Announcements

- Quiz 1 This Week!

- Upcoming deadlines:
  - Tuesday (9/11)
    - PL HW
  - Friday (9/14)
    - Written Assignment #2
Goals and Objectives

• Solve system of particles at equilibrium problems following general procedure for analysis.
Example

If the box weighs 2 kN, determine the angle of the cable at C when a horizontal force of 3 kN is applied at B to make the system in equilibrium.

Given parameters: \( W = 2 \text{ kN} \)
\( F_B = 3 \text{ kN} \)

Unknown parameter: \( \theta \).

**Eq. of Equilibrium**

\[
\Sigma F_x = F_B - T_x
\]
\[= F_B - T \cos \theta \]
\[= 0 \]
\[
\Sigma F_y = T_y - W
\]
\[= T \sin \theta - W \]
\[= 0 \]

\[ \rightarrow F_B = T \cos \theta \; \Box \]
\[ \rightarrow W = T \sin \theta \; \Box \]

\[
\frac{W}{F_B} = \frac{T \sin \theta}{T \cos \theta} = \cot \theta
\]

\[
\theta = \tan^{-1} \left( \frac{W}{F_B} \right) = \tan^{-1} \left( \frac{2 \text{ kN}}{3 \text{ kN}} \right) \]

\( \theta \)
Example

Determine the distances $x$ and $y$ for equilibrium if $F_1 = 800$ N and $F_2 = 1000$ N.

*Given:* $F_1 = 800$ N, $F_2 = 1000$ N

*Find:* $x, y$

$T_1 = T_2 = F_1$ since $F_1$ is applied directly on the cable, so the magnitude of $F_1$ equals the tension in the cable.

**Symmetry**

**Eq. of Equil.**

\[
\Sigma F_y = F_1 \sin \theta_1 - F_1 \sin \theta_2 = 0 \rightarrow F_1 \sin \theta_1 = F_1 \sin \theta_2 \rightarrow \theta_1 = \theta_2 = \theta
\]

\[
\Sigma F_x = -2F_1 \cos \theta + F_2 = 0 \rightarrow \cos \theta = \frac{F_2}{2F_1} \rightarrow \theta = \cos^{-1}\left(\frac{1000N}{2 \times 800N}\right)
\]

\[
\theta = 51.3^\circ \quad \tan \theta = \frac{y}{x}, \quad \frac{y}{2m} = 1.60m
\]
Equilibrium of a system of particles

Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton’s first law: \( \sum \mathbf{F} = \mathbf{0} \) on selected multiple free-body diagrams of particles or groups of particles.

The five ropes can each take 1500 N without breaking. How heavy can \( W \) be without breaking any?

- 5 unknown quantities:
  - 5 rope tension magnitude (\( T_1, T_2, T_3, T_4, \) & \( T_5 \))
  - \( \Sigma F = 2T, -T_2 = 0 \)
  - \( T_2 = 2T_1 \)
  - Need 5 EoE to solve for 5 unknowns.

\[ \begin{align*}
\Sigma F_y &= T_3 - 2T_1 = 0 \\
T_3 &= 2T_1 \\
\Sigma F_y &= T_1 - 2T_4 = 0 \\
T_1 &= 2T_4 \\
T_2 &= T_3 > T_1 = T_5 > T_4
\end{align*} \]
\[ \Sigma F_y = T_2 + T_4 - T_5 - W = 0 \]

\[ T_2 \text{ & } T_3 \text{ will break first under heavy load.} \]
Example

The 30-kg pipe is supported at A by a system of five cords. Determine the force in each cord for equilibrium.

Unkowns: 5

(T_{EB}, T_{EB}, T_{BA}, T_{BC}, T_{bucket})

Each FBD in 2D provides

2 EoE → 3 FBD needed
to solve the problem.

\[ \Sigma F_y = T_{bucket} - W = 0 \] \[ \Sigma F_y = T_{EB} \left( \frac{4}{5} \right) + T_{DE} \sin 30^\circ - T_{bucket} = 0 \] \[ \Sigma F_x = -T_{BC} \left( \frac{3}{5} \right) + T_{BE} \cos 30^\circ = 0 \] \[ \Sigma F_y = T_{BC} \cos 30^\circ + T_{BE} \left( \frac{3}{5} \right) - T_{BA} = 0 \] \[ \Sigma F_y = T_{BC} \sin 30^\circ - T_{BE} \left( \frac{4}{5} \right) = 0 \]
\[ \Sigma F_y = T_{BE} \sin 30^\circ - T_{SE} \left( \frac{4}{5} \right) = 0 \]

5 unknowns + 5 equations = solve linear system of equations.
Example \( \text{BAD} \)

The 30-kg pipe is supported at A by a system of five cords. Determine the force in each cord for equilibrium.

\[
0 = -T_{BA} + T_{BC} \cos 30° - T_{BE} \left( \frac{3}{5} \right) + T_{BD} \cos 30°
\]

\text{No good FBD = no good EOE}

\[ T_A = 55 \text{ N} \]
Example

Determine the tension in each cable for the system below.

How many unknowns?

→ 6 (cable tensions)

How many equations in each FBD?

→ 3 ($\Sigma F_x, \Sigma F_y, \Sigma F_z$)

→ 2 FBD needed to solve

FBD G

not a good choice since we don't need to find $F_{wall}$

FBD E

better choice since all the forces here are what we want to solve.