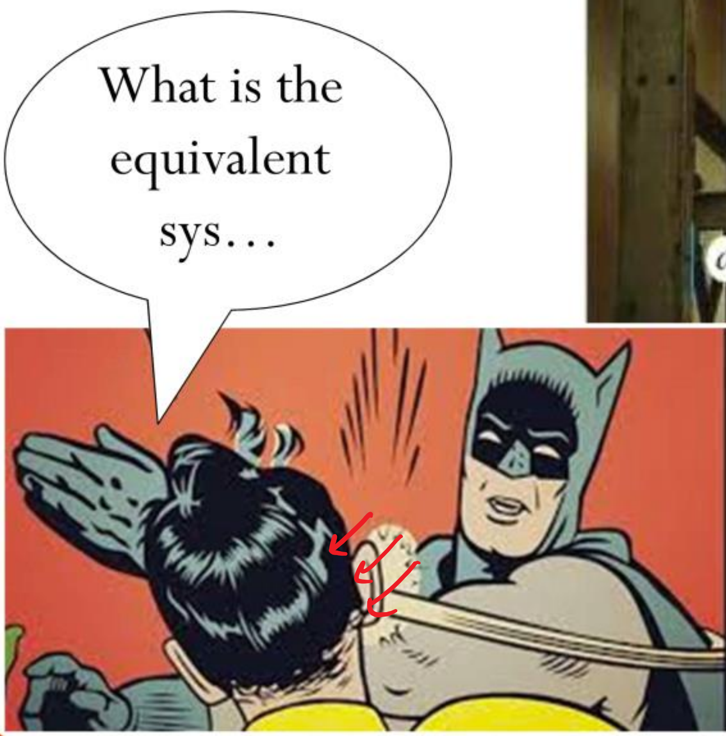
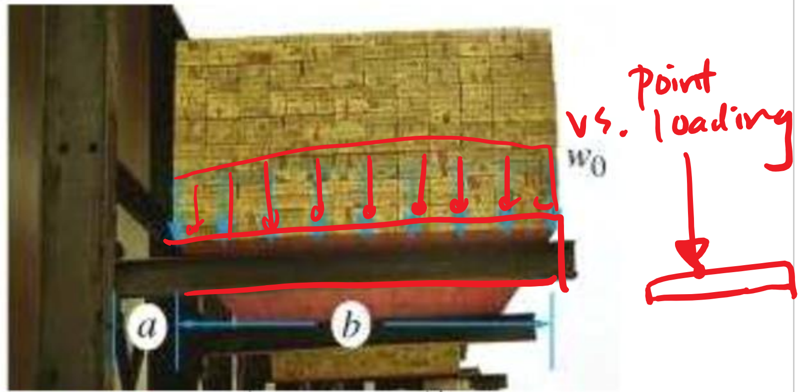
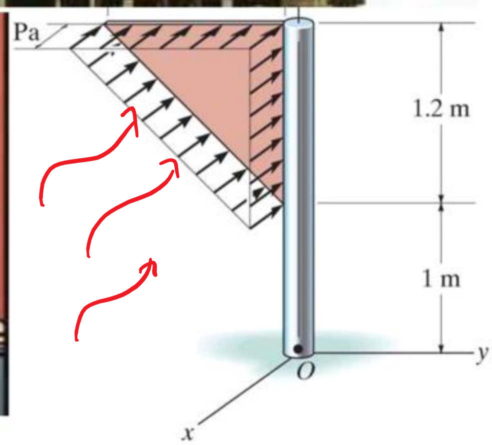
Distributed Loading







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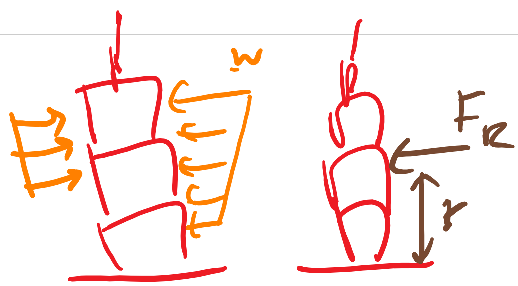
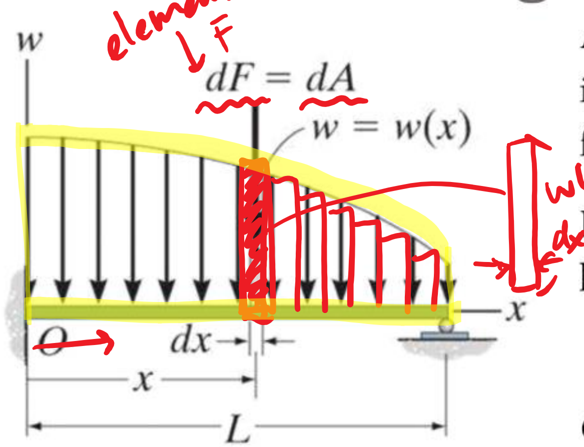
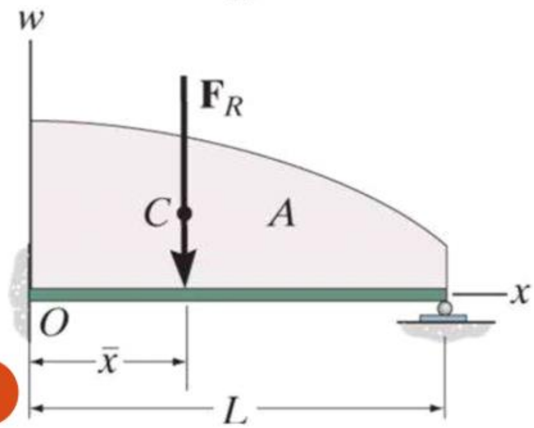


Distributed Loading





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A common case of distributed loading in a uniform load along one axis of a flat rectangular body.

In such cases, w is a function of x and has <u>units</u> of

$$[w(x)] = \frac{[N]}{[m]}$$

Consider an element of length dx. The force magnitude dF acting on it is given as

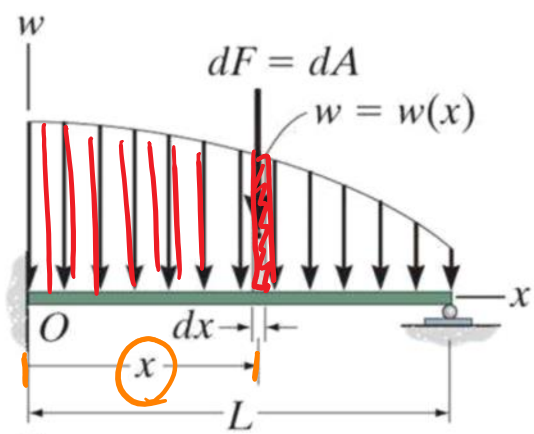
function of x.

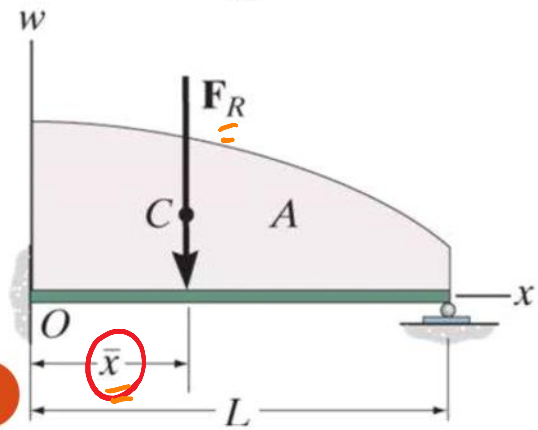
$$dF = dA = \underbrace{w(x)}_{=} dx$$

<u>The net force</u> on the beam is given by

$$F_R = \int_0^L dF = \int_0^L w(x)\,dx$$

Location of the Resultant Force





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The force dF will produce a moment about O of

if $F \perp d$

$$M_O = F d \rightarrow dM = dF \cdot x = x \cdot \underbrace{w(x)dx}_{dF}$$

The total moment about point O is

$$M_{RO} = \int dM = \int_0^L \underbrace{x}_{d} \cdot \underbrace{w(x)dx}_{dF}$$

Assuming that $\mathbf{F}_R$ acts at $\underline{x}$, it will produce the moment about point O as

$$F_R \cdot \bar{x} = M_{RO}$$

Hence,

$$\boxed{\bar{x} = \frac{M_{RO}}{F_R} = \frac{\int_0^L x\, w(x)\, dx}{\int_0^L w(x)\, dx}}$$

Rectangle Loading

Area Method

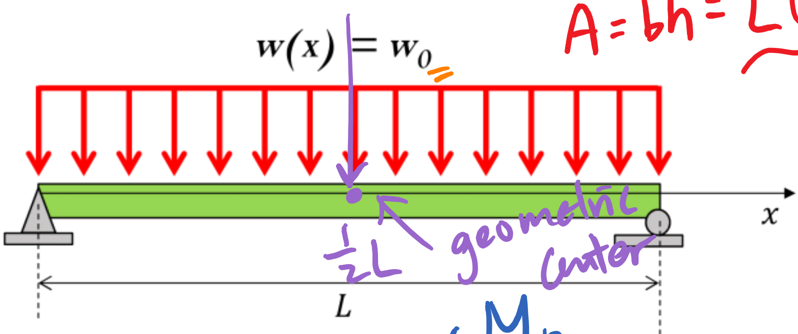
$A = bh = L(w_0) = F_R$

$F_R = \int_0^L w(x)\,dx$

$= \int_0^L w_0\,dx$

$= w_0 x \Big|_0^L = w(L-0)$

$\boxed{F_R = w_0 L}$



M_R

$$\bar{x} = \frac{\left(\int_0^L x\,w(x)\,dx\right)}{F_R}$$

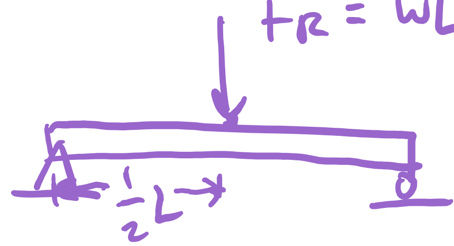
$$= \frac{\int_0^L x\,w_0\,dx}{wL}$$

$$= \frac{w_0\left(\frac{x^2}{2}\right)\Big|_0^L}{w_0 L}$$

$$= \frac{\cancel{w_0}\left(\frac{L^{\cancel{2}}}{2} - 0\right)}{\cancel{w_0}\cancel{L}} \rightarrow \boxed{\bar{x} = \frac{1}{2}L}$$

✓

$F_R = wL$



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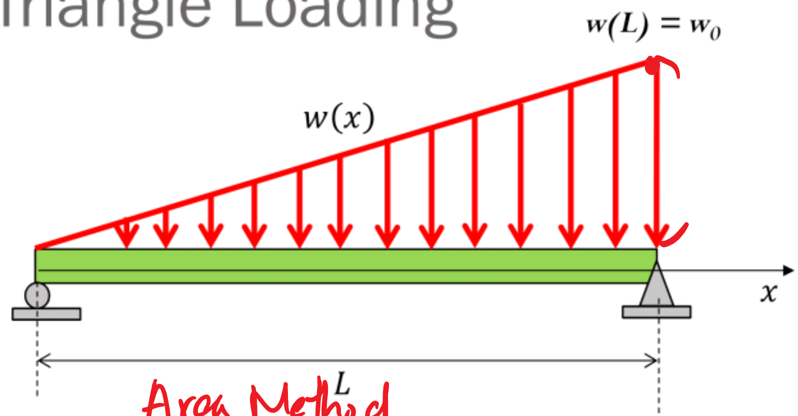
Triangle Loading

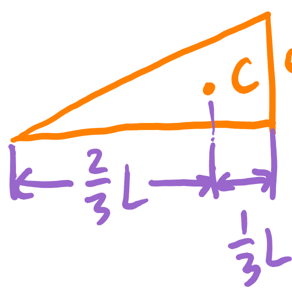


Area Method

$$F_R = \frac{1}{2}bh = \frac{1}{2}Lw_0$$

(hw: try to prove this using the integral method).

$$\bar{x} = \frac{M_R}{F_R} = \frac{2}{3}L$$



geometric center of a triangle.

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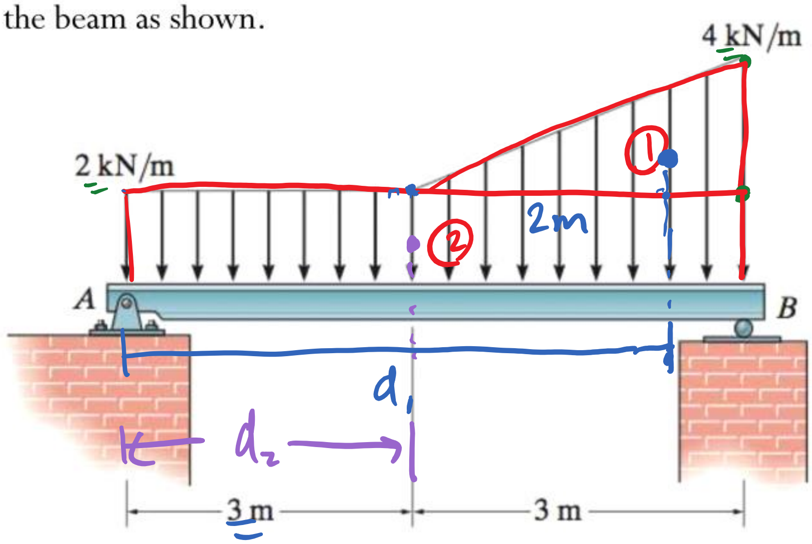
(hw: prove w/ integral method)

Example

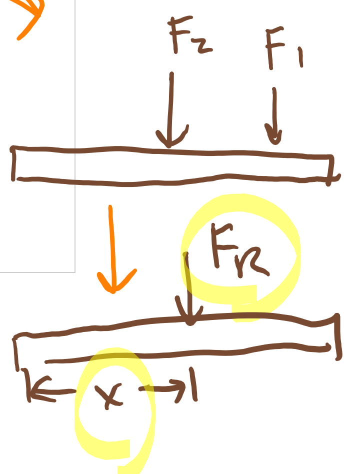
Find the equivalent force and its location from point A for the loading on the beam as shown.



I. $F_R = F_1 + F_2$

II. $M_{RA} = M_1 + M_2$

III. $\bar{x} = \dfrac{M_{RA}}{F_R}$



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I. $F_1 = \frac{1}{2}bh = \frac{1}{2}(3m)(4\,kN/m - 2\,kN/m) = 3\,kN$

$F_2 = bh = (6m)(2\,kN/m) = 12\,kN$

$F_R = F_1 + F_2 = 3\,kN + 12\,kN = 15\,kN.$

II. $M_{RA} = F_1 \cdot d_1 + F_2 \cdot d_2.$

$d_1 = 3m + 2m = 5m, \quad d_2 = \frac{1}{2}(6m) = 3m$

$\rightarrow M_{RA} = (3kN)(5m) + (12kN)(3m) = (15 + 36)\,kN \cdot m$

$M_{RA} = 51\,kN \cdot m$

III. $\bar{x} = \dfrac{M_R}{F_R} = \dfrac{51}{15}\,m.$

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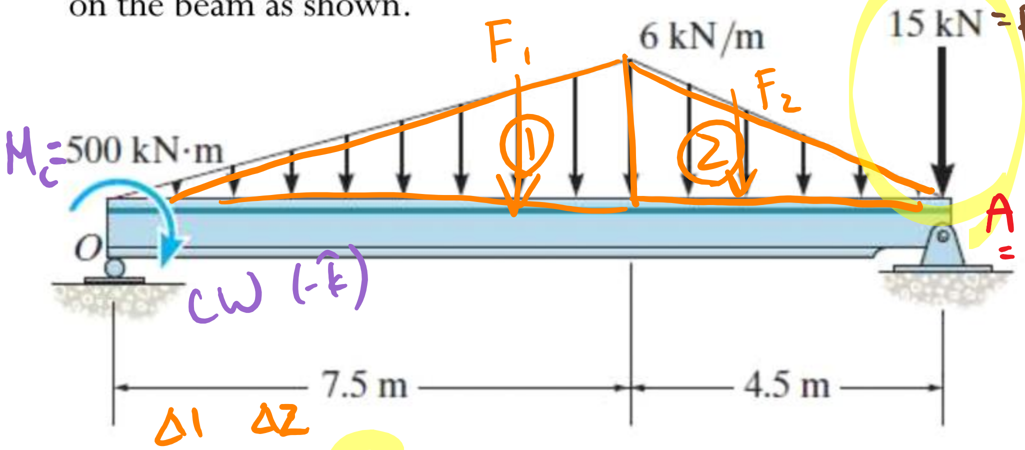
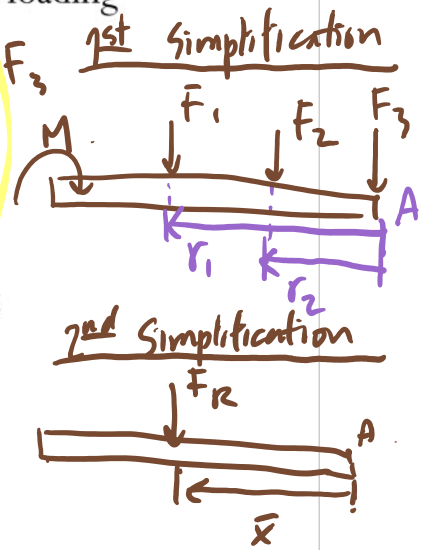
Example

Find the equivalent force and its location from point A for the loading on the beam as shown.





I. $F_R = F_1 + F_2 + F_3$

$= \left[\frac{1}{2}6(7.5) + \frac{1}{2}(4.5)(6)\right] \text{kN} + 15 \text{ kN}$

$= 51 \text{ kN}$

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II. $\vec{M}_{RA} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \cancel{\vec{r}_3 \times \vec{F}_3} + \vec{M}_c$

force that goes through A creates no moment about A.

$r_1 = 7m, \quad r_2 = 3m$

$\rightarrow \vec{M}_{RA} = (7m)(22.5 \text{kN})\hat{k} + (3m)(13.5 \text{kN})\hat{k} - 500 \text{ kN·m } \hat{k}$

$= -302 \text{ kN·m } \hat{k}$

III. $\bar{x} = \dfrac{M_{RA}}{F_R} = \dfrac{-302 \text{ kN·m}}{51 \text{ kN}} \approx -5.92 \text{ m}$ or 5.92 m to the right of A