Announcements

- Remember to register for Quiz 2

- Upcoming deadlines:
  - Friday (9/21 - Today!)
    - Written Assignment
  - Tuesday (9/25)
    - PL HW
  - Friday (9/28)
    - Written Assignment
Objective

- Distributed Loading
Distributed Loading

What is the equivalent sys…
Distributed Loading

A common case of distributed loading in a uniform load along one axis of a flat rectangular body.

In such cases, \( w \) is a function of \( x \) and has units of

\[
\left[ w(x) \right] = \frac{[N]}{[m]}
\]

Consider an element of length \( dx \). The force magnitude \( dF \) acting on it is given as:

\[
dF = dA = w(x) \, dx
\]

The net force on the beam is given by

\[
F_R = \int_0^L dF = \int_0^L w(x) \, dx
\]
Location of the Resultant Force

The force $dF$ will produce a moment about $O$ of

$$M = \int dM = \int x \cdot \frac{w(x)dx}{dF}$$

The total moment about point $O$ is

$$M = \int_0^L x \cdot \\frac{w(x)dx}{F_R}$$

Assuming that $F_R$ acts at $x$, it will produce the moment about point $O$ as

$$F_R \cdot \bar{x} = M_{O}$$

Hence,

$$\bar{x} = \frac{M_{O}}{F_R} = \frac{\int_0^L x \cdot w(x)dx}{\int_0^L w(x)dx}$$
Rectangle Loading

\[ w(x) = w_0 \]

**Area Method**

\[ A = bh = Lw_0 \]

\[ F_K = \int_0^L w(x) \, dx \]

\[ = \int_0^L w_0 \, dx \]

\[ = w_0 x \bigg|_0^L = w(L-0) \]

\[ F_K = w_0 L \]

\[ \bar{x} = \frac{1}{2} L \]

\[ \bar{x} = \frac{1}{2} L \]

\[ \text{valid} \]
Triangle Loading

\[ w(x) \]

\[ w(L) = w_0 \]

\[ F_R = \frac{1}{2} bh = \frac{1}{2} L w_0 \] (Try to prove this using the integral method)

\[ \bar{x} = \frac{MR}{F_R} = \frac{2}{3} L \]

Geometric center of a triangle.

(Prove w/ integral method)
Example

Find the equivalent force and its location from point A for the loading on the beam as shown.

I. \( F_k = F_i + F_z \)

II. \( M_{RA} = M_i + M_z \)

III. \( \bar{x} = \frac{M_{RA}}{F_k} \)

\[
\begin{align*}
I. & \quad F_i = \frac{1}{2} bh = \frac{1}{2} (3\text{m}) (4 \text{kN/m} - 2 \text{kN/m}) = 3 \text{kN} \\
& \quad F_z = bh = (6\text{m}) (2 \text{kN/m}) = 12 \text{kN} \\
\end{align*}
\]

\[
F_k = F_i + F_z = 3 \text{kN} + 12 \text{kN} = 15 \text{kN}.
\]

II. \( M_{RA} = F_i \cdot d_i + F_z \cdot d_z \)

\[
d_i = 3\text{m} + 2\text{m} = 5\text{m}, \quad d_z = \frac{1}{2} (6\text{m}) = 3\text{m}
\]

\[
\rightarrow M_{RA} = (3 \text{ kN})(5\text{m}) + (12 \text{kN})(3\text{m}) = (15 + 36) \text{kN} \cdot \text{m}
\]

\[
M_{RA} = 51 \text{kN} \cdot \text{m}
\]

III. \( \bar{x} = \frac{M_{RA}}{F_k} = \frac{51}{15} \text{ m} \)
Example

Find the equivalent force and its location from point A for the loading on the beam as shown.

\[ F_2 = F_1 + F_2 + F_3 \]
\[ = \left( \frac{1}{6} \cdot 7.5 \right) \left( \frac{1}{2} \cdot 4.5 \cdot 6 \right) \text{kN} + 15 \text{kN} \]
\[ = 51 \text{kN} \]

\[ \Rightarrow M_{RA} = F_2 \cdot r_2 + M_c \]
\[ = 7 \text{m} \cdot (22.5 \text{kN}) \hat{k} + 3 \text{m} \cdot (13.5 \text{kN}) \hat{k} - 500 \text{kN} \cdot \text{m} \hat{k} \]
\[ = -302 \text{ kN} \cdot \text{m} \hat{k} \]

\[ \Rightarrow x = \frac{M_{RA}}{F_2} = \frac{-302 \text{ kN} \cdot \text{m}}{51 \text{kN}} \approx -5.92 \text{ m} \text{ or 5.92 m to the right of A} \]