

Chapter 5 Part II – 3-D Rigid Body

Equilibrium of a rigid body



with 2D, only worried about

$$\vec{n} = [0, 0, 1]$$

tipping

$$\sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum M_x = \sum M_y = \sum M_z = 0$$

6 equations, CAN solve
for 6 unknowns!

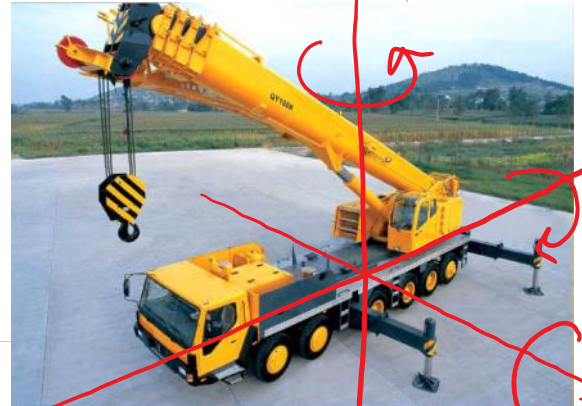









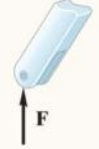
TABLE 5-2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems		
Types of Connection	Reaction	Number of Unknowns
<p>(1)</p>  <p>cable</p>		<p>One unknown. The reaction is a force which acts away from the member in the known direction of the cable.</p>
<p>(2)</p>  <p>smooth surface support</p>		<p>One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.</p>
<p>(3)</p>  <p>roller</p>		<p>One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.</p>

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems


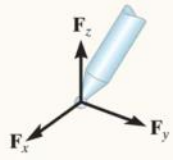

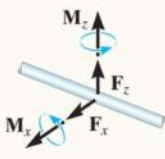
Types of Connection	Reaction	Number of Unknowns
<p>(4)</p>  <p>ball and socket</p>		<p>Three unknowns. The reactions are three rectangular force components.</p>
<p>(5)</p>  <p>single journal bearing</p>		<p>Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are <i>generally not applied</i> if the body is supported elsewhere. See the examples.</p>

TABLE 5-2 Continued


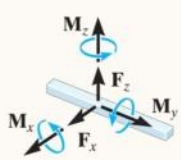

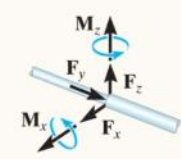



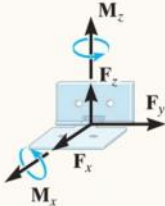

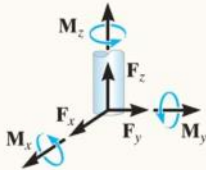
Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. <i>Note:</i> The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. <i>Note:</i> The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. <i>Note:</i> The couple moments are generally not applied if the body is supported elsewhere. See the examples.

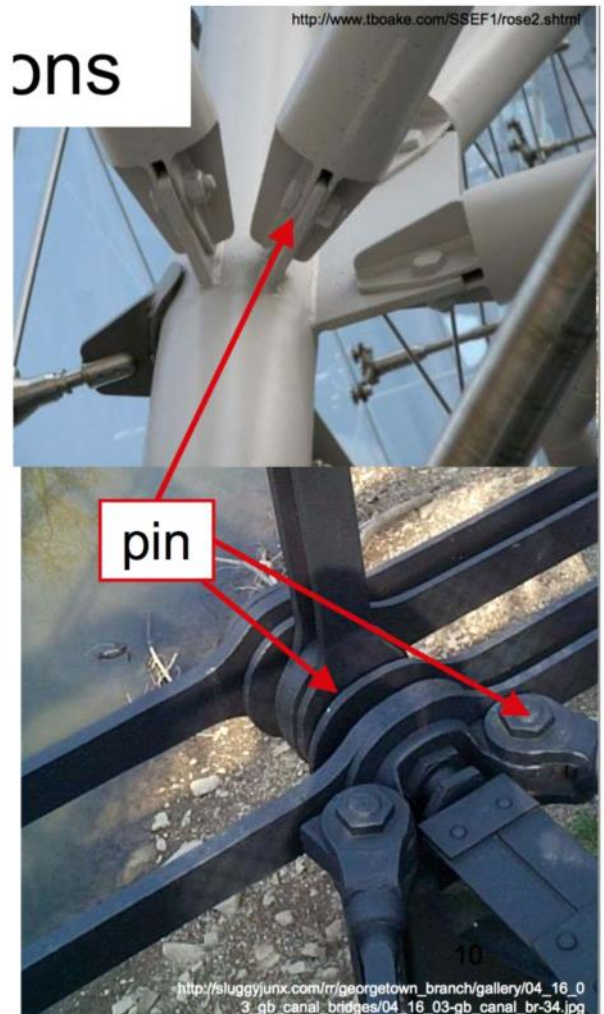
TABLE 5-2 Continued

Types of Connection	Reaction	Number of Unknowns
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. <i>Note:</i> The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

http://web.mst.edu/~ide50-3/schedule/lessons/13_Trusses.pdf

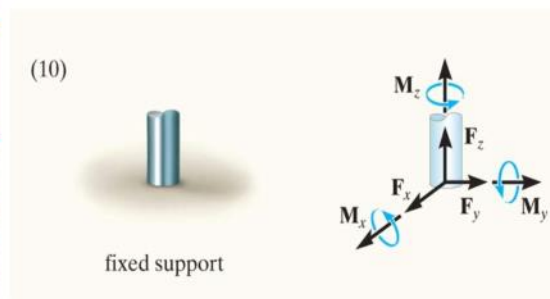
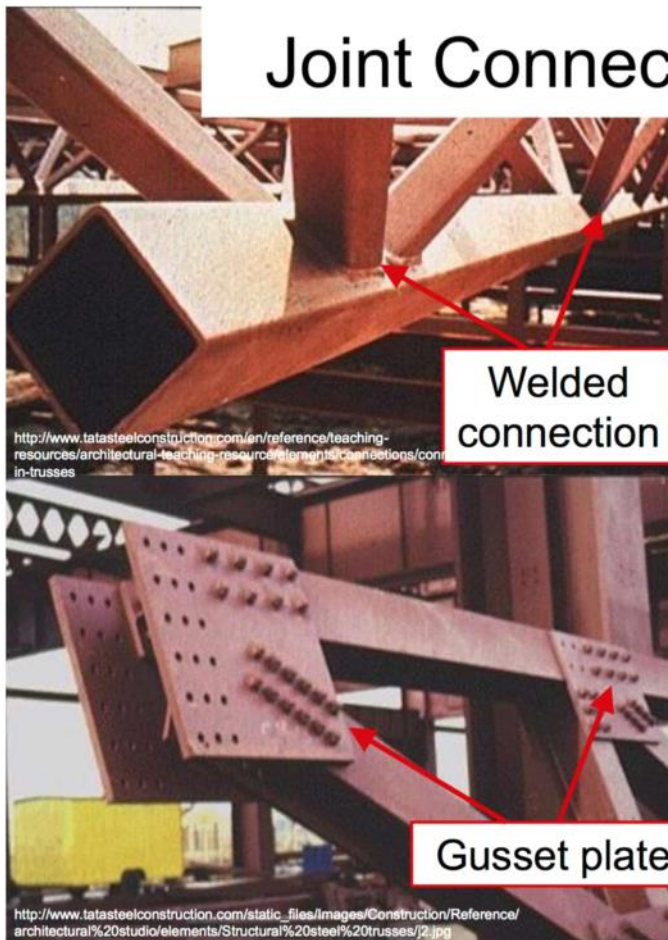


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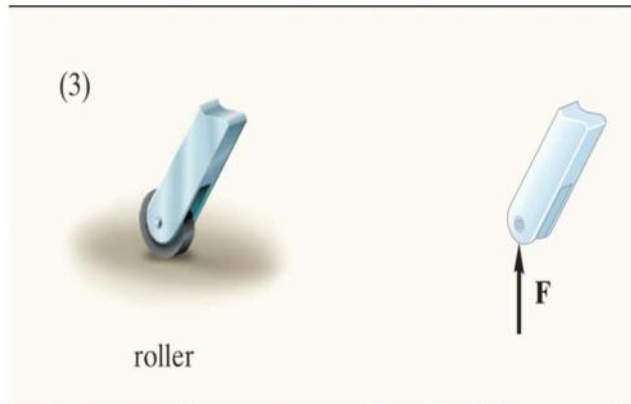


Joint Connec

http://web.mst.edu/~ide50-3/schedule/lessons/13_Trusses.pdf



Does not allow ANY
translation or rotation!
∴ 6 reaction!

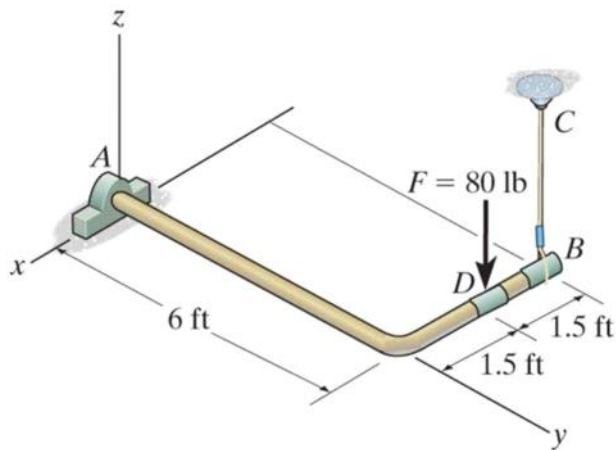


Rocker support



Roller supports

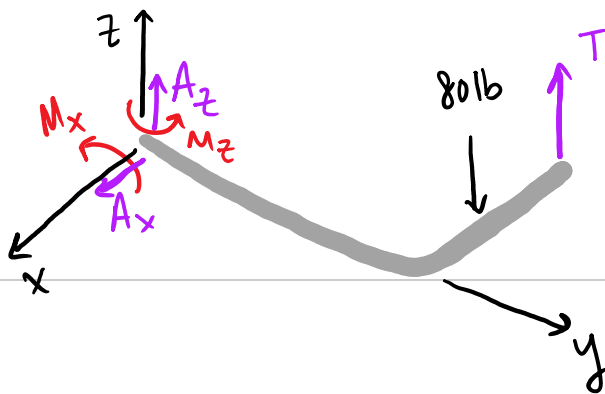




Given: The rod, supported by thrust bearing at A and cable BC, is subjected to an 80 lb force.

Find: Reactions at the thrust bearing A and cable BC.

Draw the FBD of the Rod:



there ARE 5 unknowns.

eqns. of ΣM .

$$\Sigma F_x = 0 \quad \therefore \boxed{A_x = 0}$$

$$\Sigma F_y = 0 \quad \text{none}$$

$$\Sigma F_z = 0 \quad \therefore A_z + T - 80 = 0$$

$$\sum M_y = 0 \quad \therefore -80(1.5) + T(3) = 0$$

$$T = \frac{80(1.5)}{3} = 40 \text{ lb}$$

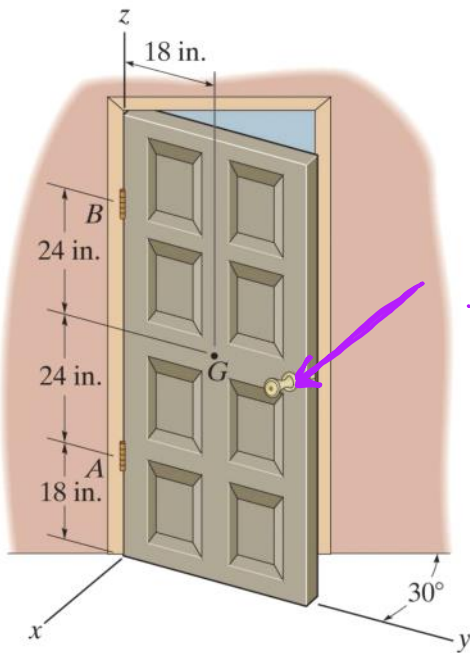
$$\therefore A_z = 80 - T = 40 \text{ lb}$$

$$\sum M_x = 0 \quad \therefore M_{A_x} + T(6) - 80(6) = 0$$

$$M_{A_x} = 6(80 - 40) = 240 \text{ lb}\cdot\text{ft (ccw)}$$

$$\sum M_z = 0 \quad \therefore$$

$$M_{A_z} = 0$$

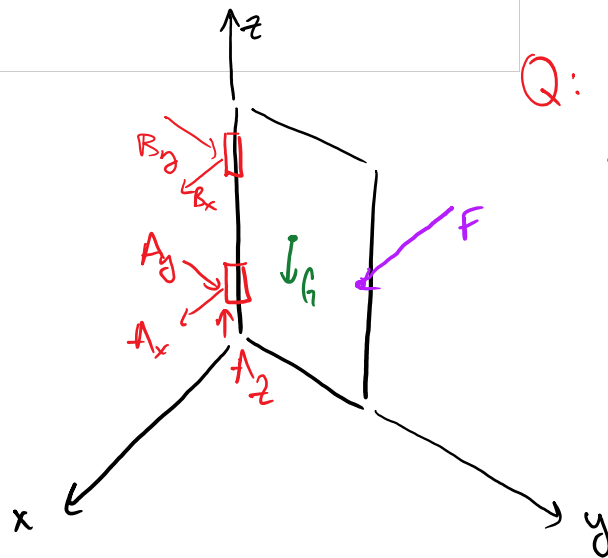


The 100 lb door has its center of gravity at G . Determine the components of reaction at hinges A and B if hinge B resists only forces in the x and y directions and A resists forces in the x , y , z directions.

$$\vec{F} = F[1, 0, 0]$$

Force at handle. F

- identify support reactions
- DRAW FBD, coordinates.



Q: unknowns?

5!
 B_x, B_y, A_x, A_y, A_z

Sum the forces and moments.

$$\sum F_x: A_x + B_x + F = 0$$

$$\sum F_y: A_y + B_y = 0$$

$$\sum F_y: A_y + B_y = 0$$

$$\sum F_x: A_x - G = 0 \Rightarrow$$

$$A_x = G$$

$$\sum M_x: -B_y(48) - (100)(18) = 0$$

$$B_y = \frac{-(100)(18)}{48} = -37.5 \text{ lb}$$

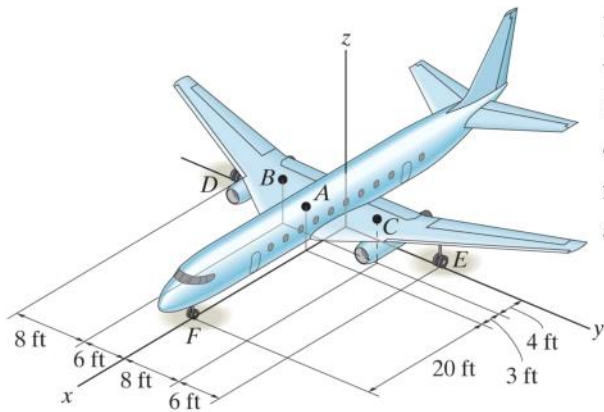
$$\sum M_y: B_x(48) + F(24) = 0$$

$$B_x = -\frac{F(24)}{48} = -\frac{F}{2}$$

now using

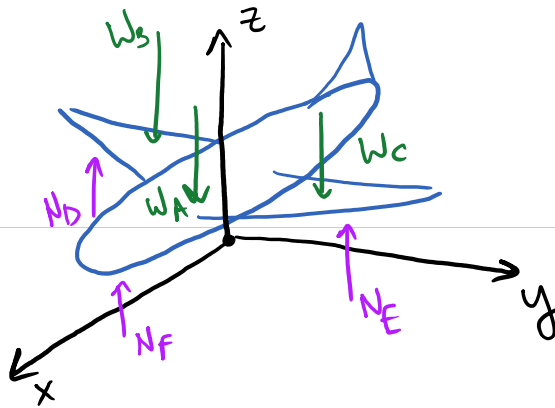
$$\sum F_y: A_y = -B_y = 37.5 \text{ lb}$$

$$\sum F_x: A_x = -B_x - F = \frac{F}{2} - F = -\frac{F}{2}$$



If these components have weights $W_A = 45000$ lb, $W_B = 8000$ lb and $W_C = 6000$ lb, determine the normal reactions of the wheels D , E , and F on the ground.

DRAW the FBD of the plane:



3 unknowns.

eqns. of $\equiv M$.

$$\sum F_x = 0 \quad \text{none}$$

$$\sum F_y = 0 \quad \text{none.}$$

$$\sum F_z = 0 \quad \therefore N_D + N_E + N_F - W_A - W_B - W_C = 0$$

$$\sum M_y = 0 \quad \therefore$$

$$-N_f(27) + W_A(7) + W_B(4) + W_C(4) = 0$$

$$N_f = \frac{W_A(7) + 4(W_B + W_C)}{27} = \boxed{13.7 \text{ kip}}$$

$$\sum M_x = 0 \therefore$$

$$W_B(6) - N_D(14) - W_C(8) + N_E(14) = 0$$

$$\text{use } \sum F_z = 0, \quad N_D = W_A + W_B + W_C - N_E - N_f$$

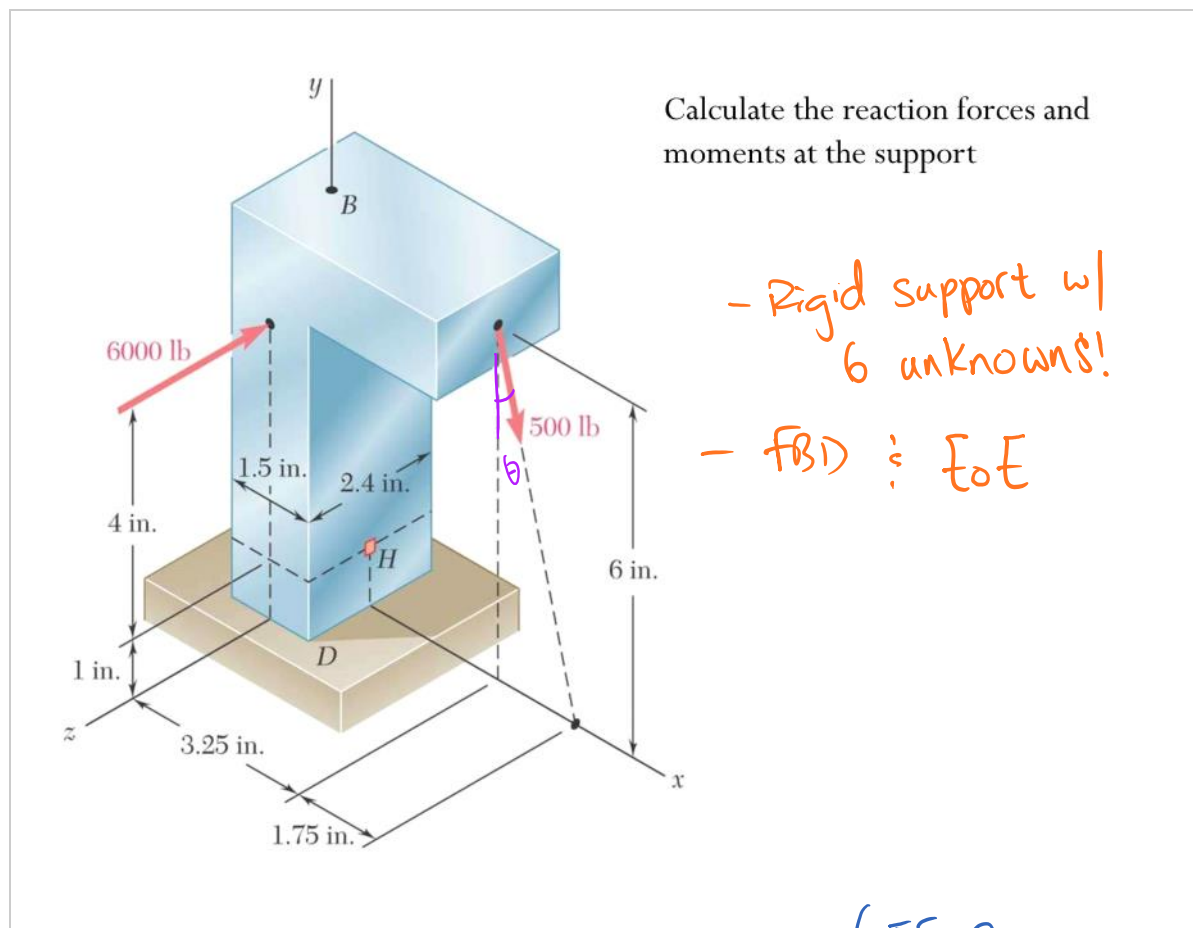
$$W_B(6) - (W_A + W_B + W_C - N_E - N_f)(14) - W_C(8) + N_E(14) = 0$$

Solve for N_E ...

$$2 N_E(14) = W_C(8) - W_B(6) + (W_A + W_B + W_C - N_f)14$$

$$\boxed{N_E = 22.6 \text{ kip}}$$

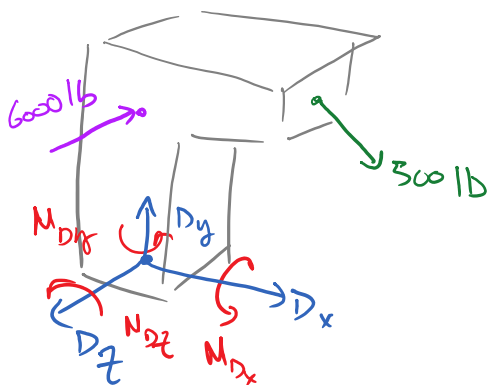
$$\boxed{N_D = 22.6 \text{ kip}}$$



DRAW the FBD:

$$\sum \vec{F} = \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

$$\sum \vec{M} = \begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases}$$



$$\vec{r}_1 = [0.5, 1.2] \text{ in}$$

$$\vec{r}_2 = [3.25, 6, 0] \text{ in}$$

$$\vec{F}_1 = 6k[0, 0, -1] \text{ lb}$$

$$\vec{F}_2 = 500[\sin\theta, -\cos\theta, 0] \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{1.75}{6}\right) = 16.3^\circ$$

eqns. of ΣM :

$$\Sigma F_x: D_x + 500\sin\theta = 0$$

$$\Sigma F_y: D_y - 500\cos\theta = 0$$

$$\Sigma F_z: D_z - 6000 = 0$$

$$\Sigma M_x: M_{Dx} - (5)6000 = 0$$

$$\Sigma M_y: M_{Dy} = 0$$

$$\Sigma M_z: M_{Dz} - (3.25)500\cos\theta - (6)500\sin\theta = 0$$

Solve system of equations!!