

Chapter 10: Moments of inertia

Main goals and learning objectives

- Determine the moment of inertia for an area

Terminology: the term **moment** in this module refers to the mathematical sense of different “measures” of an area or volume.

- The *zeroth* moment is the total mass.
- The *first* moment (a single power of position) gave us the centroid.
- The *second* moment will allow us to describe the “width.”
- An analogy that may help: in *probability* the first moment gives you the mean (the center of the distribution), and the second is the standard deviation (the width of the distribution).

Applications



Many structural members like beams and columns have cross sectional shapes like an I, H, C, etc..

Why do they usually not have solid rectangular, square, or circular cross sectional areas?

What primary property of these members influences design decisions?

Applications

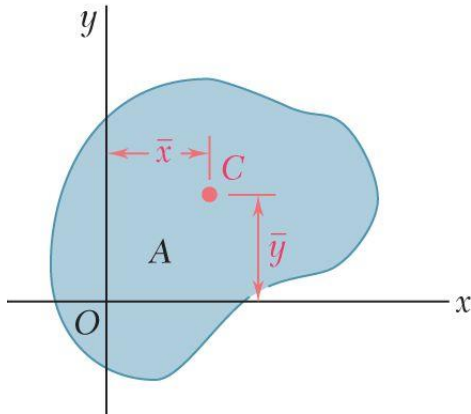


Many structural members are made of tubes rather than solid squares or rounds. **Why?**

This section of the book covers some parameters of the cross sectional area that influence the designer's selection.

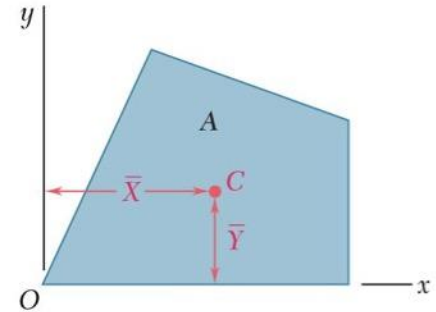
Recap: First moment of an area (centroid of an area)

- The first moment of the area A with respect to the x -axis is given by $Q_x = \int_A y dA$
- The first moment of the area A with respect to the y -axis is given by $Q_y = \int_A x dA$
- The centroid of the area A is defined as the point C of coordinates \bar{x} and \bar{y} , which satisfies the relation



$$\int_A x dA = A \bar{x}$$

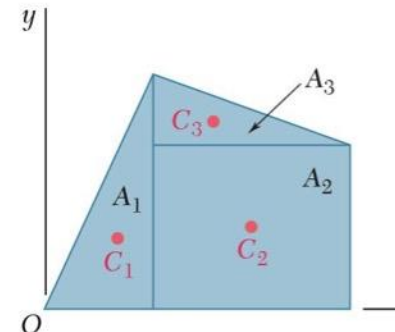
$$\int_A y dA = A \bar{y}$$



- In the case of a composite area, we divide the area A into parts

$$A_{total} \bar{X} = \sum_i A_i \bar{x}_i \quad A_{total} \bar{Y} = \sum_i A_i \bar{y}_i$$

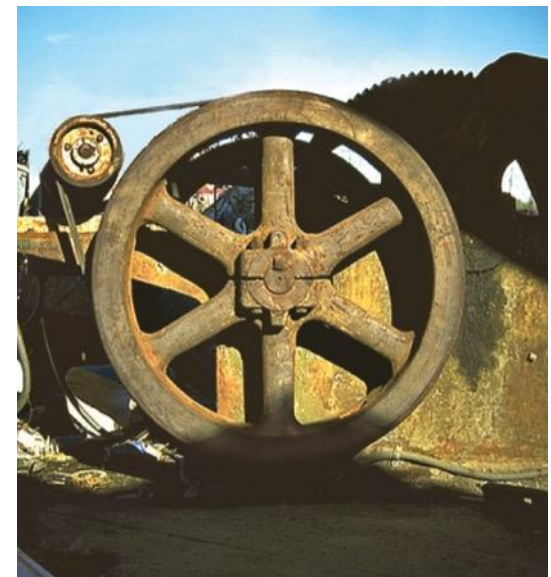
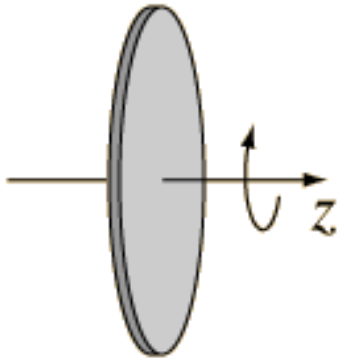
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Mass Moment of Inertia

- **Mass moment of inertia** is the mass property of a rigid body that determines the torque needed for a desired angular acceleration about an axis of rotation.
- A larger mass moment of inertia around a given axis requires more torque to increase the rotation, or to stop the rotation, of a body about that axis
- Mass moment of inertia depends on the shape and density of the body and is different around different axes of rotation.

Torque-acceleration relation:

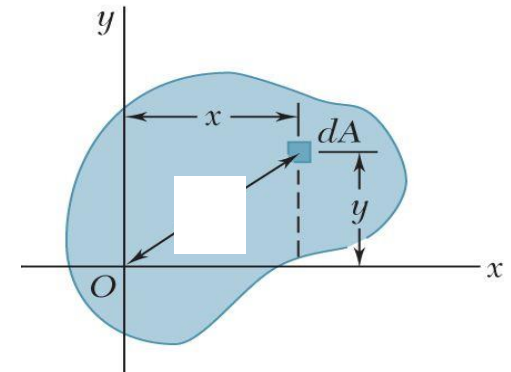
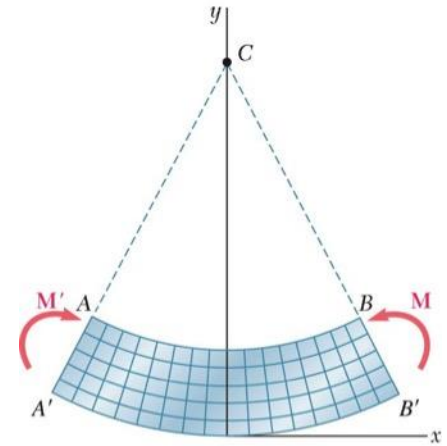


Second moment of area

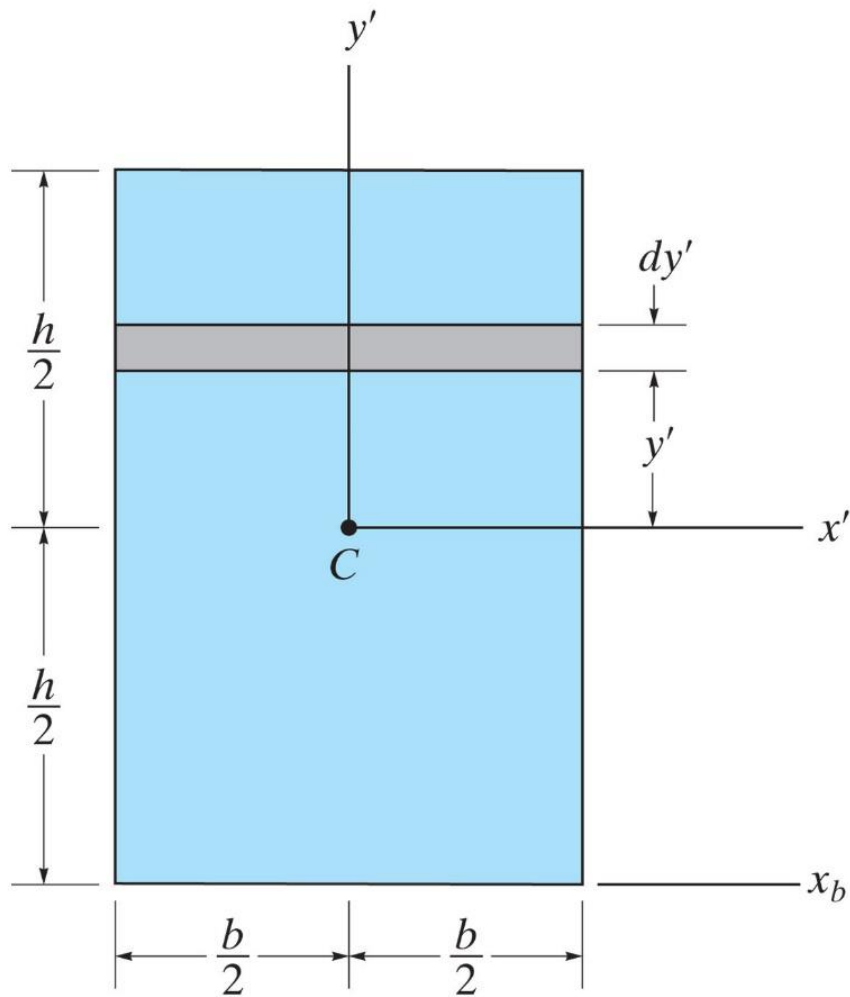
Moment of inertia is the property of a deformable body that determines the moment needed to obtain a desired curvature about an axis.

Moment of inertia depends on the shape of the body and may be different around different axes of rotation.

- The moment of inertia of the area A with respect to the x -axis is given by
- The moment of inertia of the area A with respect to the y -axis is given by
- The moment of inertia of the area A with respect to the origin is given by (Polar MoI)

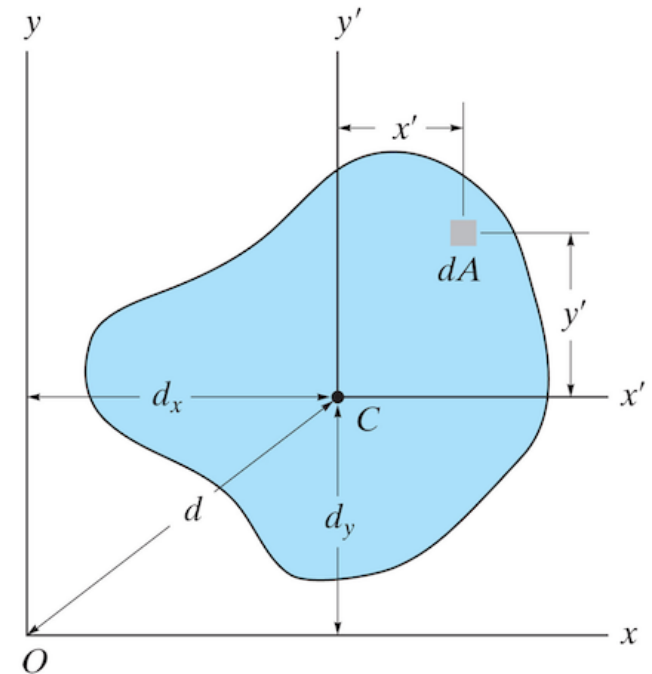


Determine the moment of inertia for the rectangular area shown w.r.t. the centroidal axis x' .

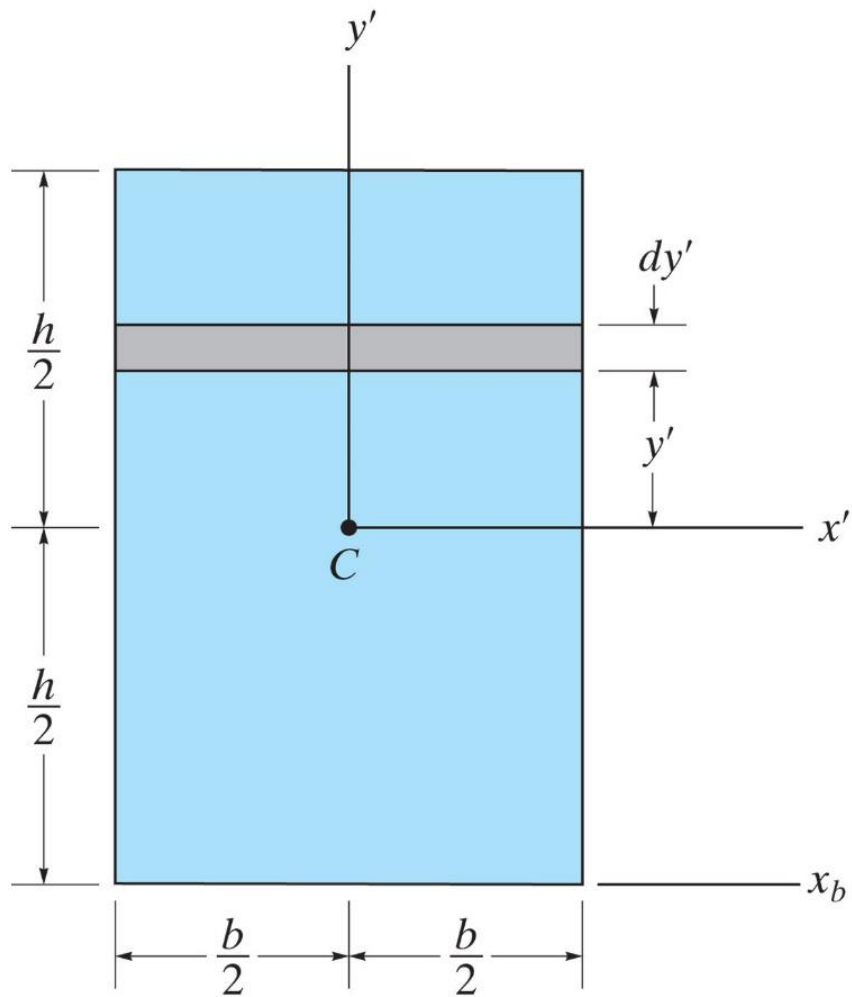


Parallel axis theorem

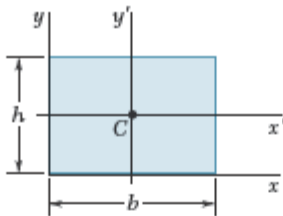
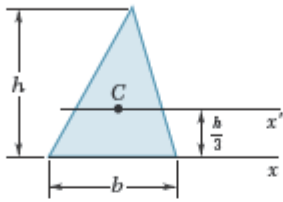
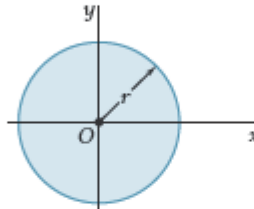
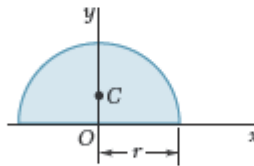
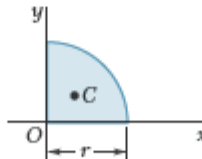
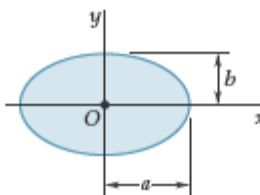
- Often, the **moment of inertia** of an area is known for an axis passing through the **centroid**; e.g., x' and y' :
- The moments around other axes can be computed from the known $I_{x'}$ and $I_{y'}$:



Note: the integral over y' gives zero *when done through the centroid axis*.



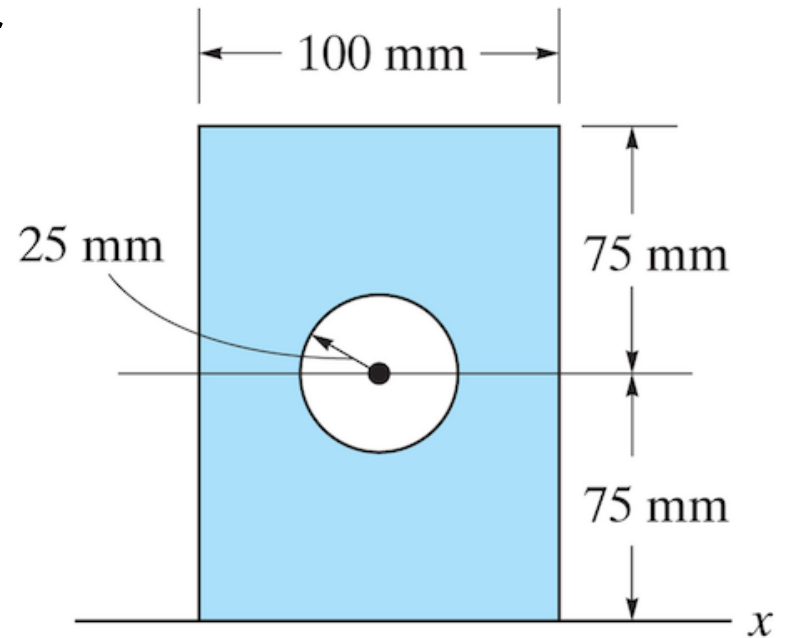
Determine the moment of inertia for the rectangular area shown w.r.t. the axis passing through the base of the rectangle x_b .

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Moment of inertia of composite

- If individual bodies making up a **composite** body have individual areas A and moments of inertia I computed through their centroids, then the **composite area** and **moment of inertia** is a sum of the individual component contributions.
- This requires the **parallel axis theorem**
- Remember:
 - The position of the centroid of each component **must** be defined with respect to the **same origin**.
 - It is allowed to consider **negative areas** in these expressions. Negative areas correspond to holes/missing area. **This is the one occasion to have negative moment of inertia.**

Determine the moment of inertia for the shaded area about the x -axis.



Find the moment of inertia about its centroid:

