Announcements

- Quiz 1 Next Week!
- If this is your first week – check out the course website for all the logistics you need to know:
  https://courses.engr.illinois.edu/tam210

- Upcoming deadlines:
  - Friday (9/7 – TODAY!)
    - Written Assignment #1
  - Tuesday (9/11)
    - PL HW
  - Friday (9/14)
    - Written Assignment #2

1 L5 - Force along a line Cross product
Chapter 3: Equilibrium of a particle

If geometry included

\[ \Sigma F = 0 \text{ rotation would occur} \]

\[ \Sigma F = 0 \Rightarrow \text{no translation} \]

- no geometry

\[ \text{NO} \]

\[ \Rightarrow \Sigma F = 0 \]
Goals and Objectives

• Practice following general procedure for analysis.

• Introduce the concept of a free-body diagram for an object modeled as a particle.

• Solve particle equilibrium problems using the equations of equilibrium.
Applications

For a spool of given weight, how would you find the forces in cables AB and AC?

If designing a spreader bar (BC) like this one, you need to know the forces to make sure the rigging (A) doesn’t fail.
General procedure for analysis

1. Read the problem carefully; write it down carefully.

2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.

3. Apply principles needed.

4. Solve problem symbolically. Make sure equations are dimensionally homogeneous

5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).

6. See if answer is reasonable.
Free body diagram

Account for all the forces involved in the problem.

- The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables AB and AC as a function of θ.

* Be strategic about choosing the right “body” for the problem
* Only include external forces acting on the “body” in the diagram
* Include geometry & coordinate system
Idealizations

Pulleys are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side.

Frictionless pulley

Other pulley assumptions
- massless
- particle
(unless stated otherwise)
Idealizations

Springs are (usually) regarded as linearly elastic; then the tension is proportional to the change in length $s$.

$$F = ks = k(l-l_0)$$

Linearly elastic spring
Idealizations

Contact force in smooth surface:

Force (normal) will always be perpendicular to the surface.

L5 - Force along a line Cross product
Free Body Diagram Example

- Assume the box is a particle.

- $k = 200 \text{ N/m}$

- 1 unknown (magnitude)
- 2 unknowns (magnitude & direction)
- Given by particle assumption
- 1 unknown (magnitude)

- FBD
- 45°

- a "slanted" coordinate system is preferred
Free Body Diagram Example

Note: Cables have negligible mass assumption is usually applied for this course unless specified otherwise.

* Do NOT include internal forces.
Equilibrium of a particle

According to Newton’s first law of motion, a particle will be in equilibrium (that is, it will remain at rest or continue to move with constant velocity) if and only if

\[ \sum F = 0 \]

In three dimensions, equilibrium requires:

\[ \sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0 \]

**Coplanar forces:** if all forces are acting in a single plane, such as the “xy” plane, then the equilibrium condition becomes (20)

\[ \sum F_x = 0 \quad \sum F_y = 0 \]
Example

If the spring $DB$ has an unstretched length of 2 m, determine the stiffness of the spring to hold the 40-kg crate in the position shown.

Given: $l_0 = 2$ m, $l = \sqrt{13}$ m

$m = 40$ kg

Find: $k$

\[ \Sigma F_x = 0 = -T_x + F_s \cos \theta \]

\[ \sigma = -T \left( \frac{\sqrt{3}}{2} \right) + t_s \left( \frac{3}{\sqrt{3}} \right) = 0 \]

\[ \rightarrow T = t_s \left( \frac{3}{\sqrt{3}} \frac{\sqrt{3}}{2} \right) = t_s \left( \frac{6}{2\sqrt{3}} \right) = 0 \]

\[ \Sigma F_y = 0 = T_y + F_{sy} - W \]

\[ = -T \sin 45^\circ + t_s \sin \theta - W \]
or \[ T \left( \frac{L}{2} \right) + F_s \left( \frac{2}{\sqrt{3}} \right) - W = 0 \] \[ \tag{2} \]

Substitute (1) \rightarrow (2)

\[ F_s \left( \frac{6}{\sqrt{13}} \right) + F_s \left( \frac{2}{\sqrt{13}} \right) - W = 0 \]

\[ F_s \left( \frac{3}{\sqrt{13}} + \frac{2}{\sqrt{13}} \right) = W \rightarrow F_s = \frac{W}{\left( \frac{5}{\sqrt{13}} \right)} \] \[ \tag{3} \]

Apply linear spring assumption: \( F_s = k_s \).

\[ s = l - l_0 = \sqrt{13} m - 2 m \]

\[ F_s = k \left( \sqrt{13} m - 2 m \right) \] \[ \tag{4} \]

Substitute (4) \rightarrow (3)

\[ k \left( \sqrt{13} m - 2 m \right) = W \left( \frac{\sqrt{13}}{5} \right) \]

\[ k = \frac{W \sqrt{13}}{5 \left( \sqrt{13} m - 2 m \right)} \]

Substitute in numbers

\[ k = \frac{(40 \text{ kg})(9.81 \text{ m/s}^2) \sqrt{13}}{5 \left( \sqrt{13} - 2 \right) m} \]

\[ k = 176 \text{ N/m} \]