



Announcements

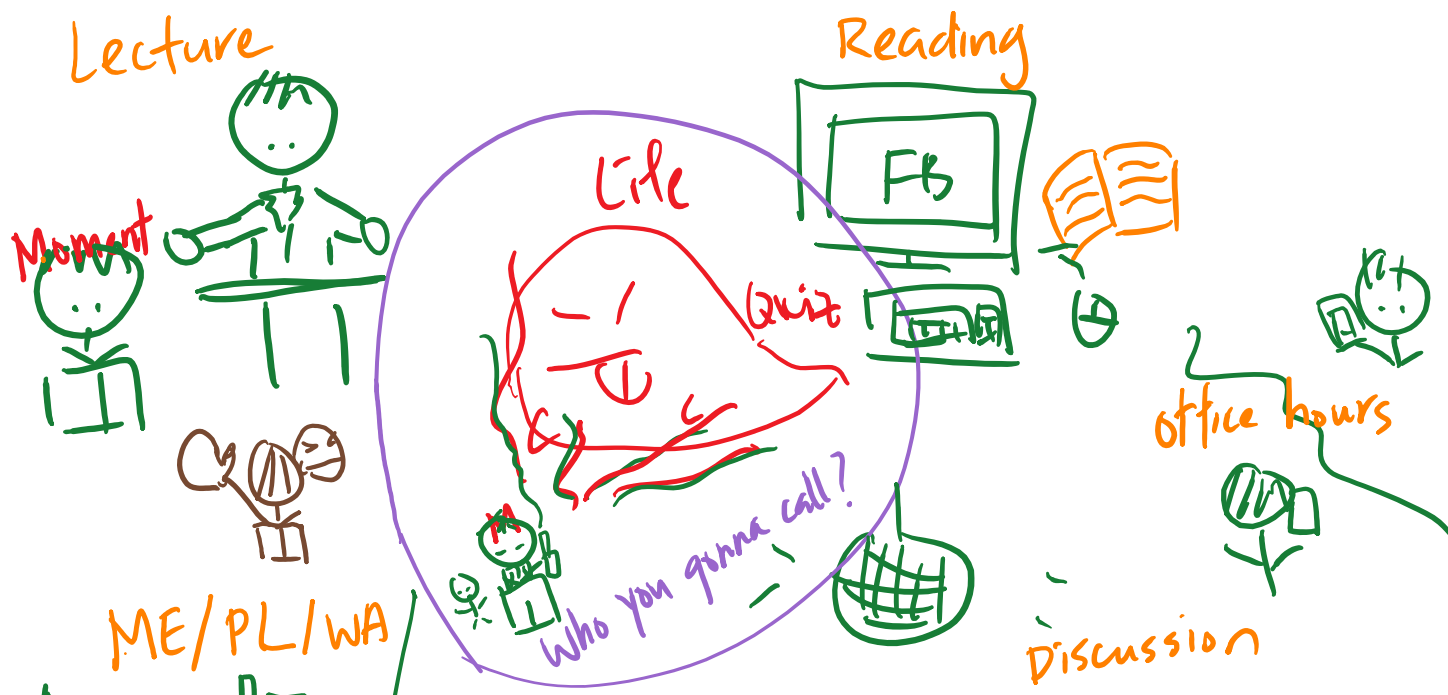
- No quiz next week ☺
- Have you been on Piazza lately?

□ Upcoming deadlines:

- Thursday (9/21)
 - ME HW7
- Friday (9/22)
 - Writing Assignment 1 (FBD only)
- Tuesday (9/26)
 - PL HW8



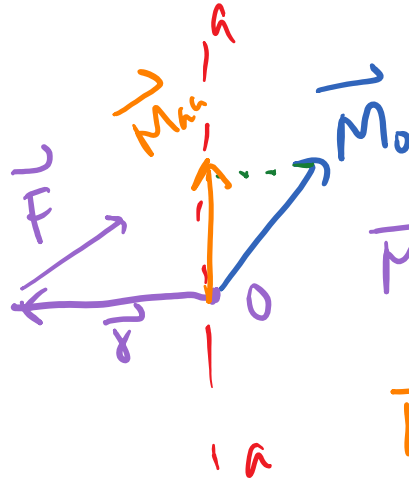
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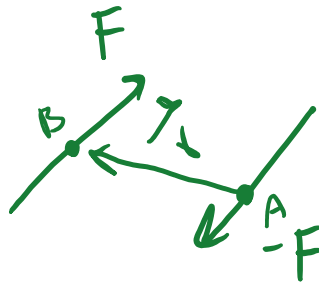
Recap

- Moment of a force
- About a point
- About an axis
- Couple moment



$$\vec{M}_O = \vec{r} \times \vec{F}$$

$$\vec{M}_a = \hat{u}_a \cdot (\vec{r} \times \vec{F})$$

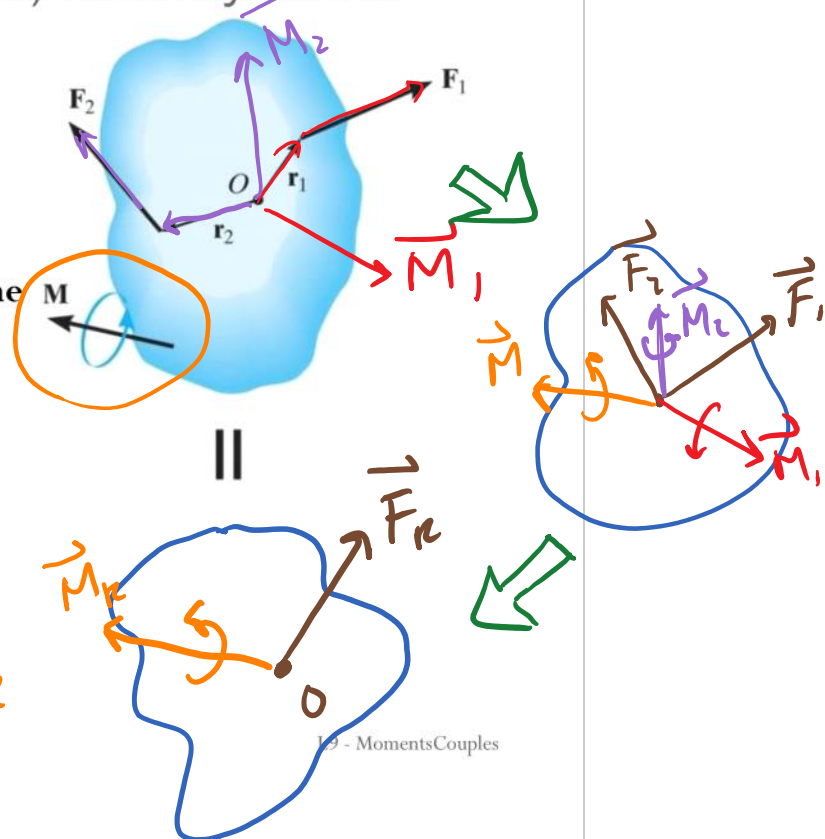


$$\vec{M}_F = \vec{r}_{AB} \times \vec{F}$$

Equipollent (or equivalent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point P .

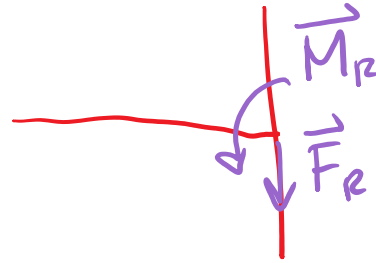
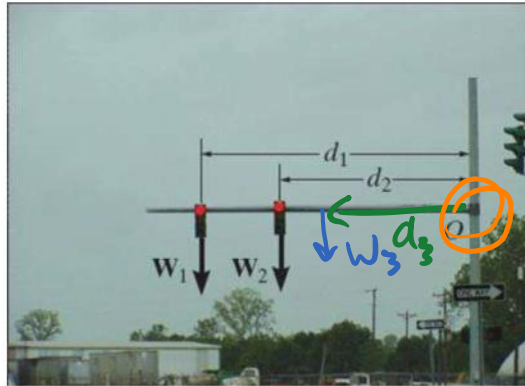


$$\left\{ \begin{array}{l} \sum \vec{F} = \vec{F}_R \\ \sum \vec{M}_c + \sum \vec{M}_o = \vec{M}_R \end{array} \right.$$

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19 - Moments/Couples

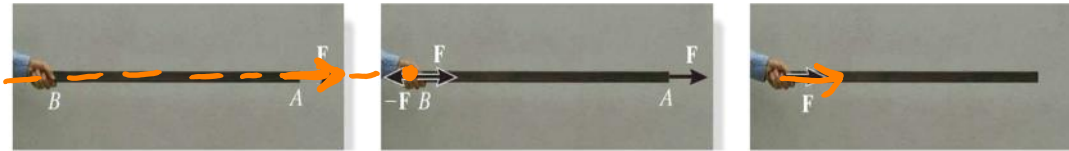
What is the equivalent system?



$$\vec{F}_R = \vec{W}_1 + \vec{W}_2 + \vec{W}_3$$

$$\vec{M}_R = (W_1 d_1 + W_2 d_2 + W_3 d_3) (+\hat{k})$$

Moving a force on its line of action



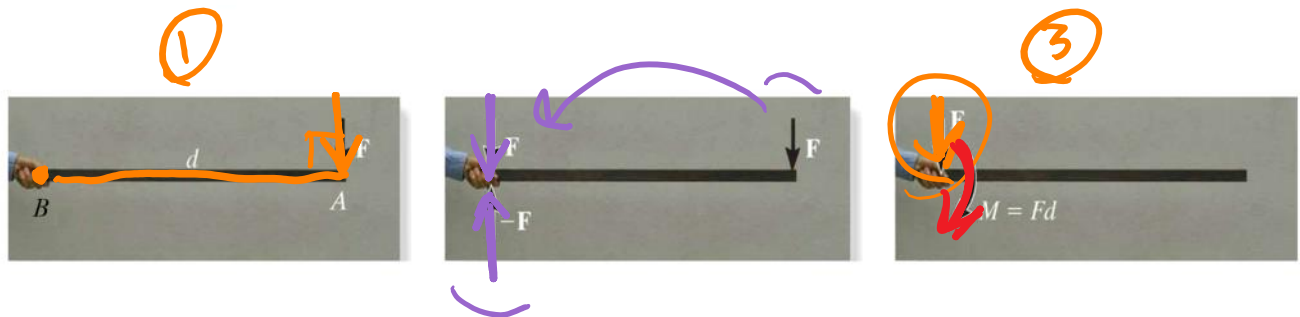
Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

Hence, a force vector is called a **sliding vector**.

However, the **internal effect** of the force on the body does depend on where the force is applied.



Moving a force off of its line of action



$$M = Fd$$

or

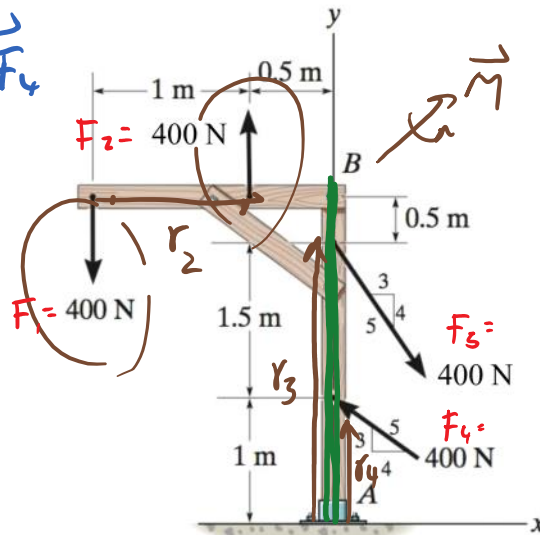
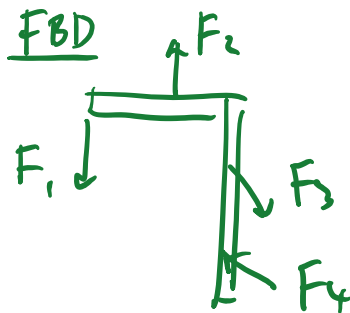
$$\vec{M} = \vec{r} \times \vec{F}$$

Example – 2D Equivalent System

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.

Given: $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$

Find: \vec{F}_R, d



$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \quad \left(\begin{aligned} F_{Rx} &= F_{1x} + F_{2x} + F_{3x} + F_{4x} \\ F_{Ry} &= F_{1y} + F_{2y} + F_{3y} + F_{4y} \end{aligned} \right)$$

$$\vec{M}_R = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 + \vec{r}_4 \times \vec{F}_4$$

$$\vec{r}_1 \times \vec{F}_2 = Fd(+\hat{k})$$

We want $\vec{M}_R = \vec{r}_R \times \vec{F}_R$

$$M_R = d F_{Rx} \Rightarrow d = \frac{M_R}{F_{Rx}}$$

$$F_{Rx} = F_3 \left(\frac{3}{5} \right) - F_4 \left(\frac{4}{5} \right) = -80 \text{ N} \quad \left| \vec{F}_R = -80\hat{i} - 80\hat{j} \text{ N} \right|$$

$$F_{Rx} = -F_3\left(\frac{4}{5}\right) + F_4\left(\frac{3}{5}\right) = -80\text{ N}$$

$$M_R = r_2 F_2 - r_3 F_3x + r_4 F_4x$$

$$= (1\text{ m})(400\text{ N}) - (2.5\text{ m})(400\text{ N}) + (1\text{ m})(400\text{ N})$$

$$\vec{M}_R = -200\text{ N}\cdot\text{m} \hat{k}$$

$$d = \frac{M_R}{F_{Rx}} = \frac{200\text{ N}\cdot\text{m}}{80\text{ N}}$$

$$d = 2.5\text{ m}$$