## Announcements

- CBTF Quiz 6 next week
- 211 students DO NOT take 210 final, or you will get a zero on 211 final!!!
$\square$ Upcoming deadlines:
- Tuesday (11/14)
- PL HW22
- Thursday (11/16)
- ME HW23



## Center of Gravity Application

The suspended sign is a homogeneous flat plate that has a mass of 130 kg . Determine the support reaction at $B$.


## Example



Where is the centroid of the area?

## Example



Where is the centroid of the bar?

## Example



## Composite bodies

A composite body consists of a series of connected simpler shaped bodies.
Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.


## Centroid of typical 2D shapes

| Shape | Figure | $\bar{x}$ | $\bar{y}$ | Area |
| :---: | :---: | :---: | :---: | :---: |
| Right-triangular area |  | $\frac{b}{3}$ | $\frac{h}{3}$ | $\frac{b h}{2}$ |
| Quarter-circular area |  | $\frac{4 r}{3 \pi}$ | $\frac{4 r}{3 \pi}$ | $\frac{\pi r^{2}}{4}$ |
| Semicircular area | $\frac{+}{4}$ | 0 | $\frac{4 r}{3 \pi}$ | $\frac{\pi r^{2}}{2}$ |
| Quarter-elliptical area |  | $\frac{4 a}{3 \pi}$ | $\frac{4 b}{3 \pi}$ | $\frac{\pi a b}{4}$ |
| Semielliptical area |  | 0 | $\frac{4 b}{3 \pi}$ | $\frac{\pi a b}{2}$ |

## Composite bodies - Analysis Procedure

1. Divide the body into finite number of simple shapes
2. Consider "holes" as "negative" parts
3. Establish coordinate axes
4. Determine centroid location by applying the equations

$$
\begin{array}{ll}
\bar{x}=\frac{\sum \tilde{x} W}{\sum W} & \bar{x}=\frac{\sum \tilde{x} A}{\sum A} \\
\bar{y}=\frac{\sum \tilde{y} W}{\sum W} & \bar{y}=\frac{\sum \tilde{y} A}{\sum A} \\
\bar{z}=\frac{\sum \tilde{z} W}{\sum W} & \bar{z}=\frac{\sum \tilde{z} A}{\sum A}
\end{array}
$$



## Example



What is the centroid of the resultant area?

## Example

Locate the centroid of the cross section area. ${ }^{y}$



The truss is made from five members, each having a length of 4 m and a mass of 7 kg / m . Determine the distance $d$ to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.

Determine the location of the center of gravity of the three-wheeler. If the three-wheeler is symmetrical with respect to the $\mathrm{x}-\mathrm{y}$ plane, determine the normal reaction each of its wheels exerts on the ground.


## Second moment of area - Applications



Many structural members like beams and columns have cross sectional shapes like an I, H, C, etc..

Why do they usually not have solid rectangular, square, or circular cross sectional areas?

What primary property of these members influences design decisions?

## Second moment of area

Moment of inertia is the property of a deformable body that determines the moment needed to obtain a desired curvature about an axis.

Moment of inertia depends on the shape of the body and may be different around different axes of rotation.

- The moment of inertia of the area A with respect to the x -axis is given by

- The moment of inertia of the area A with respect to the y -axis is given by


