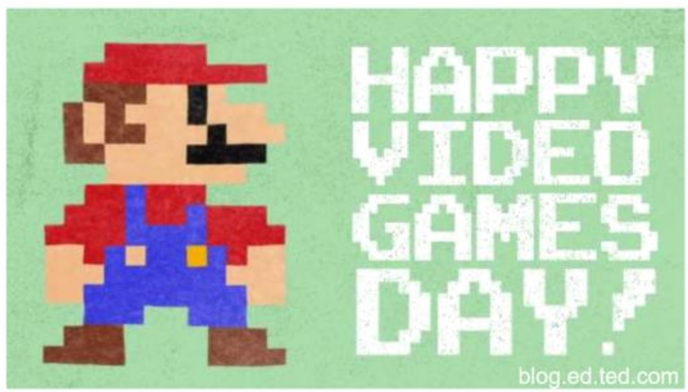
Announcements

- Morning Office Hours: Mon/Wed, 10–11am in 220H MEB
- Quiz 1 starts tomorrow

❑ Upcoming deadlines:

- Friday (9/14)
  - WA#2
- Tuesday (9/18)
  - PL HW3



1

Chapter 4: Force System Resultants

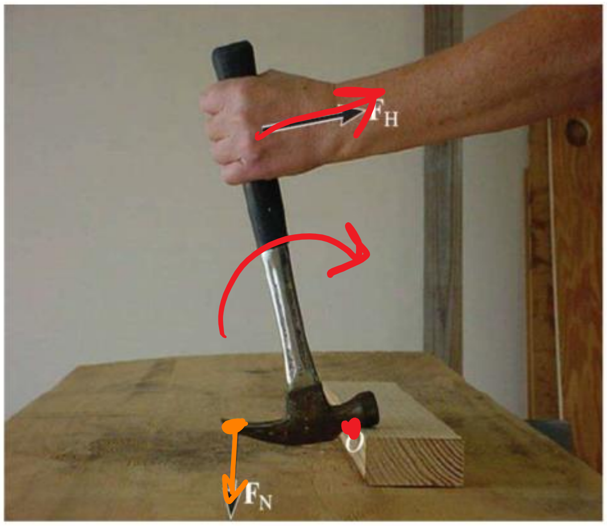
Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions

~ no more particle assumption.
→ where the force is applied on a body matters.

3

Applications



Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?

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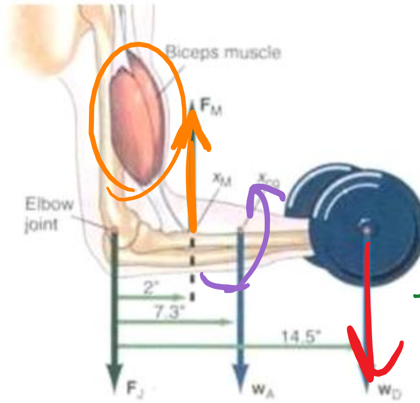
Particle

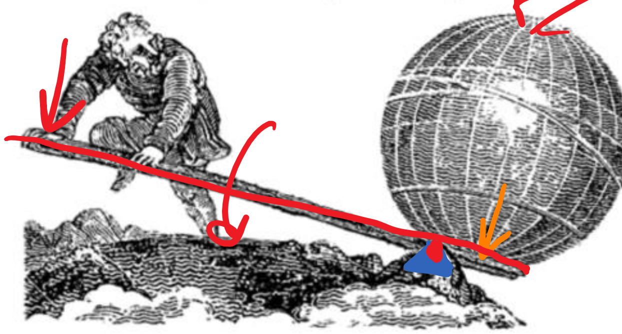
vs.

Rigid Body



F

d



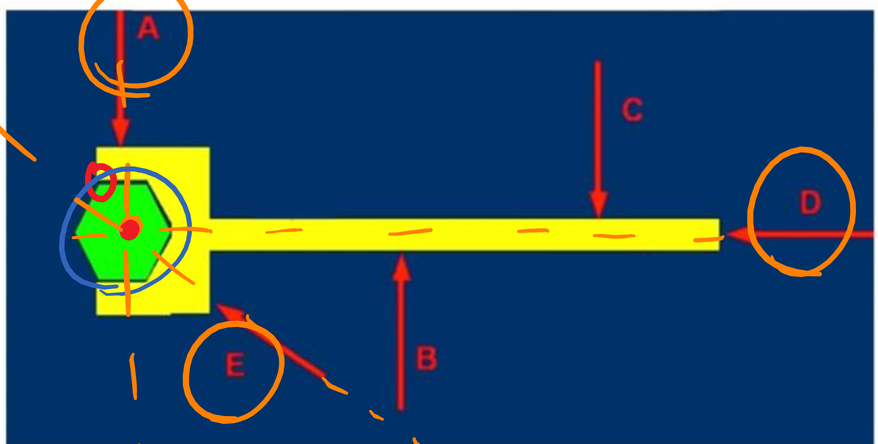
Moment 1.a very brief period of time. An Exact point in time. 2. importance. 3. A turning Effect produced by a force acting at a distance on An object.

(torque)

Wednesday, September 12, 2018 10:23 AM

l.o.a. A

Moment of a Force



l.o.a. D

Which force(s) have NO turning effect?

- Ones with their line of action going through point O

line of action E

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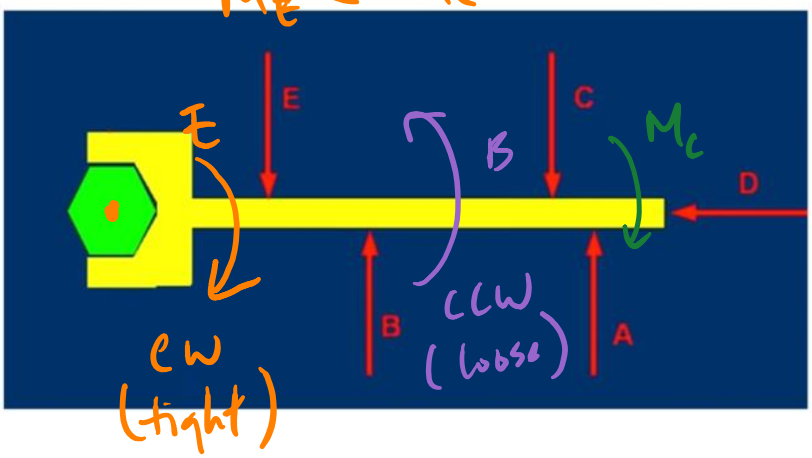
Moment of a Force

$M_E < M_C$



(−) (clockwise)

(counter clockwise) (+)

1) Which force(s) yields a "tighty" effect?

1) Which force(s) yields a "loosey" effect?

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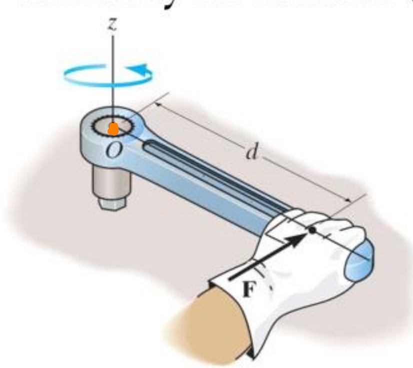
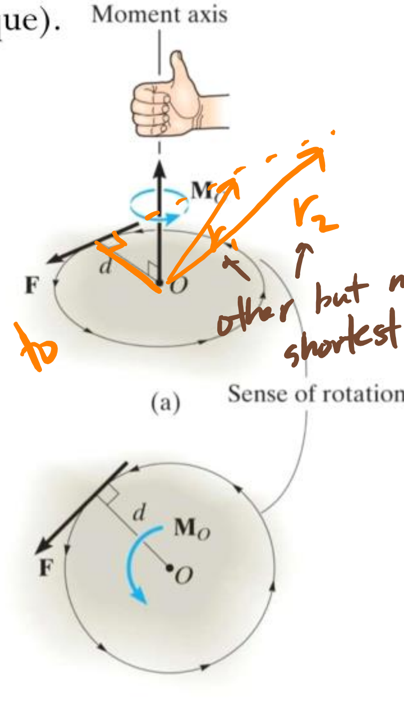
Moment of a force – scalar formulation (magnitude)

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).



$M_O = Fd$

shortest distance from O to $\vec{F}$.



Direction: follows right hand rule.

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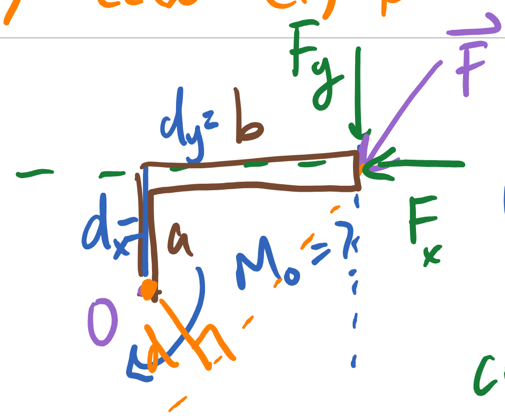
2D: CW: (−) $\hat{k}$

(x, y) CCW: (+) $\hat{k}$

• Break down into components.

$M_O = M_{Ox} + M_{Oy}$

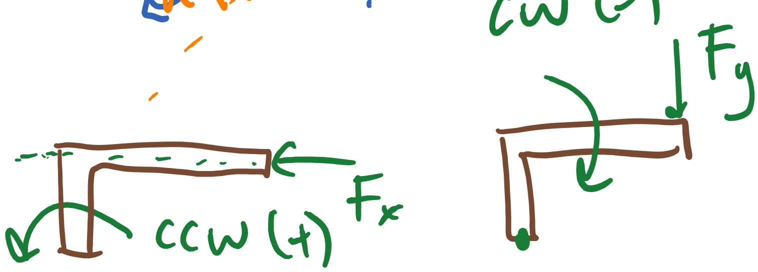
(since d is not readily available)



$M_O = Fd$

$M_{Ox} = F_x d_x = F_x a$ (+)

$M_{Oy} = F_y d_y = F_y b$. (−)

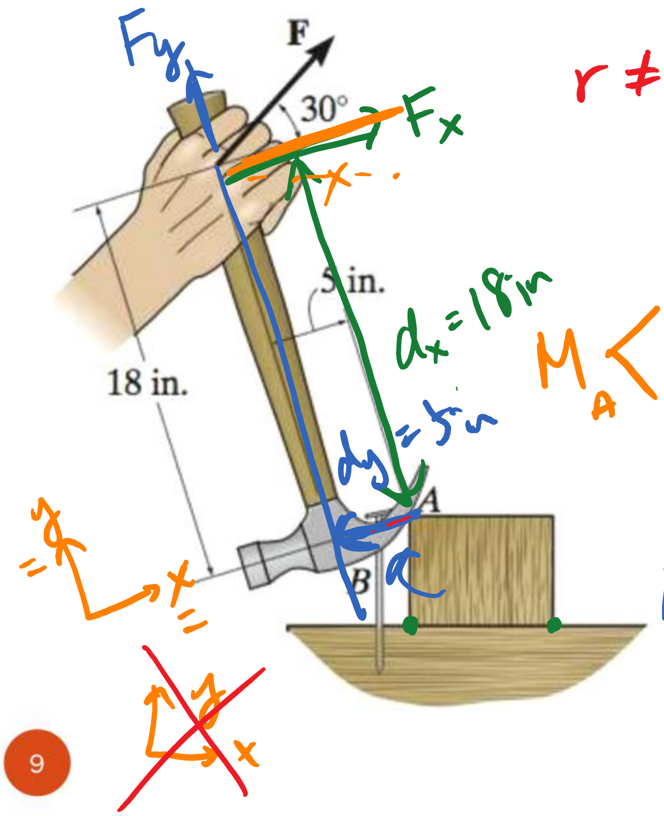


$\rightarrow M_O = +M_{Ox} - M_{Oy}$

$= F_x a - F_y b$

Example – Scalar Formulation

Determine the moment of this force about the point A as a function of $\mathbf{F}$.



$r \neq d$, $M \neq Fr$, $M = Fd$

$$M_A \begin{cases} M_{Ax} = (-) F_x (18in) = F\cos 30° (18in) \\ M_{Ay} = (-) F_y (5in) = F\sin 30° (5in) \end{cases}$$

$$M_A = -F\cos 30° (18in) - F\sin 30° (5in)$$

$$\underline{M_A = -18.1\,F}$$

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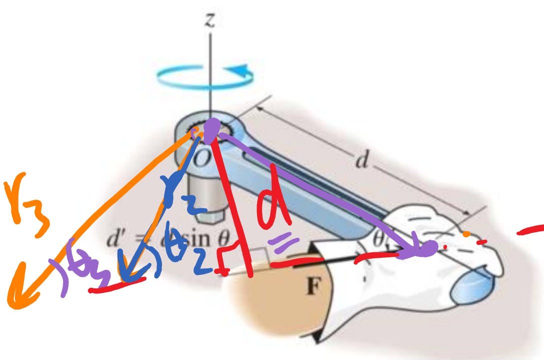
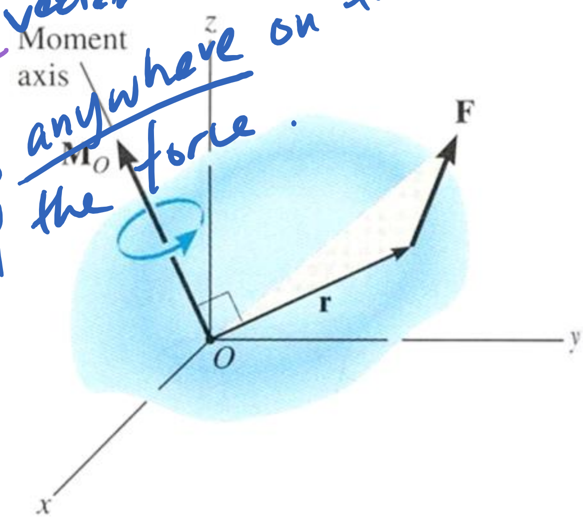
Moment of a force – vector formulation

The moment of a force **F** about point **O**, or actually about the moment axis passing through O and perpendicular to the plane containing **O** and **F**, can be expressed using the cross (vector) product, namely:



$$\vec{M} = \vec{r} \times \vec{F}$$

position vector from pt. of ref. to anywhere on the line of action of the force.



$|M| = F r \sin\theta$

$r \sin\theta = d$

$\rightarrow |M| = Fd$ ✓

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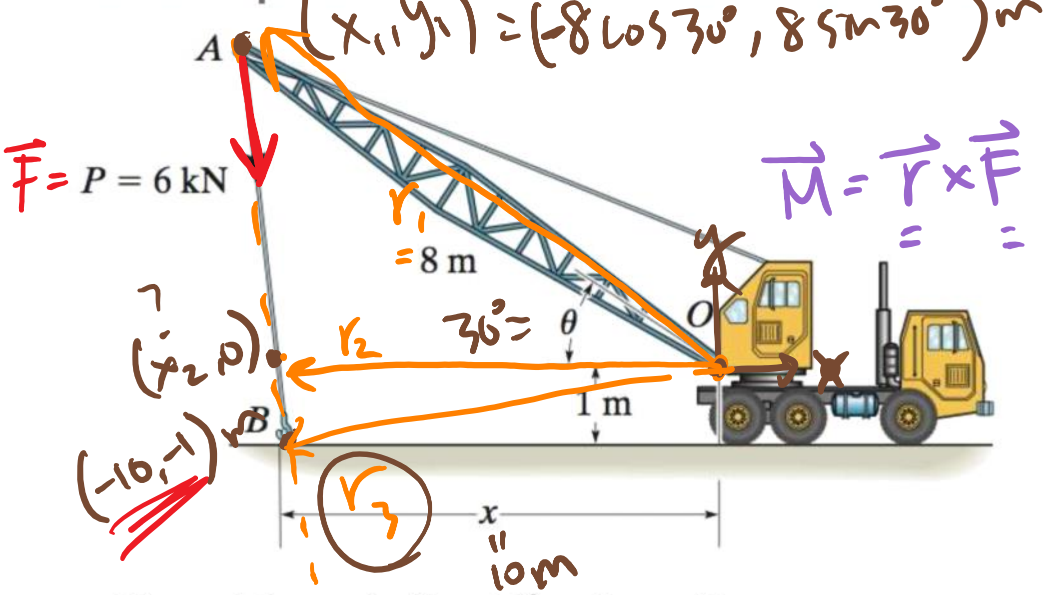
$r_3 \sin\theta_3 = r_2 \sin\theta_2 = d$

Example – Vector Formulation



Given: The angle $\theta = 30°$ and $x = 10$ m.

Find: The moment by **P** about point O.

$\vec{M} = \vec{r}_1 \times \vec{F}$ or $\vec{M}_1 = \vec{r}_2 \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -10 & -1 & 0 \\ F_x & F_y & 0 \end{vmatrix}$$

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$\vec{F} = F(\hat{u}_{AB}) = -3.13\hat{i} - 5.11\hat{j}$ kN

$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{r_{AB}} = \frac{[-10 - (-8\cos 30°)]\hat{i} + [(-1) - 8\sin 30°]\hat{j}}{\sqrt{r_x^2 + r_y^2}}$$

$$= \frac{-3.07\hat{i} - 5\hat{j}}{5.87}$$

$\vec{M}_o = \vec{r}_1 \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8\cos 30° & 8\sin 30° & 0 \\ F_x & F_y & 0 \end{vmatrix}$$

$$\vec{M}_o = \underbrace{[10F_y - F_x(-1)]}_{\text{magnitude}} \hat{k}$$

$\hat{k}$ ← direction.

$\vec{M}_o = 48.2\hat{k}$ kN·m

$$- \begin{vmatrix} -8\cos 30^\circ & 8\sin 30^\circ & 0 \\ F_x & F_y & 0 \end{vmatrix}$$

$$= \left(-8\cos 30\, F_y - 8\sin 30\, F_x\right) \hat{k}$$

$$\underline{\vec{M}_o = 48.2\, \hat{k} \ \text{kN}\cdot\text{m}}$$

magnitude

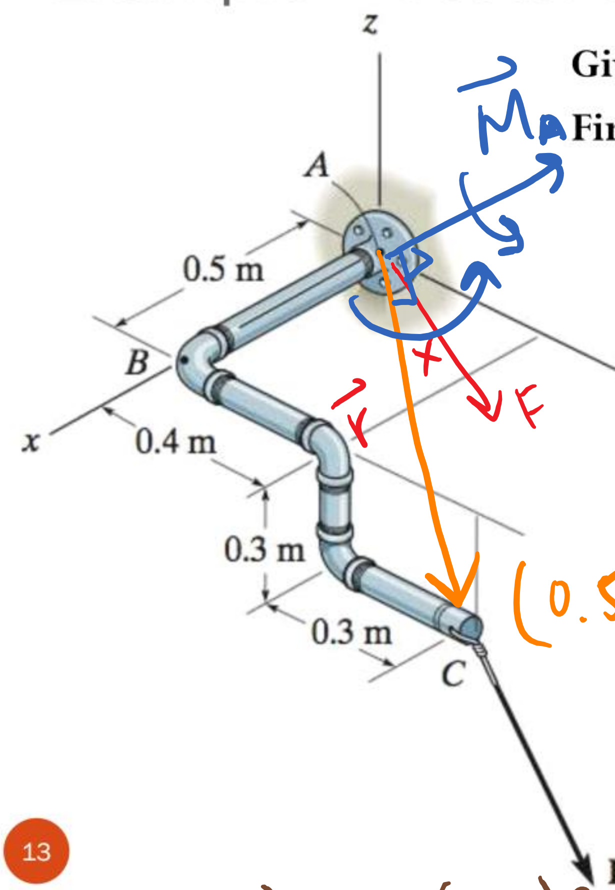
$$\underline{\vec{M}_o = 48.2\, \hat{k} \ \text{kN}\cdot\text{m}}$$

Same

Example – Vector Formulation

Given: F = {600**i** + 800**j** - 500**k**} N

$\vec{M}_A$ **Find:** Moment of the force about point A.



$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{r} = (0.5\hat{i} + 0.7\hat{j} - 0.3\hat{k})\,m$$

$(0.5, 0.7, -0.3)$

$$\vec{M}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

$$= 0.7(-500) - 800(-0.3)\hat{i} - [0.5(-500) - (-0.3)(600)]\hat{j}$$

$$+ [0.5(800) - 0.7(600)]\hat{k}$$

$$\vec{M}_A = (-110\hat{i} + 70\hat{j} - 20\hat{k})\,N \cdot m$$