Announcements

- Quiz 1 This Week!

❑ Upcoming deadlines:

- Tuesday (9/11)
  - PL HW
- Friday (9/14)
  - Writtein Assignment #2

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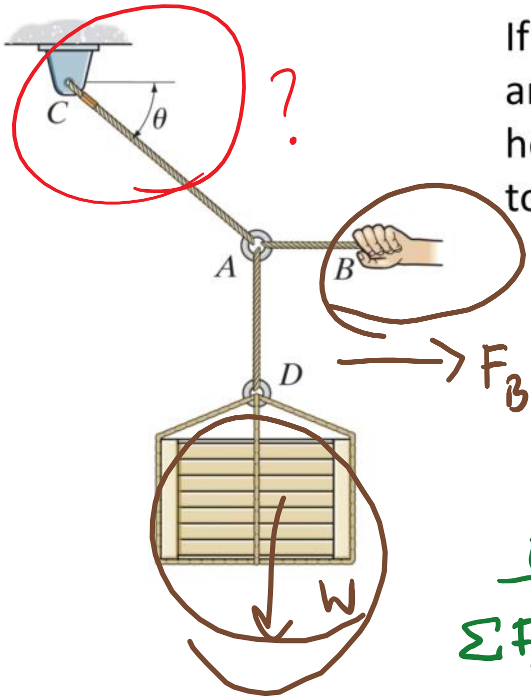
Goals and Objectives

- Solve system of particles at equilibrium problems following general procedure for analysis.

Example



If the box weighs 2 kN, determine the angle of the cable at *C* when a horizontal force of 3 kN is applied at *B* to make the system in equilibrium.

Given parameters : $W = 2\,kN$

$F_B = 3\,kN$

Unknown parameter : θ .

Eq. of Equilibrium

$\Sigma F_x = F_B - T_x$

$= F_B - T\cos\theta$

$= 0$

$\rightarrow F_B = T\cos\theta$ ①

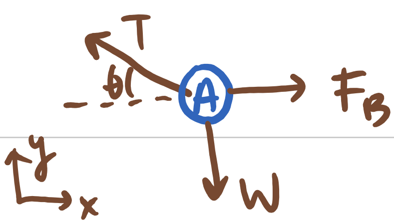
$\Sigma F_y = T_y - W$

$= T\sin\theta - W$

$= 0$

$\rightarrow W = T\sin\theta$ ②

FBD (at A)



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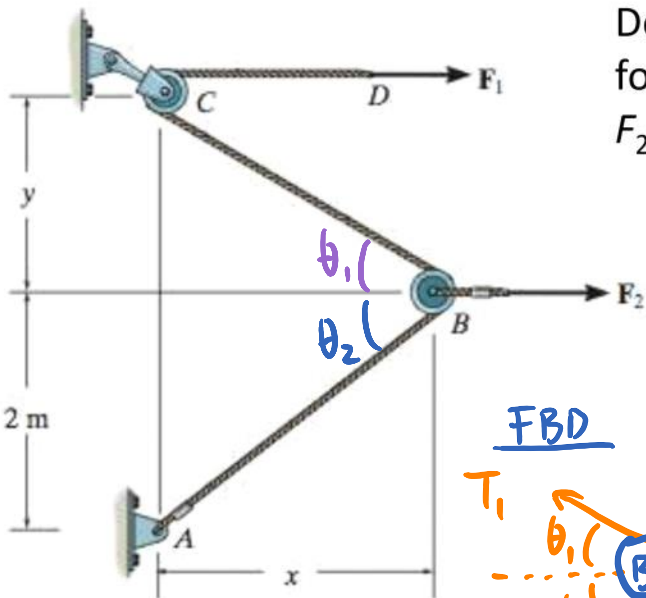
$\rightarrow \dfrac{②}{①} \Rightarrow \dfrac{W = \cancel{T}\sin\theta}{F_B = \cancel{T}\cos\theta} \Rightarrow \dfrac{W}{F_B} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\rightarrow \theta = \tan^{-1}\left(\dfrac{W}{F_B}\right) = \boxed{\tan^{-1}\left(\dfrac{2\,\cancel{kN}}{3\,\cancel{kN}}\right) = \theta}$

Example

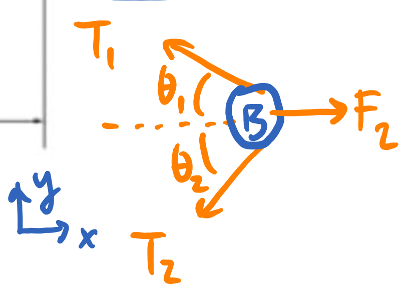
Determine the distances x and y for equilibrium if F_1 = 800 N and F_2 = 1000 N.



Given: $F_1 = 800\text{N}$, $F_2 = 1000\text{ N}$

Find: x, y

FBD



$T_1 = T_2 = F_1$ since F_1 is applied directly on the cable, so the magnitude of F_1 equals the tension in the cable.

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Eq. of Equil.

symmetry

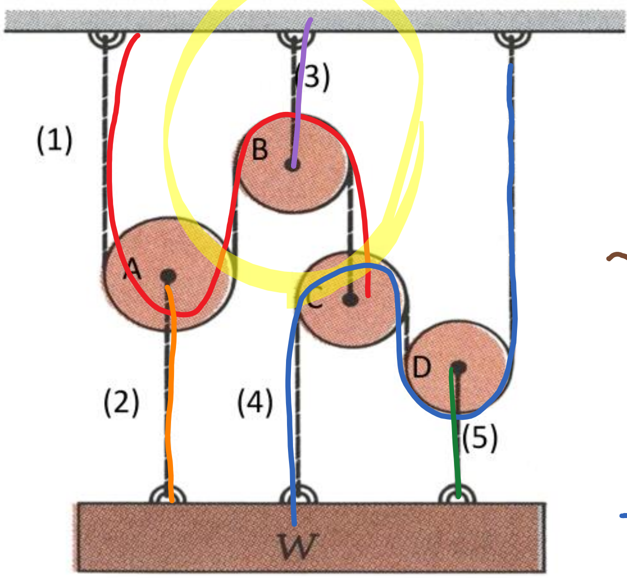
$$\Sigma F_y = F_1 \sin\theta_1 - F_1 \sin\theta_2 = 0 \longrightarrow F_1 \sin\theta_1 = F_1 \sin\theta_2 \longrightarrow \theta_1 = \theta_2 = \theta$$

$$\Sigma F_x = -2F_1 \cos\theta + F_2 = 0 \longrightarrow \cos\theta = \frac{F_2}{2F_1} \longrightarrow \theta = \cos^{-1}\left(\frac{1000\text{N}}{2\cdot 800\text{N}}\right)$$

$$\theta \approx 51.3^\circ \quad \tan\theta = \frac{y}{x}, \quad \boxed{y = 2\text{m}}, \quad \boxed{x = 1.60\text{m}}$$

Equilibrium of a system of particles

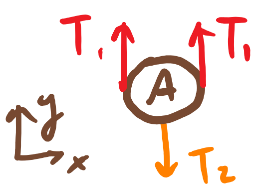
Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton's first law: $\Sigma \mathbf{F} = \mathbf{0}$ on selected multiple free-body diagrams of particles or groups of particles.



The five ropes can each take 1500 N without breaking. How heavy can W be without breaking any?

~ 5 unknown quantities:
5 rope tension magnitude
(T_1, T_2, T_3, T_4, & T_5)

FBD A



EoE

$\Sigma F_y = 2T_1 - T_2 = 0$

$\boxed{T_2 = 2T_1}$

~ each FBD gives 1 EoE

→ Need 5 EoE to solve for 5 unknowns.

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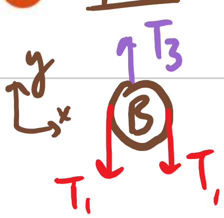
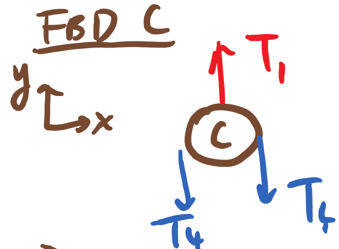
FBD B



$\Sigma F_y = T_3 - 2T_1 = 0$

$\boxed{T_3 = 2T_1}$

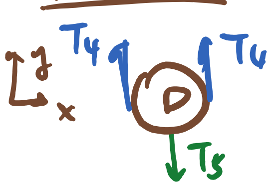
FBD C



$\Sigma F_y = T_1 - 2T_4 = 0$

$\boxed{T_1 = 2T_4}$

FBD D

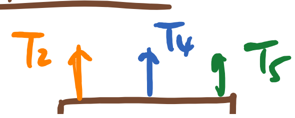


$\Sigma F_y = 2T_4 - T_5 = 0$

$\boxed{T_5 = 2T_4}$

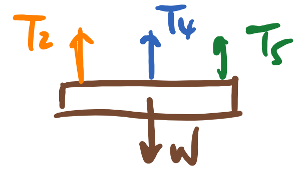
→ $T_2 = T_3 > T_1 = T_5 > T_4$

FBD W



T_2 & T_3 will break first

load

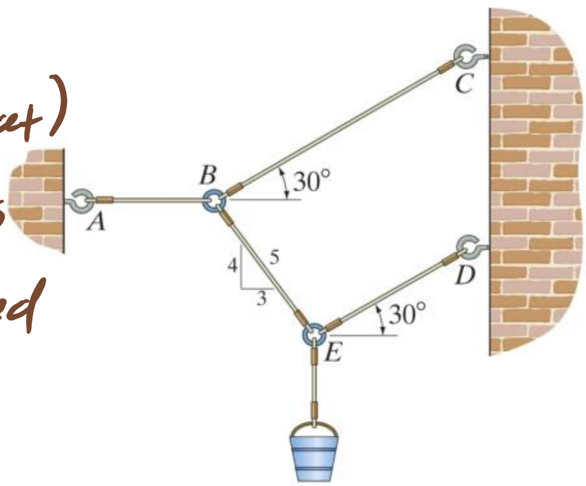


$$\sum F_y = T_2 + T_4 + T_5 - W = 0$$

T_2 & T_3 will break first under heavy load.

Example

The 30-kg ~~pipe~~ bucket is supported at A by a system of five cords. Determine the force in each cord for equilibrium.



unknowns: 5

$(T_{ED}, T_{EB}, T_{BA}, T_{BC}, T_{bucket})$

- Each FBD in 2D provides 2 EoE → 3 FBD needed to solve the problem.

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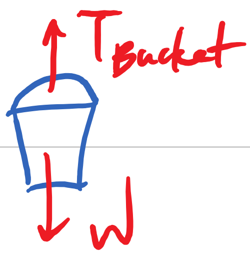
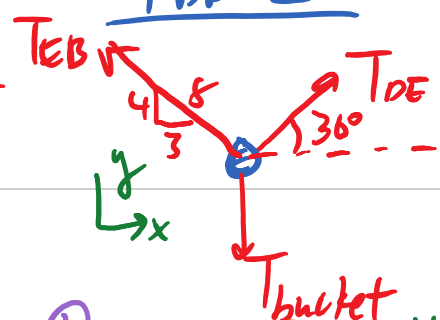
FBD 1



FBD 2



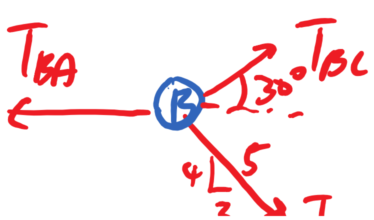
$\Sigma F_y = T_{bucket} - W = 0$ 1

1 $T_{bucket} = W$

$\Sigma F_y = +T_{EB}\left(\frac{4}{5}\right) + T_{DE}\sin 30° - T_{bucket} = 0$ (2, 3)

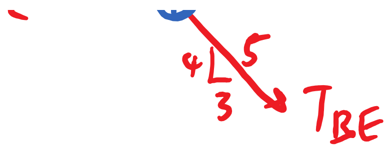
$\Sigma F_x = -T_{EB}\left(\frac{3}{5}\right) + T_{DE}\cos 30° = 0$ 3

FBD 3



$\Sigma F_x = T_{BC}\cos 30° + T_{BE}\left(\frac{3}{5}\right) - T_{BA} = 0$ 4 (4, 5)

$\Sigma F_y = T_{BC}\sin 30° - T_{BE}\left(\frac{4}{5}\right) = 0$ 5



$$\Sigma F_y = T_{BC}\sin 30^\circ - T_{BE}\left(\frac{4}{5}\right) = 0 \quad 5$$

5 unknowns + 5 equations = solve linear system of equations.

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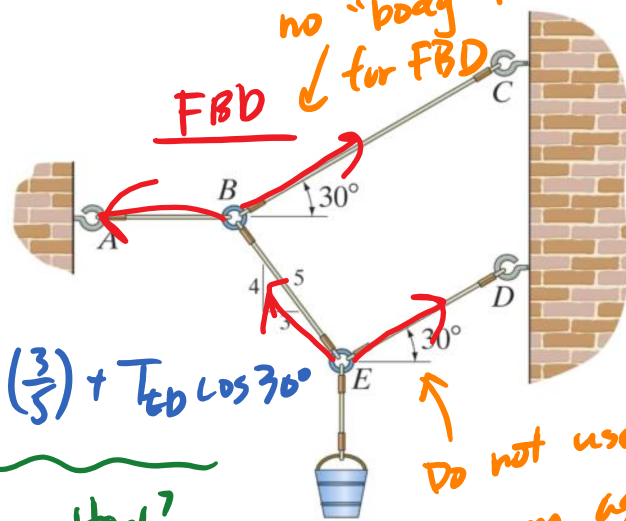
Example (BAD)

The 30-kg pipe is supported at *A* by a system of five cords. Determine the force in each cord for equilibrium.

no "body" identified for FBD

FBD



$$0 = -T_{BA} + T_{BC}\cos 30° - T_{EB}\left(\frac{3}{5}\right) + T_{ED}\cos 30°$$

How?

Do not use original diagram as FBD.

No good FBD = no good EoE

$$\boxed{T_A = 55N}$$

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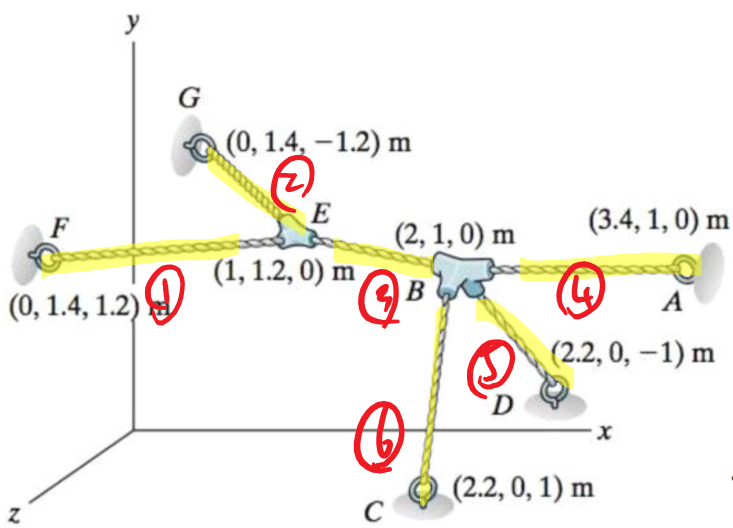
Example

Determine the tension in each cable for the system below.



How many unknowns?

→ 6 (cable tensions)

How many equations in each FBD?

→ 3 $(\Sigma F_x, \Sigma F_y, \Sigma F_z)$

→ 2 FBD needed to solve

FBD G

F_{wall} ← not a good choice since we don't need to find F_{wall}

G

T_{GE}

FBD E

T_{EG} ← better choice since all the forces here are what we want to solve.

E

T_{EF}

T_{EB}