

To do ...

- Quiz 7 – good luck!
 - Last day of office hours and piazza help: **Wed, Dec 13**
 - No discussion sections next week
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- HW 27 ME due **Sat – Last one!!**
 - HW 26 PL due **Tues (Dec 12) – Last one!!**

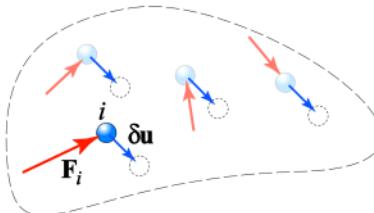
Virtual Displacements

A **virtual displacement** is a conceptually possible **displacement or rotation** of all or part of a system of particles. The movement is assumed to be possible, but actually does not exist.

Principle of Virtual Work

The **principle of virtual work** states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,

$$\delta U = \mathbf{F} \cdot \delta \mathbf{x} + M \delta \theta = 0$$

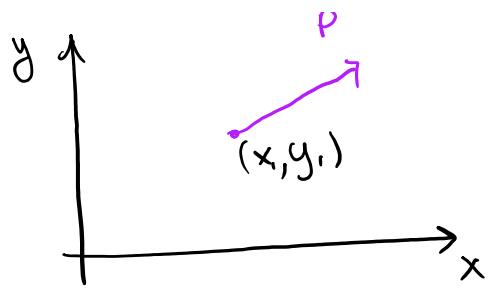


- Proposed by Swiss Mathematician Jean Bernoulli in 18th century.
- Effective for solving equilibrium problems of several connected rigid bodies
- do not consider internal forces since rigid bodies do not deform
- we consider problems with single degree of freedom!

Consider the force $\vec{P} = [P_x, P_y]$

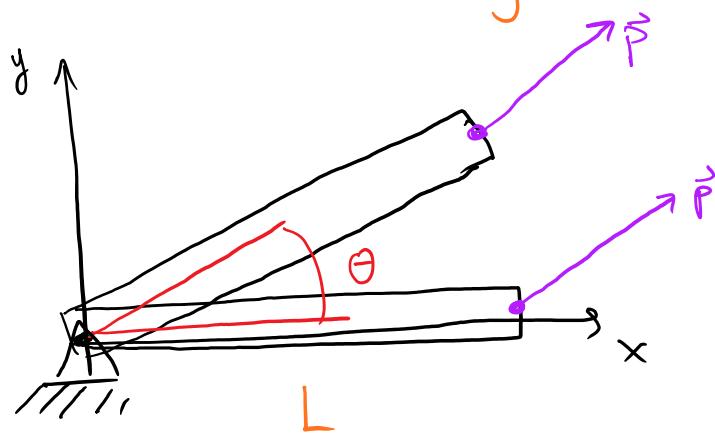


2 degrees of freedom needed



2 degrees of freedom needed
to describe location of \vec{P}

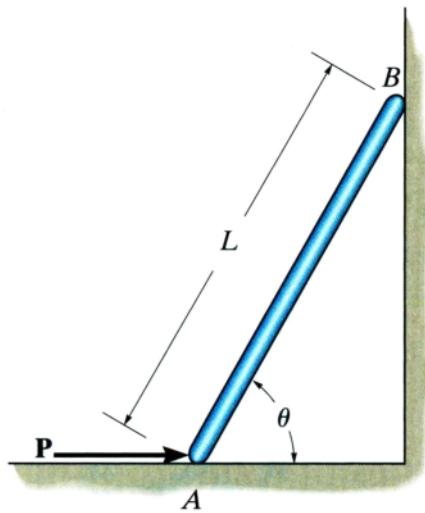
NOW CONSIDER A beam of length L AND force $\vec{P} = [P_x, P_y]$



- single degree of freedom θ will
describe the location

$$(x, y) = (L \cos \theta, L \sin \theta)$$

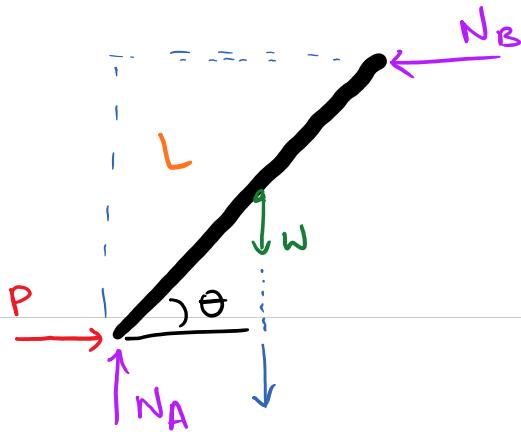
of \vec{P}



The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.

* Draw FBD

* EOE



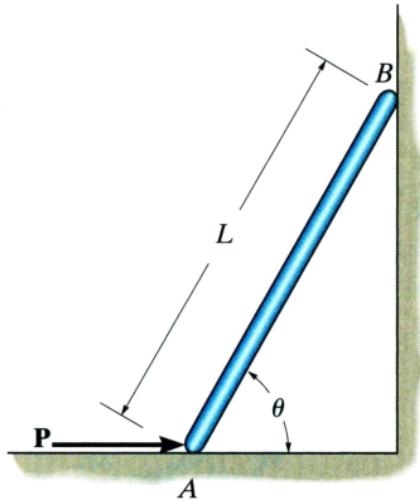
$$\sum F_x: P - N_B = 0 \quad P = N_B$$

$$\sum F_y: N_A - W = 0$$

$$\sum M_A: (L \sin \theta) N_B - \left(\frac{L}{2} \cos \theta \right) W = 0$$

$$PL \sin \theta = \frac{L}{2} W \cos \theta$$

$$P = \frac{W}{2} \frac{1}{\tan \theta}$$

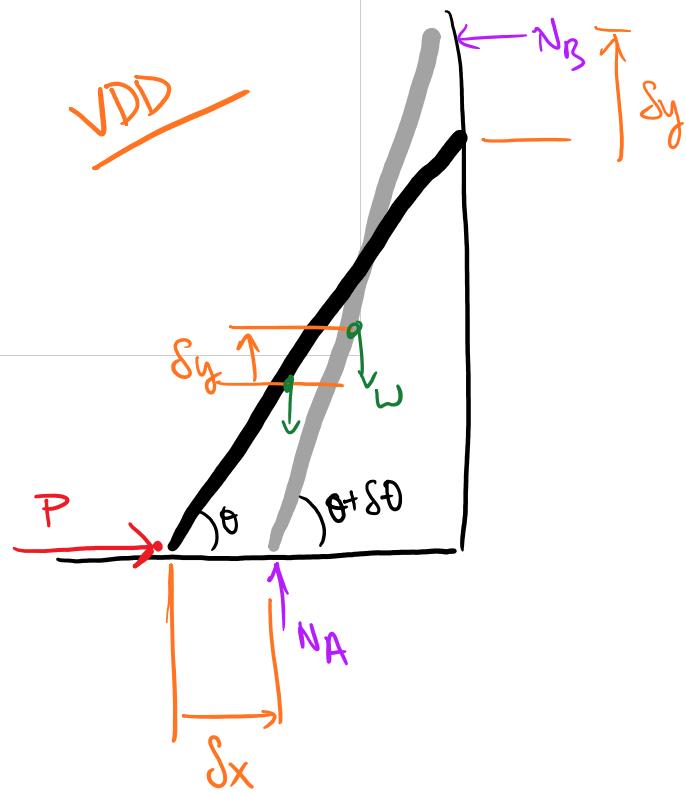
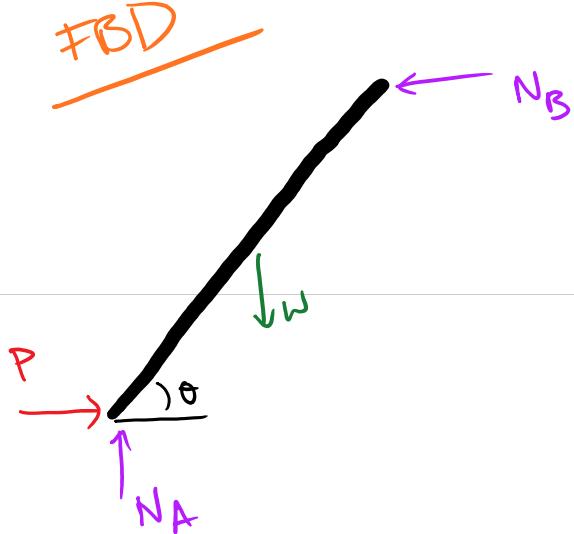


The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.

* DRAW FBD

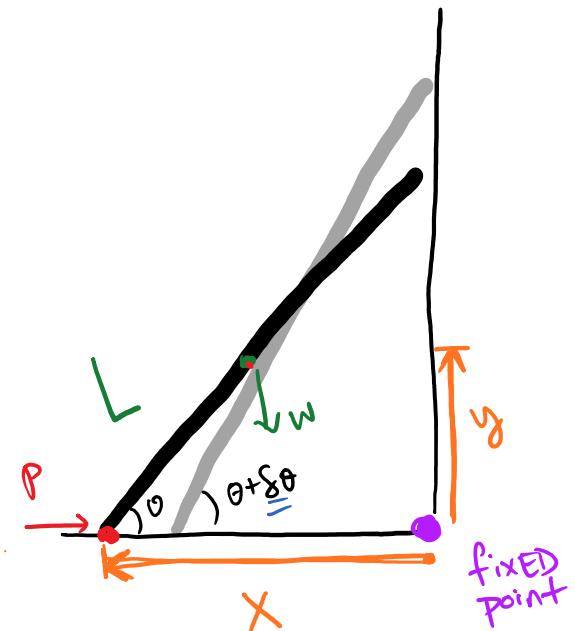
* label Forces

* Virtual Displacement diagram



Active forces! $\rightarrow P, W =$

Coordinates from fixed pt.



to Active forces.

$$x = L \cos \theta$$

$$\delta x = -L \sin \theta \delta \theta$$

$$y = \frac{L}{2} \sin \theta$$

$$\delta y = \frac{L}{2} \cos \theta \delta \theta$$

Virtual Work eqn.

$$\delta U = -P \delta x - W \delta y = 0$$

$$-P(-L \sin \theta \delta \theta) - W\left(\frac{L}{2} \cos \theta \delta \theta\right) = 0$$

$$PL \sin \theta \delta \theta - \frac{WL}{2} \cos \theta \delta \theta = 0$$

$$(P \sin \theta - \frac{W}{2} \cos \theta)L \delta \theta = 0$$

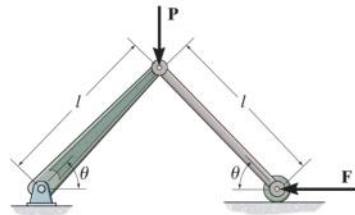
solve this term = 0

solve this term = 0

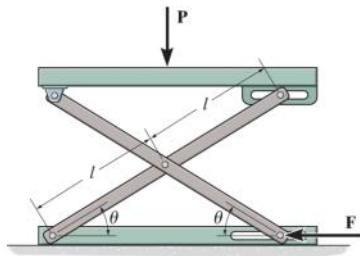
$$P \sin \theta - \frac{W}{2} \cos \theta = 0$$

$$\boxed{P = \frac{W}{2} \frac{\cos \theta}{\sin \theta} = \frac{W}{2} \frac{1}{\tan \theta}}$$

So what's the point....



* systems will single degree of freedom θ



* many equilibrium positions ; many connected members

Important points and procedure:

- force does work when displacement occurs in direction of force
- virtual displacement is imaginary!
↳ displacement is in the positive direction of a position coordinate

PROCEDURE:

- DRAW FBD ; define coordinate
- Sketch deflected position for positive

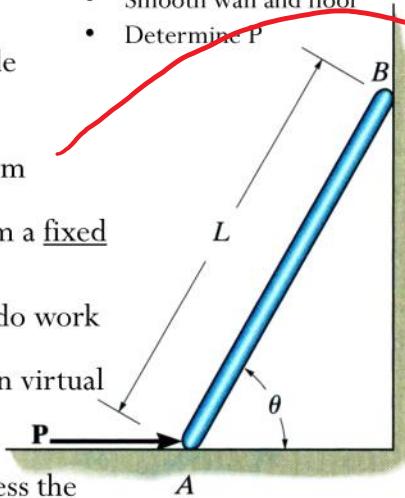
virtual displacement

- Indicate position coordinates from a fixed point for **Active forces** only
- coordinate Axes should be parallel to the line of action of the force
- Relation each position to the single degree of freedom
 - ↳ take partial derivative
- Write virtual work equation, determine correct sign (+/- work)
- Solve!

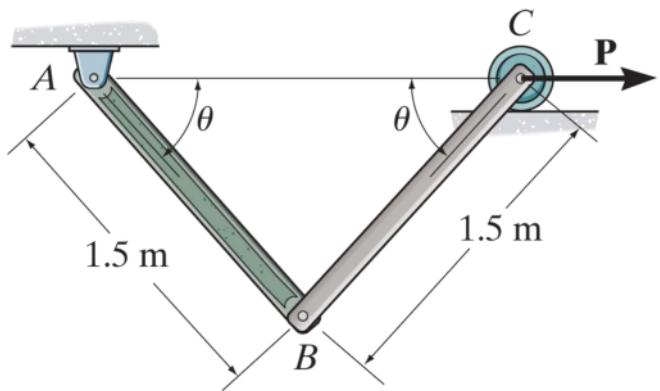
Procedure for Analysis

1. Draw FBD of the entire system and provide coordinate system
2. Sketch the “deflected position” of the system
3. Define position coordinates measured from a fixed point and select the parallel line of action component and remove forces that do no do work
4. Differentiate position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/ couple moment
6. Factor out the comment virtual displacement term and solve

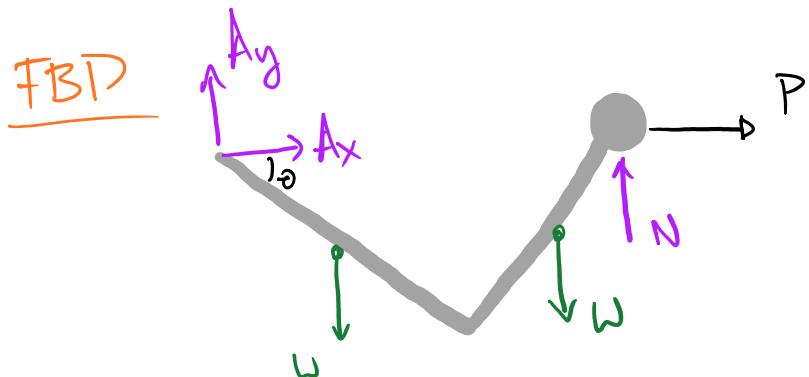
- Thin rod of weight W
- Smooth wall and floor
- Determine P



- USE $+δθ$
- find active forces



Determine the required magnitude of force P to maintain equilibrium of the linkage at an angle of 60 degrees. Each member has a mass of 20 kg.

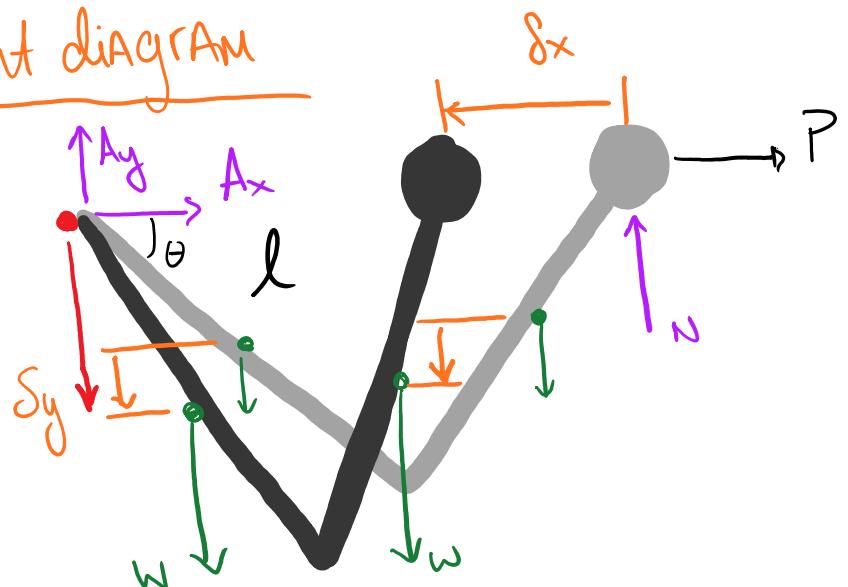


virtual displacement diagram

* Consider $+ \delta\theta$

* fixed point

* 3 active forces!



coordinates:

1

$$x = 2l \cos \theta$$

$$y = \frac{k}{2} \sin \theta$$

$$\delta x = -2l \sin \theta \delta \theta$$

$$\delta y = \frac{l}{2} \cos \theta \delta \theta$$

Virtual Work eqn.

$$\delta U = P \delta x + W_1 \delta y + W_2 \delta y = 0$$

$$P(-2l \sin \theta \delta \theta) + 2W\left(\frac{l}{2} \cos \theta \delta \theta\right) = 0$$

$$(-2P \sin \theta + W \cos \theta) l \delta \theta = 0$$

Solve this = 0

$$-2P \sin \theta + W \cos \theta$$

$$P = \frac{W}{2} \frac{\cos \theta}{\sin \theta}$$