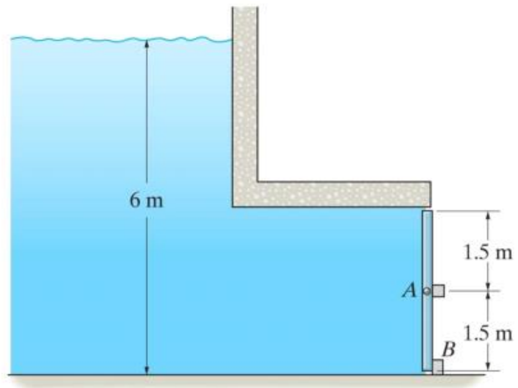
Lecture 36

Monday, December 4, 2017 18:16

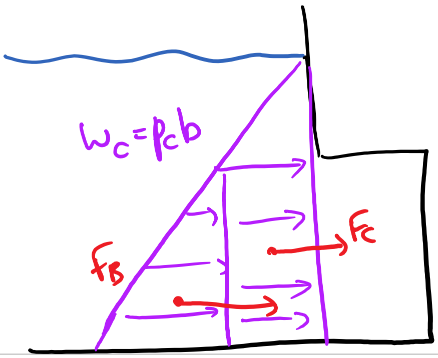
To do ...

- Quiz 7 next week
- Last day of office hours and piazza help: **Wed, Dec 13**
- No discussion sections next week

- HW 24 PL due **Tues**
- HW 27 ME due **Sat**



The 2-m-wide rectangular gate is pinned at its center A and is prevented from rotating by the block at B. Determine the reactions at these supports due to hydrostatic pressure.



$w_B = p_B b$

$$F_c = w_c h = p_c b h = \rho g b h z_c$$

$$F_B = \frac{1}{2}(w_B - w_c)h = \frac{1}{2}\rho g b h (z_B - z_c)$$

$$F_R = F_c + F_B = \rho g b h \left(\frac{z_B + z_c}{2}\right)$$

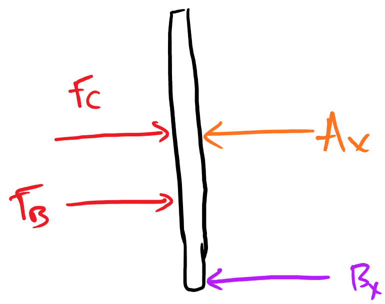
Sum the moment About A.

$$\Sigma M_A: \quad F_B(0.5) - B_x(1.5) = 0$$

$$\frac{3}{2} B_x = \frac{1}{2} F_B \quad \Rightarrow \quad \underline{B_x = \frac{1}{3} F_B}$$

$\underline{29.4 \text{ kN}}$

Sum the forces in x



$$\Sigma F_x: \quad F_C + F_B - A_x - B_x = 0$$

$$A_x = F_C + F_B - B_x = \underline{235\ \text{kN}}$$

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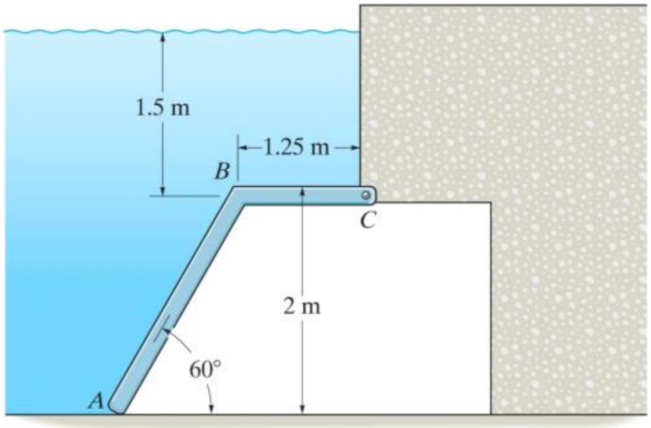
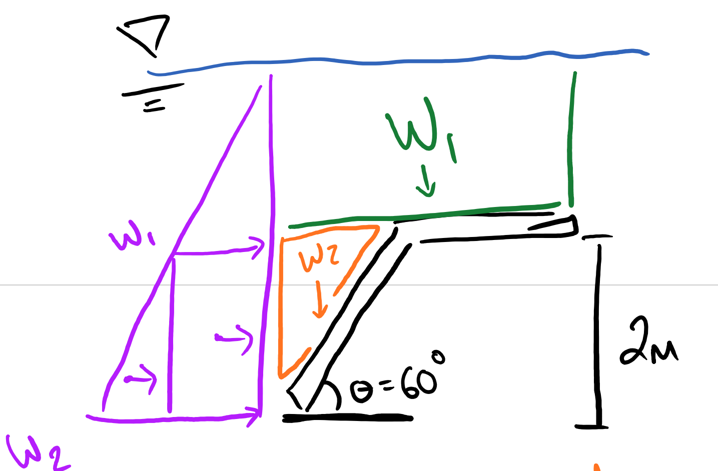


Determine the magnitude of the resultant force acting on the gate ABC due to hydrostatic pressure. The gate has a width of 1.5 m and the density of the water is 1000 kg/m^3.

DRAW the FBD:



the resultant force is

$$|F_R| = \sqrt{F_x^2 + F_y^2}$$

$$F_x = F_1 + F_2 = (2m)w_1 + \frac{1}{2}(2m)(w_2 - w_1)$$

$$F_x = (2m)w_1 + w_2 - w_1 = w_2 - w_1 = (1.5m)(P_2 - P_1)$$

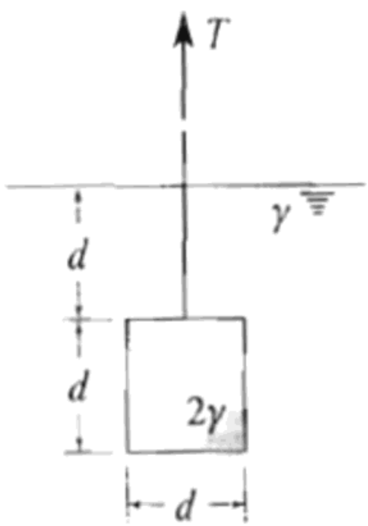
$$F_x = (1.5\,m)(\rho g)(z_2 - z_1) = (1.5\,m)(\rho g)(z_M) = \underline{74\text{ kN}}$$

$$F_y = W_1 + W_2 = M_1 g + M_2 g = \rho g(\Omega_1 + \Omega_2)$$

$$F_y = (1.5\,m)(\rho g)\left[(1.5)\left(1.25 + \frac{2}{\tan 60}\right) + \left(\frac{1}{2}\right)(2)\left(\frac{2}{\tan 60}\right)\right]$$

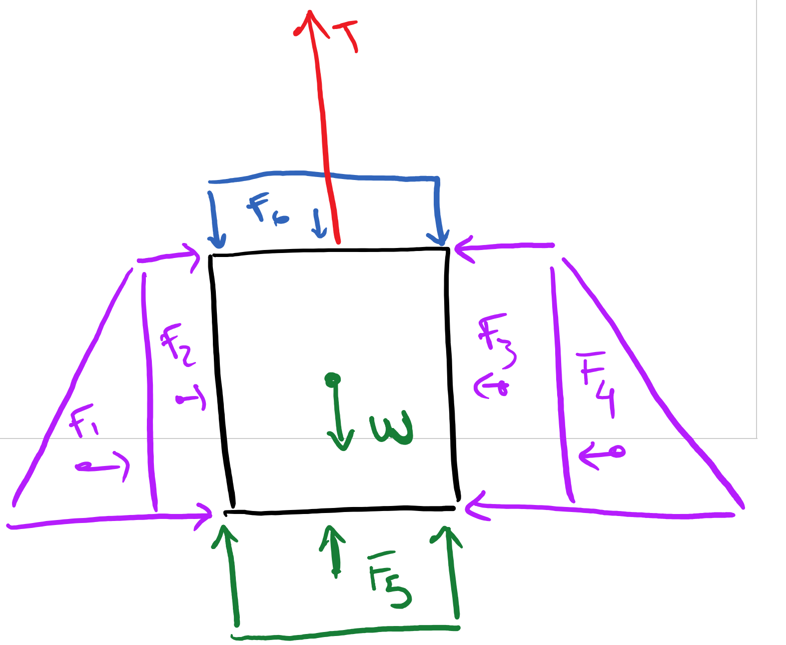
$$\underline{F_y = 70\text{ kN}}$$

$$F_R = \sqrt{F_x^2 + F_y^2} = \underline{102\text{ kN}}$$



A cube of material with edge lengths d and specific weight 2γ is suspended by a cable and is submerged to a depth d in a fluid having specific weight γ. Determine the force T in the cable.

DRAW the FBD of the cube.



Sum the forces:

$$\Sigma F_x = 0$$

$$F_1 + F_2 - F_3 - F_4 = 0$$

$$\Sigma F_y = 0$$

$$T + F_5 - W - F_6 = 0$$

$$T = W + F_6 - F_5$$

$$T = Mg + dw_6 - dw_5$$

$$T = \rho_2 \Omega g + d p_6 d - d p_5 d$$

$$T = \rho_2 g d^3 + (\rho_1 g d) d^2 - (\rho_1 g \, 2d) d^2$$

$$T = \rho_2 g d^3 + \rho_1 g d^3 - 2 \rho_1 g d^3$$

$$\gamma \equiv \text{specific gravity} \equiv \rho g$$

$$T = 2\gamma d^3 - \gamma d^3 - 2\gamma d^3$$

$$\boxed{T = \gamma d^3}$$

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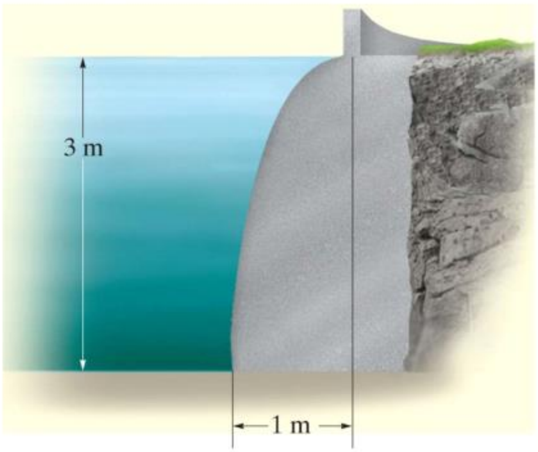
Wednesday, November 9, 2016 11:10

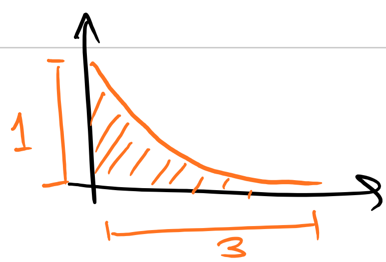


Determine the magnitude and location of the resultant hydrostatic force acting on the surface of a seawall shaped in the form of a parabola. The wall is 5 m. The density of the sea water is 1020 kg/m^3

$w_B = p_B b = \rho g z b$

$F_{R_1} = \frac{1}{2} w_B z = \frac{1}{2} \rho g b z^2 = 225.1$ kN

Weight



weight $= \rho g V = \rho g A \cdot b$

$A = \frac{1}{3} ab$

$W = \rho g b \left(\frac{1}{3} (1)(3) \right) = \rho g b = \underline{50 \text{ kN}}$

the resultant force is

$F_R = \sqrt{F_h^2 + W^2} = \sqrt{(225)^2 + (50)^2} = \underline{231 \text{ kN}}$

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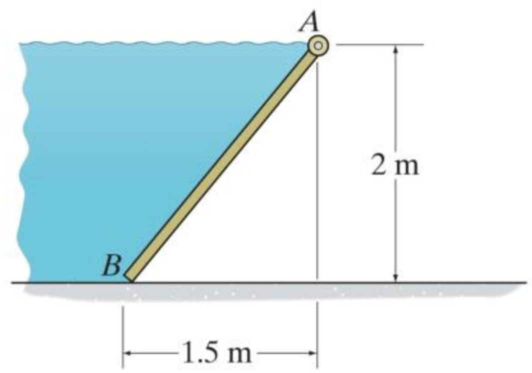
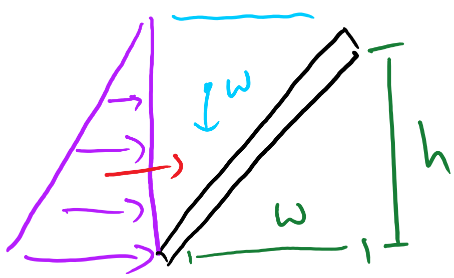


Determine the magnitude of the hydrostatic force acting on gate AB which has a width of 1.5 m.

$$F_H = \frac{1}{2} w_B h = \frac{1}{2} \rho g h b h = \frac{\rho g b h^2}{2}$$

$$F_V = \rho g V = \rho g A b = \rho g b \frac{1}{2} h w$$

$$F_V = \frac{1}{2} \rho g b h w$$



$$w_B = p_B b = \gamma h b$$

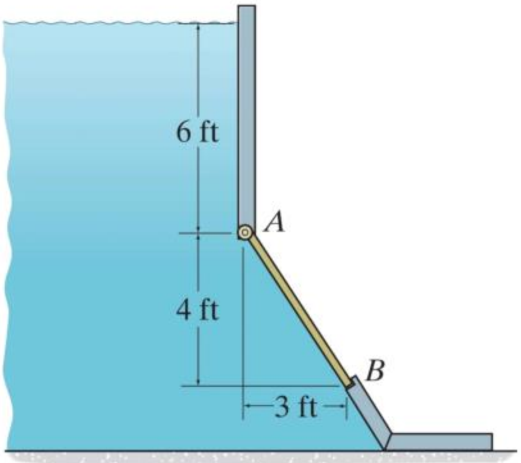
$$F_R = \sqrt{F_R^2 + W^2} = \sqrt{\left(\frac{\rho g b h^2}{2}\right)^2 + \left(\frac{\rho g b w h}{2}\right)^2}$$

$$F_R = \frac{\rho g b h}{2} \underbrace{\sqrt{h^2 + w^2}}$$

$$a = \sqrt{h^2 + w^2}$$

a - length of gate!

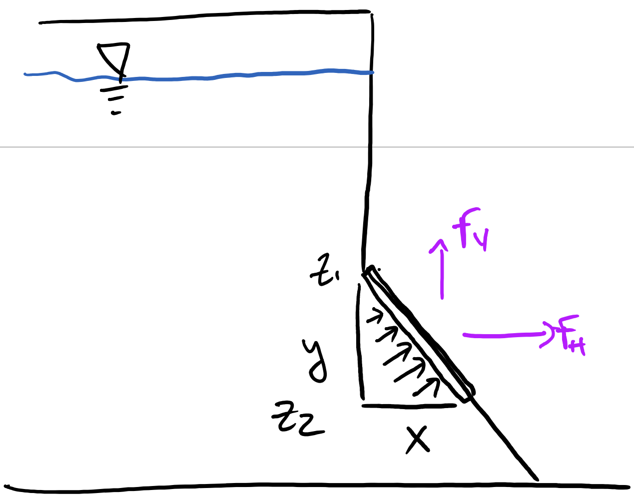
$$\boxed{F_R = \frac{1}{2} \rho g b h a}$$

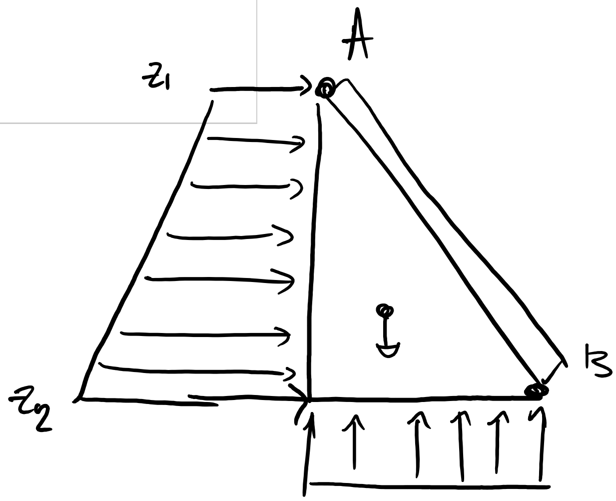


Determine the magnitude of the hydrostatic force acting on gate AB which has a width of 2 m. The specific weight of water is 62.4 lb/ft^3.

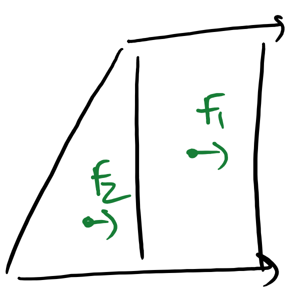
width $b = 2\,m$

$\gamma = 62.4\ lb/ft^3$





the horizontal component is:



$$F_H = F_1 + F_2 = w_1(z_2 - z_1) + \frac{1}{2}(z_2 - z_1)(w_2 - w_1)$$

$$F_H = p_1 b(z_2 - z_1) + \frac{1}{2}(z_2 - z_1) b (p_2 - p_1)$$

$$F_H = \rho g b z_1 (z_2 - z_1) + \frac{1}{2} \rho g b (z_2 - z_1)(z_2 - z_1)$$

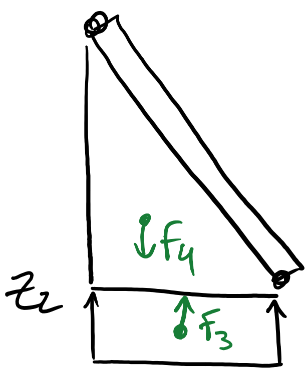
$$F_H = \rho g b\, z_1(z_2-z_1) + \frac{1}{2}\rho g b (z_2-z_1)^2$$

$$F_H = \rho g b (z_2 - z_1)\left[z_1 + \frac{1}{2}(z_2 - z_1)\right]$$

$$F_H = \frac{1}{2}\rho g b (z_2 - z_1)(z_2 + z_1)$$

the vertical component is found using:

method I



$$F_V = F_3 - F_4 = w_2 X - \rho g V$$

$$F_V = P_2 b x - \rho g b\left(\frac{1}{2} x (z_2 - z_1)\right)$$

$$F_V = \rho g b x z_2 - \frac{1}{2}\rho g b x (z_2 - z_1)$$

$$F_V = \rho g b x \left(z_2 - \frac{1}{2}(z_2 - z_1)\right)$$

$$F_V = \frac{1}{2}\rho g b x (z_1 + z_2)$$

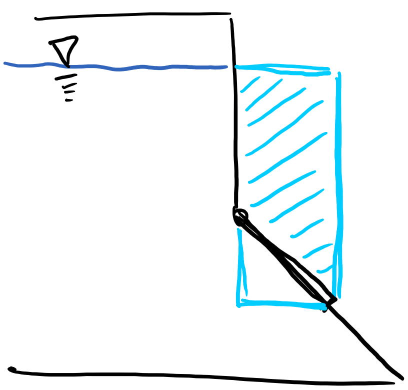
the magnitude of the resultant force is:

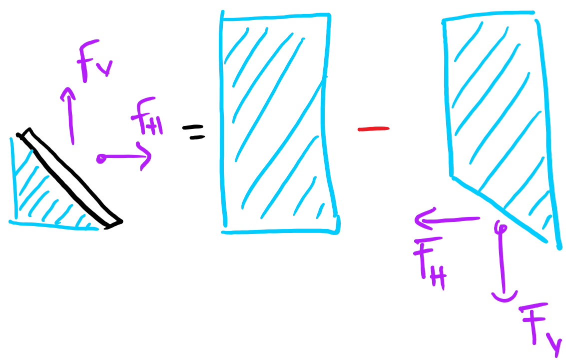
$$\|\vec{F}_R\| = \sqrt{\vec{F}_H^{\,2} + \vec{F}_V^{\,2}}$$

$$\|\vec{F}_R\| = \sqrt{\left(\tfrac{1}{2}\rho g b(z_1+z_2)(z_2-z_1)\right)^2 + \left(\tfrac{1}{2}\rho g b(z_1+z_2)\,x\right)^2}$$

$$\boxed{\|\vec{F}_R\| = \tfrac{1}{2}\rho g b(z_1+z_2)\sqrt{(z_2-z_1)^2 + x^2}}$$

Method II: use composite areas!





The horizontal component is the same as above.

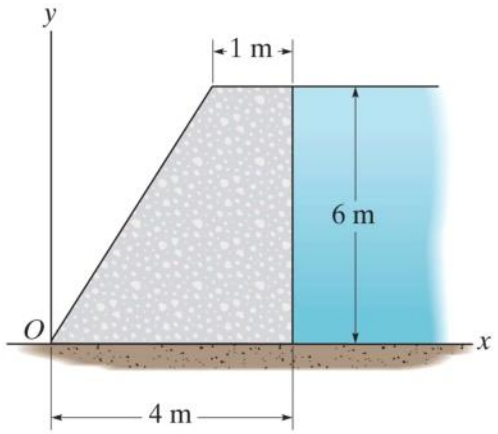
The vertical component is the weight of "water" above the gate.

$$F_V = \rho g V$$

$$F_V = \rho g b\left(x z_1 + \tfrac{1}{2}x(z_2 - z_1)\right)$$

$$F_V = \rho g b \left(x z_1 + \frac{1}{2} x (z_2 - z_1) \right) \quad 1 \; z_2$$

$$F_V = \frac{1}{2} \rho g b x (z_1 + z_2)$$ $\leftarrow$ SAME AS ABOVE!

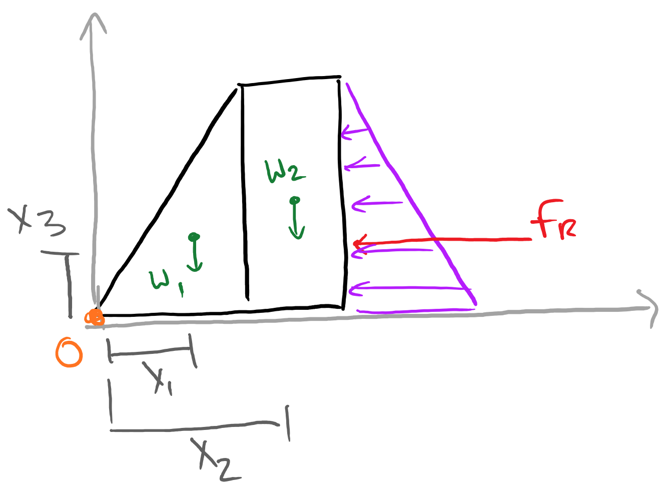


The factor of safety for tipping of the concrete dam is defined as the ratio of the stabilizing moment due to the dam's weight divided by the overturning moment about O due to the water pressure. Determine this factor if the concrete has a density of 2500 kg/m^3.

b is the width which is not given.

$$W_1 = \rho_1 g V = \rho_1 g b\left(\tfrac{1}{2}(3)(6)\right) = 9\rho_1 g b$$

$$W_2 = \rho_1 g V = \rho_1 g b(1\cdot 6) = 6\rho_1 g b$$



$$F_R = \tfrac{1}{2} w_0\, 6 = \tfrac{1}{2} \rho_0 b\, 6$$

$$F_R = \tfrac{1}{2}(6) b \rho_2 g (6) = 18\rho_2 g b$$

$$x_1 = \tfrac{2}{3}(3) = 2 \text{ m}$$

$$x_2 = 3 + \tfrac{1}{2} = 3.5 \text{ m}$$

$$x_3 = \left(\tfrac{1}{3}\right)6 = 2 \text{ m}$$

the overturning moment about O from the hydrostatic pressure is

$$M_{OT} = x_3 F_R$$

the stabilizing moment from the weight of the concrete dam

$$M_S = x_1 W_1 + x_2 W_2$$

the factor of safety is:

$$F.S. = \frac{M_S}{M_{OT}} = \frac{x_1 W_1 + x_2 W_2}{x_3 F} = \frac{x_1(9\rho_1 g b) + x_2(6\rho_1 g b)}{x_3(18\rho_2 g b)}$$

$$F.S. = \frac{(3\cancel{gb})(\rho_1(3x_1 + 2x_2))}{(3\cancel{gb})(6\rho_2 x_3)}$$

$$F.S. = \frac{\rho_1}{\rho_2}\left(\frac{13}{12}\right) = \left(\frac{2500}{1000}\right)\left(\frac{13}{12}\right) = \boxed{2.71}$$

Chapter 11: Virtual Work

Main goals and learning objectives

- Introduce the principle of virtual work
- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

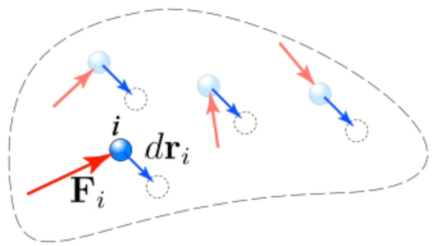
Definition of Work

Work of a force

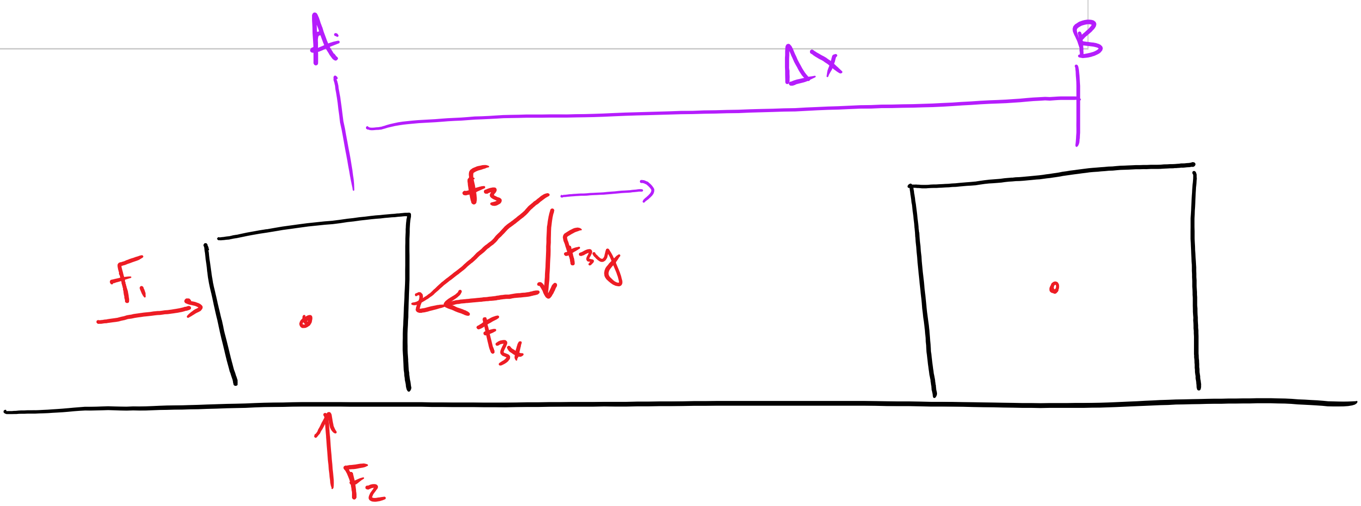
A force does work when it undergoes a displacement in the direction of the line of action.

The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $\boldsymbol{dr}$ is given by

$$\boxed{dU = \mathbf{F} \cdot d\mathbf{r}}$$



* work is a scalar
↳ mag → pos/neg
↳ units → N·m = Joules



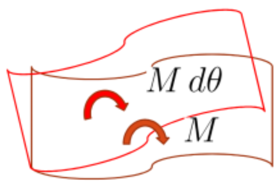
$W_1 = F_1 \cdot \Delta x = F_1 \Delta x \cos\theta = F_1 \Delta x$ pos

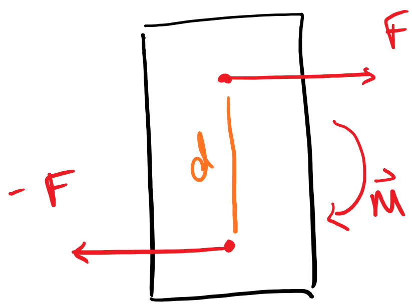
$W_2 = F_2 \cdot \Delta x = F_2 \Delta x \cos\theta = 0$ 0

$W_3 = F_3 \cdot \Delta x = F_3 \Delta x \cos\theta = -F_{3x} \Delta x$ neg

Definition of Work

Work of a couple $dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$





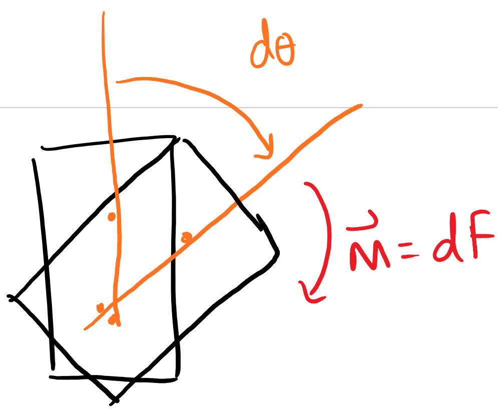
$\Sigma\vec{F} = 0$

no translation

$\Sigma\vec{M}$: dF

only rotation

M = vector



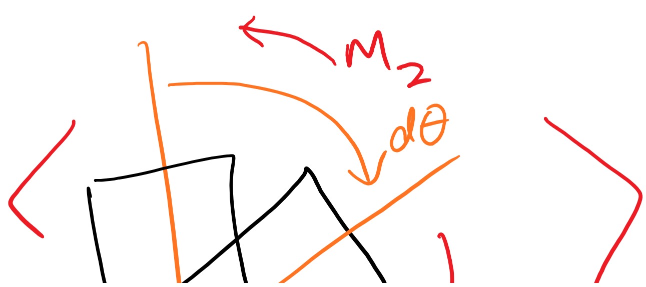
$dU = M\,d\theta$ → scalar

↳ mag → pos/neg

↳ units → N·m = Joule

rad

$\vec{d} \times \vec{F}$ = N·m



M_1 → pos work

M_2 neg

neg
work