Lecture 04 (09/05/18)

Announcements

- Extra notes and example available on Compass2g
- Sign-up for Quiz 1
  - Practice quiz available on PL (format only)
    - lectures 1 ~ 5

Eat Me

- Upcoming deadlines:
- Thursday (9/6)
  - Prairie Learn HW1
- Friday (9/7)
  - Written Assignment 1

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Lecture Objectives

- Vector Projection
- Vector Cross Product
- Engineering Idealization
- General Procedure of Analysis

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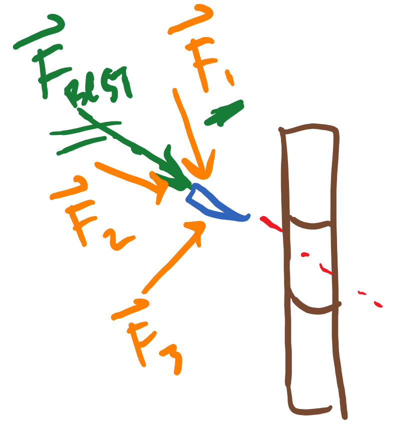
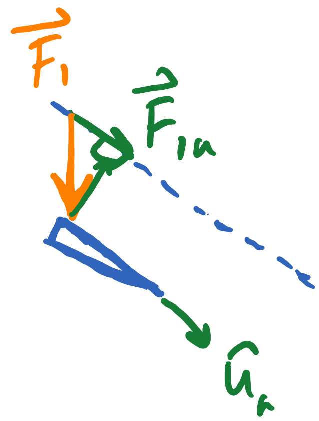
Vector Projections

https://www.youtube.com/watch?v=pP92q68KG24



$$\text{proj}(\vec{F}_1, \hat{u}_a) = \underbrace{(\vec{F}_1 \cdot \hat{u}_a)}_{\text{mag.}} \underbrace{\hat{u}_a}_{\text{dir.}} = \vec{F}_{1a}$$



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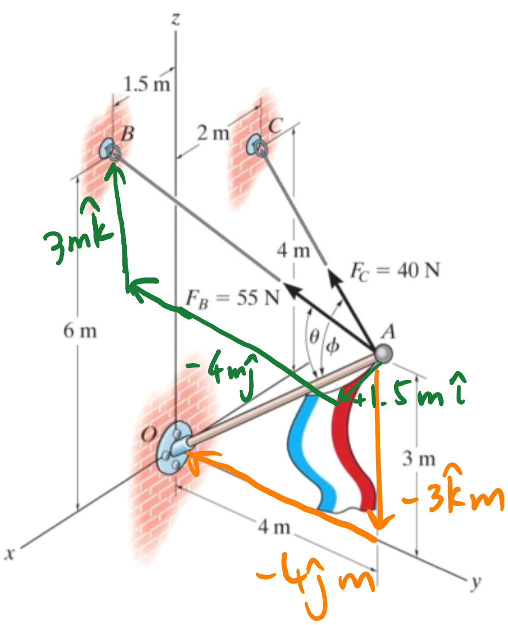
Example

a) Determine the angle between *AB* and the flag pole.



$$\vec{r}_{AB} = 1.5\hat{i} - 4\hat{j} + 3\hat{k} \text{ m} \quad (A_x,\ A_y,\ A_z)$$

$$\vec{r}_{AO} = -4\hat{j} - 3\hat{k} \text{ m} \quad (B_y,\ B_z)$$

$$\theta = \cos^{-1}\left(\frac{\vec{r}_{AB} \cdot \vec{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{1.5(0) + (-4)(-4) + (3)(-3)}{\sqrt{1.5^2+4^2+3^2} \cdot \sqrt{4^2+3^2}}\right) \frac{\cancel{m}}{\cancel{m}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{7}{26.1}\right) \approx 74.4^\circ}$$

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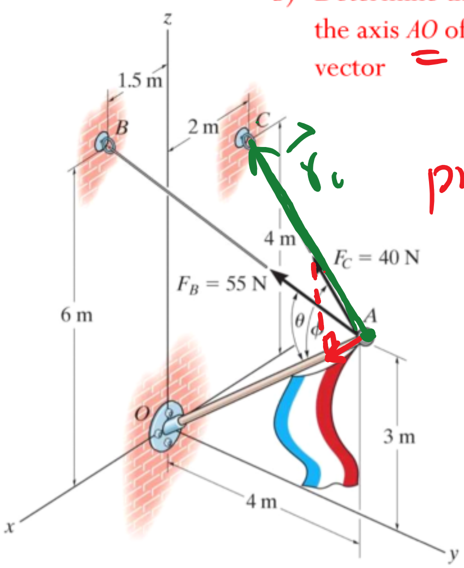
Example

a) Determine the angle between AB and the flag pole.

b) Determine the projected component of the force vector F_C along the axis AO of the flag pole. Express your result as a Cartesian vector



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$$proj(\vec{F}_c, \vec{r}_{AO}) = (\vec{F}_c \cdot \hat{u}_{AO}) \cdot \hat{u}_{AO} = \vec{F}_{c\parallel}$$

(1) $\vec{F}_c$, (2) $\hat{u}_{AO}$; $(\vec{F}_c \cdot \hat{u}_{AO})$: magnitude; $\hat{u}_{AO}$: direction

$$\vec{F}_c = F_c \hat{u}_c \qquad \hat{u}_c = \frac{\vec{r}_c}{r_c}, \quad \vec{r}_c = \vec{C} - \vec{A}$$

$$\vec{C} = -2\hat{i} + 4\hat{k} \text{ m}, \quad \vec{A} = 4\hat{j} + 3\hat{k} \text{ m}$$

$$\vec{r}_c = (-2-0)\hat{i} + (0-4)\hat{j} + (4-3)\hat{k} \text{ m} = -2\hat{i} - 4\hat{j} + \hat{k} \text{ m}$$

$$r_c = \sqrt{2^2+4^2+1^2} \text{ m} = \sqrt{21} \text{ m}$$

(1) $\vec{F}_c = F_c \hat{u}_c = 40N\left(\frac{-2}{\sqrt{21}}\hat{i} + \frac{-4}{\sqrt{21}}\hat{j} + \frac{1}{\sqrt{21}}\hat{k}\right)$

$$\vec{u}_{AO} = \frac{\vec{r}_{AO}}{r_{AO}} = \frac{-4\hat{j} - 3\hat{k} \text{ m}}{\sqrt{4^2+3^2} \text{ m}} = -\frac{4}{5}\hat{j} - \frac{3}{5}\hat{k} \quad (2)$$

$$\vec{F}_{c\parallel} = (\vec{F}_c \cdot \hat{u}_{AO})\hat{u}_{AO} = \left[\left(\frac{-80}{\sqrt{21}} \cdot 0\right) + \left(\frac{-160}{\sqrt{21}} \cdot \frac{-4}{5}\right) + \left(\frac{40}{\sqrt{21}} \cdot \frac{-3}{5}\right)\right]\left(-\frac{4}{5}\hat{j} - \frac{3}{5}\hat{k}\right) N$$

$$= \frac{104}{\sqrt{21}}\left(-\frac{4}{5}\hat{j} - \frac{3}{5}\hat{k}\right)N = \boxed{\frac{-416}{5\sqrt{21}}\hat{j} - \frac{312}{5\sqrt{21}}\hat{k} \text{ N} = \vec{F}_{c\parallel}}$$

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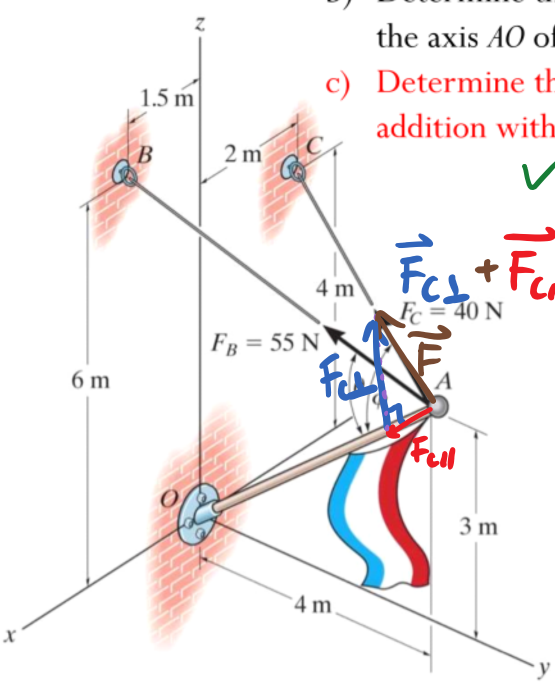
Example

a) Determine the angle between *AB* and the flag pole.

b) Determine the projected component of the force vector $\boldsymbol{F}_C$ along the axis *AO* of the flag pole.

c) Determine the perpendicular component such at its vector addition with the projected component equals $\boldsymbol{F}_C$.



$$\vec{F}_{C\perp} + \vec{F}_{C\parallel} = \vec{F}_C \longrightarrow \vec{F}_{C\perp} = \vec{F}_C - \vec{F}_{C\parallel}$$

$$\vec{F}_{C\perp} = \left(-\frac{80}{\sqrt{21}}\hat{i} - \frac{160}{\sqrt{21}}\hat{j} + \frac{40}{\sqrt{21}}\hat{k}\right)\text{N} - \left(-\frac{416}{5\sqrt{21}}\hat{j} - \frac{312}{5\sqrt{21}}\hat{k}\right)\text{N}$$

$$\boxed{\vec{F}_{C\perp} = \left(-\frac{80}{\sqrt{21}}\hat{i} - \frac{384}{5\sqrt{21}}\hat{j} + \frac{512}{5\sqrt{21}}\hat{k}\right)\text{N}}$$

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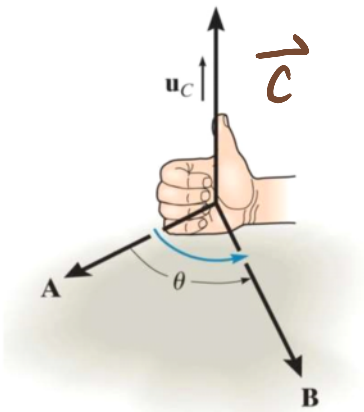
Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written



$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = C\hat{u}_c$$

The magnitude of vector **C** is given by:

$$C = AB\sin\theta$$

The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

$$\vec{C} \perp \vec{A} \;\&\; \vec{C} \perp \vec{B}$$

$$\vec{C} = (AB\sin\theta)\,\hat{u}_c$$

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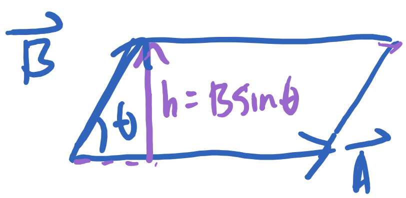

$$\text{Area} = Ah = AB\sin\theta$$

$$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$$

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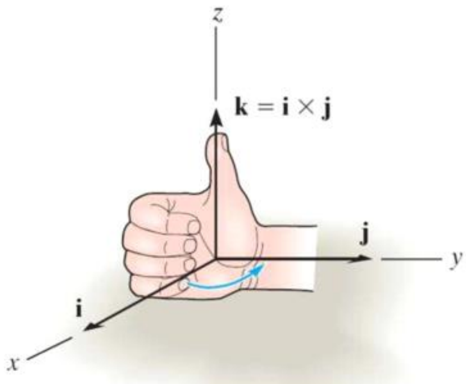
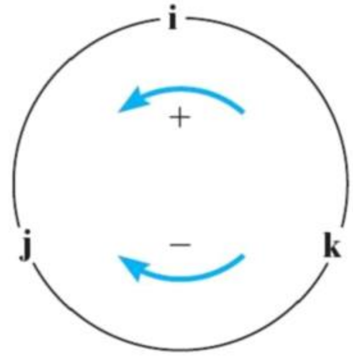
Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$

$\theta = 0°$

$\sin 0° = 0$





Considering the cross product in Cartesian coordinates

$$\begin{aligned}
\boldsymbol{A} \times \boldsymbol{B} &= (A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}) \times (B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k}) \\
&= +A_x B_x (\boldsymbol{i} \times \boldsymbol{i}) + A_x B_y (\boldsymbol{i} \times \boldsymbol{j}) + A_x B_z (\boldsymbol{i} \times \boldsymbol{k}) \\
&\quad +A_y B_x (\boldsymbol{j} \times \boldsymbol{i}) + A_y B_y (\boldsymbol{j} \times \boldsymbol{j}) + A_y B_z (\boldsymbol{j} \times \boldsymbol{k}) \\
&\quad +A_z B_x (\boldsymbol{k} \times \boldsymbol{i}) + A_z B_y (\boldsymbol{k} \times \boldsymbol{j}) + A_z B_z (\boldsymbol{k} \times \boldsymbol{k}) \\
&= (A_y B_z - A_z B_y) \boldsymbol{i} - (A_x B_z - A_z B_x) \boldsymbol{j} + (A_x B_y - A_y B_x) \boldsymbol{k}
\end{aligned}$$

+k

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Cross (or vector) product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb.$$

Each component can be determined using 2 × 2 determinants.

$$\hat{i}: \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \hat{j}: \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \hat{k}: \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$(A_yB_z - B_yA_z)\hat{i} - (A_xB_z - B_xA_z)\hat{j} + (A_xB_y - B_xA_y)\hat{k} = \vec{A} \times \vec{B}$$

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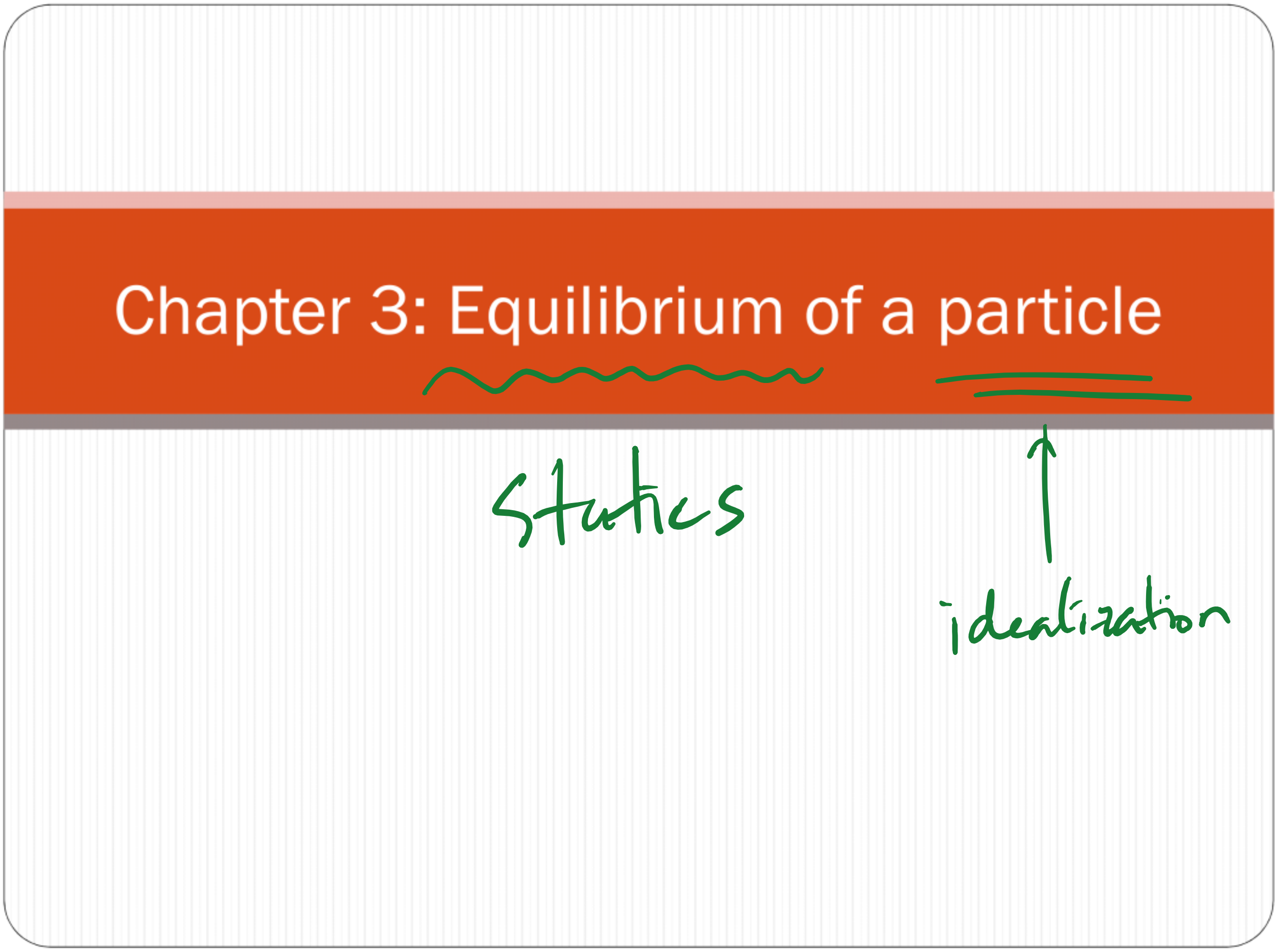

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Fundamental concepts

Idealizations:

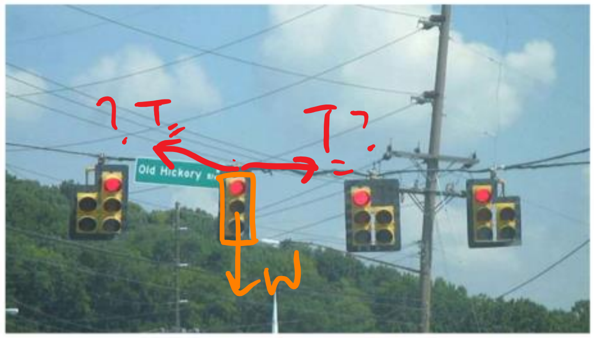
- Particle:

  may or may not have mass, & neglect size (geometry)

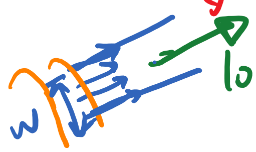
- Rigid Body:

  • no deformation

  • a system of Particles with fixed distance from each other {geometry matters}



- Concentrated Force:



loading assume to be acting on a point.

Understanding and applying these things allows for amazing achievements in engineering!

General procedure for analysis

1. Read the problem carefully; write it down carefully.

2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.

3. Apply principles needed.

4. Solve problem symbolically. Make sure equations are dimensionally homogeneous.

5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).

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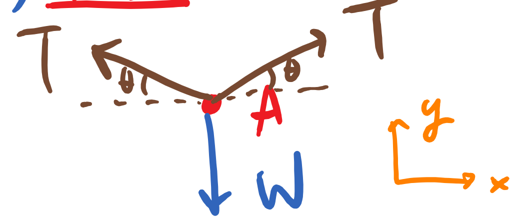
Example: Determine the required strengths of the cables holding up a traffic light with $W = 200\ \text{lb}$, $\theta = 10°$.

1.) Given: W

Find: T

2.) FBD



3.) Governing Eqn: Eq. of Equilibrium

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

4) $$\left.\begin{aligned} \Sigma F_x &= -T\cos\theta + T\cos\theta = 0 \\ \Sigma F_y &= 2T\sin\theta - W = 0 \end{aligned}\right\} \longrightarrow T = \frac{W}{2\sin\theta}$$

5) $$T = \frac{W}{2\sin\theta} = \frac{200\ \text{lb}}{2\sin 10^\circ} \approx 576\ \text{lb}.$$

$$\boxed{T = 576\ \text{lb}}$$