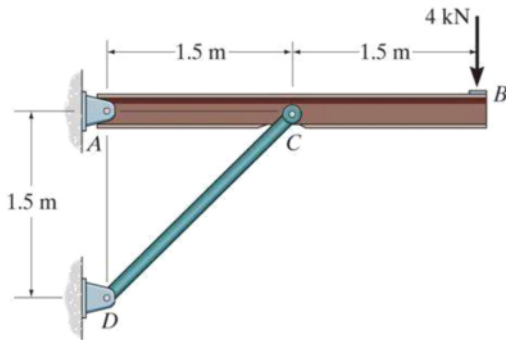


## To do ...

- Enter your student netID in Mastering Engineering **DUE Monday, Oct 2**
- **Quiz 3 – in class – Monday Oct 2**
- HW 10 PL due **Tues**
- HW 11 due **Thurs**

Calculators?

A) Yes



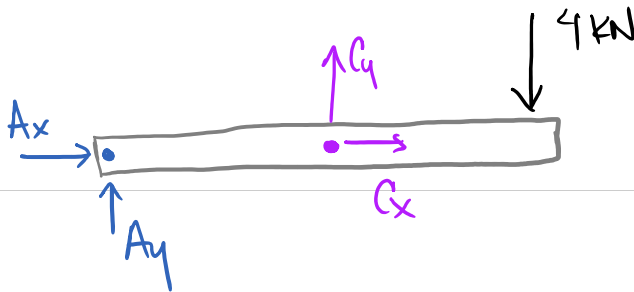
Given the 4 kN load at B of the beam is supported by pins at A and C. Find the support reactions at A and C.

- idealized model
- identify 2-force and 3-force members

- DRAW FBD

- determine the number of unknowns.

FBD of BEAM AB



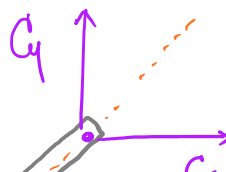
$$\sum F_x: A_x + C_x = 0$$

$$\sum F_y: A_y + C_y - 4 \text{ kN} = 0$$

$$\sum M_A: (1.5 \text{ m}) C_y - (3 \text{ m})(4 \text{ kN}) = 0$$

\* 4 unknowns,  
 $A_x, A_y, C_x, C_y$   
 Cannot solve!!

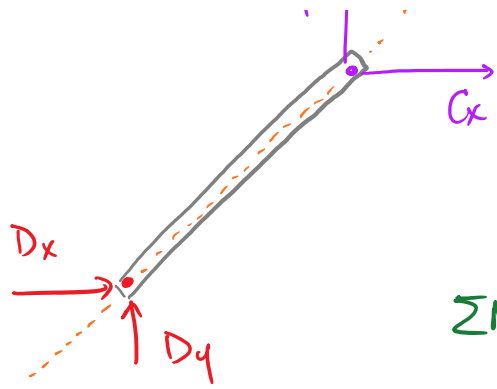
DRAW the FBD of the link DC



$$\sum F_x: C_x + D_x = 0$$

$$C_x = -D_x$$

$$C_y = -D_y$$



$$\sum F_x: C_x + D_x = 0$$

$$\sum F_y: C_y + D_y = 0$$

$$C_y = -D_y$$

$$\sum M_B: (1.5\text{m})C_x - (1.5\text{m})C_y = 0 \therefore C_x = C_y$$

$$\Rightarrow C_x = C_y = -D_x = -D_y$$

$$- |\vec{C}| = |\vec{D}|$$

$$- \vec{C} + \vec{D} = 0$$

$$- \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \pm 45^\circ$$

It is a 2-force member!!

So you know direction, not magnitude!

using  $\sum M_A$  from above...

$$\sum M_A: (1.5\text{m})F_c \sin(45) - 3(4\text{kN}) = 0$$

$$F_c = 11.3 \text{ kN}$$

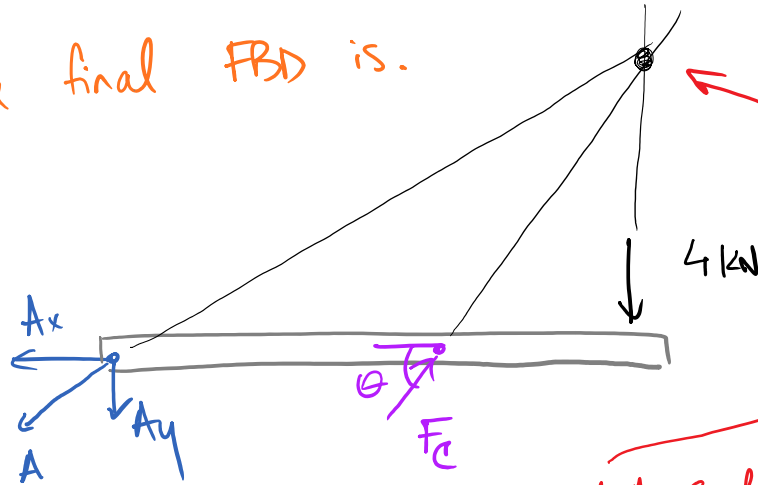
$$\sum F_x: A_x = -F_c \cos(45) = \underline{-8.00 \text{ kN}}$$

$$\sum F_y: A_y = 4\text{kN} - F_c \sin(45) = \underline{-4.00 \text{ kN}}$$

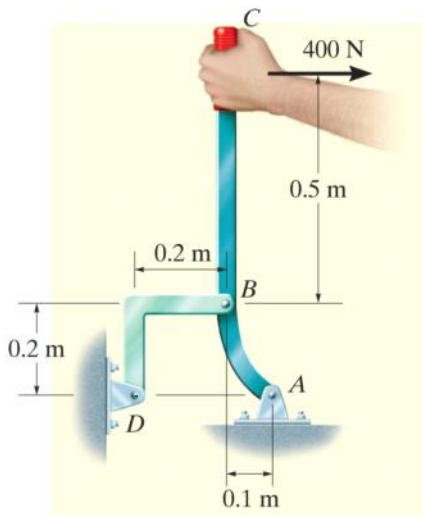
\* negative sign means the sense is in

the other direction:

the final FBD is.



A 3-force member  
in  $\Sigma M$  has all lines  
of action meet at a  
point!



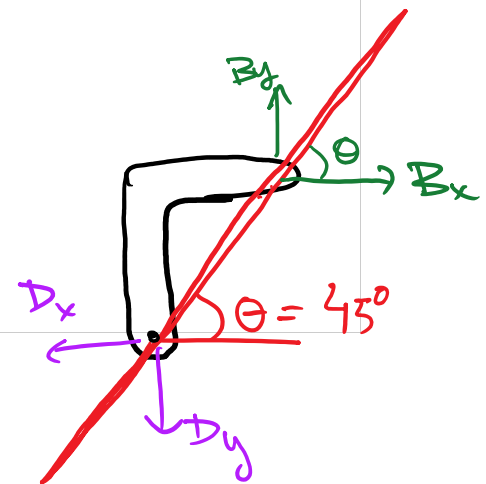
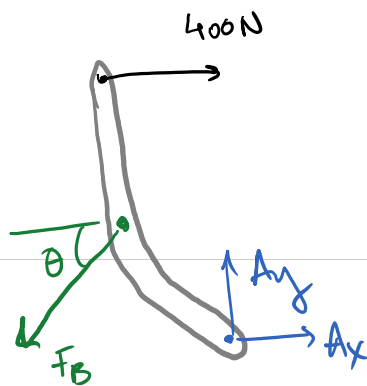
The lever  $ABC$  is pin supported at  $A$  and connected to a short link  $BD$ . If the weight of the members is negligible, determine the reaction forces at pins  $D$  and  $A$ .

Q: Why?

- idealized model

- note:  $DB$  is a 2 force member  
 $ABC$  is a 3 force member

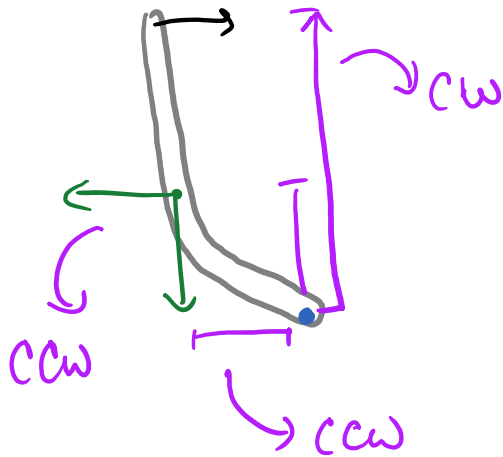
- DRAW FBD.



- Now, sum the forces and moments for  $ABC$ :

$$\sum F_x: 400 \text{ N} + A_x - F_B \cos(45) = 0$$

$$\sum F_y: A_y - F_B \sin(45) = 0$$



$$\sum \vec{M}_A: (0.1) F_B \sin(45) + (0.2) F_B \cos(45) - (0.7) 400 = 0$$

$$F_B = \frac{(0.7)(400)}{(0.1)\sin 45 + (0.2)\cos 45} = \boxed{1.32 \text{ kN}}$$

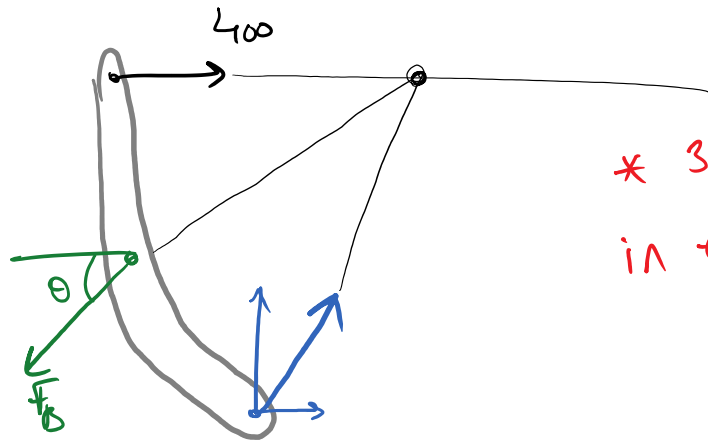
now solve for  $A_x$  &  $A_y$ :

$$\sum F_x: A_x = F_B \cos(45) - 400 = 533 \text{ N}$$

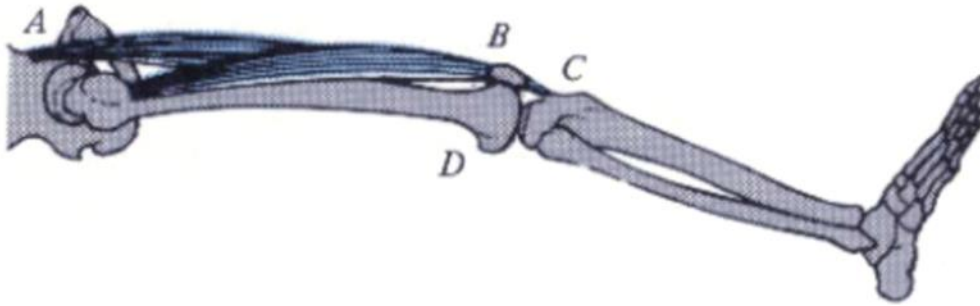
$$\sum F_y: A_y = F_B \sin(45) = 933 \text{ N}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} = \underline{1.07 \text{ kN}}$$

$$\theta_A = \tan^{-1} \left( \frac{A_y}{A_x} \right) = \underline{60.3^\circ}$$



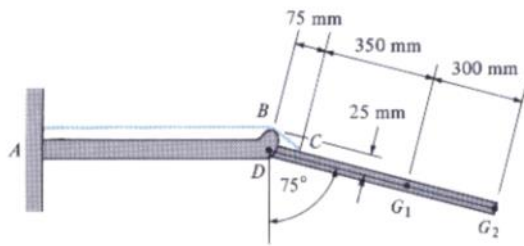
\* 3-force member  
in equilibrium!



A skeletal diagram of the lower leg is shown. Model the lower leg and determine the tension  $T$  in the quadriceps and the magnitude of the resultant force at the femur (pin) at D in order to hold the lower leg in the position shown. The lower leg has a mass of 3.2 kg and the foot has a mass of 1.6 kg.

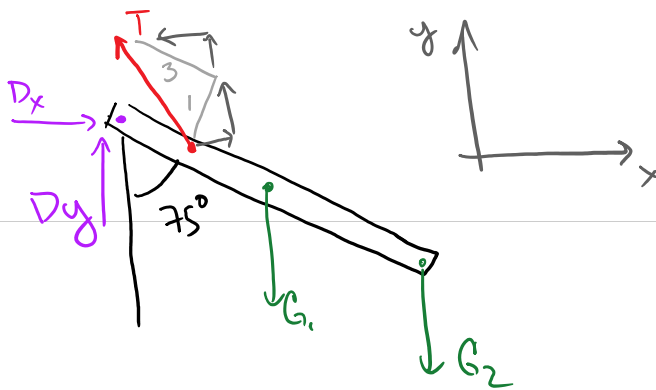
Q: What is the idealized model?





Determine the tension  $T$  in the quadriceps and the magnitude of the resultant force at the femur (pin) at D. The lower leg has a mass of 3.2 kg and the foot has a mass of 1.6 kg.

-DRAW the FBD



Sum the forces and moments:

$$\sum F_x: D_x - \left(\frac{3}{\sqrt{10}} T\right) \cos(15) + \left(\frac{1}{\sqrt{10}} T\right) \sin(15) = 0$$

$$\sum F_y: D_y - G_1 - G_2 + \left(\frac{1}{\sqrt{10}} T\right) \cos(15) + \left(\frac{3}{\sqrt{10}} T\right) \sin(15) = 0$$

$$\sum M_D: \left(\frac{1}{\sqrt{10}} T\right)(0.075 \text{ m}) - (31.4 \text{ N})(0.425 \sin(75)) - (15.7 \text{ N})(0.725 \sin(75)) = 0$$

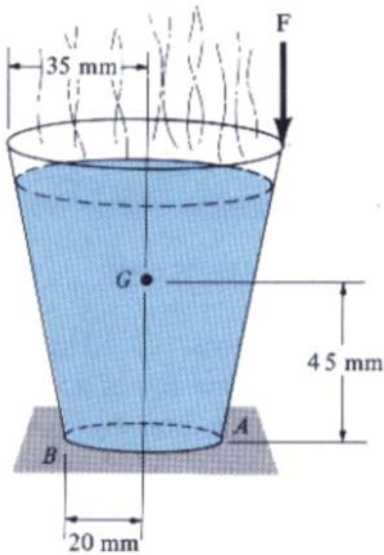
Solve eqn  $\sum M_D$  for  $T$ , giving

$$T = 1007 \text{ N}$$

Solve eqn  $\Sigma F_x$  And  $\Sigma F_y$  for  $D_x$  ;  $D_y$

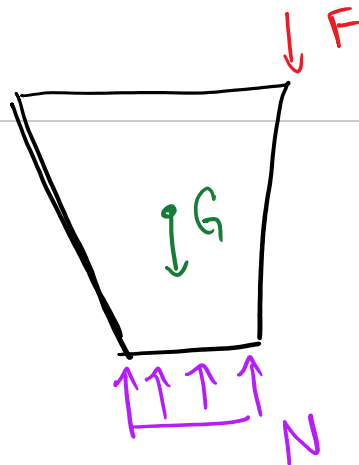
$$D_x = \frac{T}{\sqrt{10}} (3\cos(15) - \sin(15)) = \boxed{982 \text{ N}}$$

$$D_y = G_1 + G_2 - \frac{T}{\sqrt{10}} (3\sin(15) + \cos(15)) = \boxed{508 \text{ N}}$$



The cup is filled with 125 g of liquid. The mass center is located at G. If a vertical force  $F$  is applied to the rim of the cup, determine its magnitude so the cup is on the verge of tipping over.

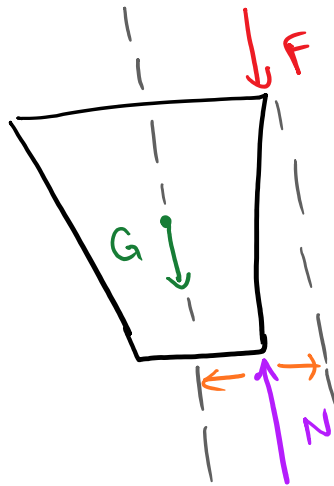
DRAW the FBD :



But this is NOT in equilibrium unless  $F=0$ ! (Check it!)

Also, "on the verge of tipping" MEAS that  $N$  acts at a specific point

MEAS



Sum forces and moments!

$$\sum F_x: 0$$

$$\sum F_y: N - G - F = 0$$

$$\sum M_A: G x_1 - (x_2 - x_1) F = 0$$

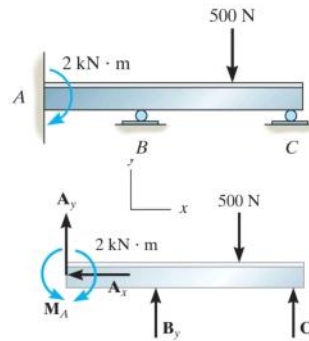
$$F = \left( \frac{x_1}{x_2 - x_1} \right) G$$

$$F = \left( \frac{20 \text{ mm}}{15 \text{ mm}} \right) (.125 \text{ kg} \cdot 9.81 \text{ m/s}^2) = 1.635 \text{ N}$$

## Constraints

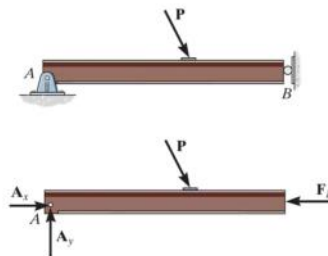
To ensure equilibrium of a rigid body, it is not only necessary to satisfy equations of equilibrium, but the body must also be properly constrained by its supports

- **Redundant constraints:** the body has more supports than necessary to hold it in equilibrium; the problem is STATICALLY INDETERMINATE and cannot be solved with statics alone
- **Improper constraints:** In some cases, there may be as many unknown reactions as there are equations of equilibrium. However, if the supports are not properly constrained, the body may become unstable for some loading cases.



Q: How many reaction forces?

4 forces, 1 couple moment



Q: How many reaction forces?

3 forces.