To do ...

- Quiz 2 this week (ends on today!)
- WA 1 due TODAY @ 11:59 pm
- HW 8 due Tues
- HW 9 due Thurs

- In class quiz 3 MONDAY, Oct 2

- Thank you for your feedback!!

- Happy Autumn Equinox!!
What is the equivalent system?

\[ \vec{F}_R \perp \vec{M}_R \]

\[ \vec{x} = \frac{\vec{M}_R}{F_R} \]
Replace the force system acting on the post by a resultant force and resultant moment about point A, and specify where its line of action intersects the post AB measured from point A.

- Sum forces
- Find and sum moments

\[
\sum F_x = \frac{4}{5} \times 250 - 300 - 500 \cos(30) = -533 \text{ N}
\]

\[
\sum F_y = 500 \sin(30) - \frac{3}{5} \times 250 = 100 \text{ N}
\]

\[
\left| \overrightarrow{F} \right| = \sqrt{F_x^2 + F_y^2} = 542 \text{ N}
\]

\[
\theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = 10.6^\circ
\]

\[
\overrightarrow{M_R} = \sum M = (1 \times 300) + (2 \times 500 \cos(30)) - (0.2 \times 500 \sin(30))
\]

\[
- (0.5 \times \frac{3}{5} \times 250) - (3 \times \frac{4}{5} \times 250) = \overrightarrow{x \cdot \overrightarrow{F_{Z_x}}}
\]

only \(x\)-component creates

\[
\overrightarrow{x} = 0.827 \text{ m}
\]
moment
Reduction of a simple distributed load

vs. force applied at single point.
Reduction of a simple distributed load

In structural analysis, we often are presented with a **distributed load** \( w(x) \) (force/length) and we need to find the equivalent loading \( F \).

Example of such forces are winds, fluids, or the **weight** of items on the body's surface.

\[
w(x) = p(x) \cdot b = \frac{N}{M^2} \cdot m = \frac{N}{M} = \frac{\text{force}}{\text{length}}
\]
\[
w(x) = \frac{dF}{dx}
\]

**Replace coplanar parallel force system with a single equivalent resultant force**

\[
\overrightarrow{F_r} = \sum df = \int_0^L w(x) dx = \text{Area under the curve}
\]

\[
\overrightarrow{M_r} = \sum x \cdot df(x) = \int_0^L x \cdot df(x) = \int_0^L x \cdot w(x) dx = \text{Resultant moment}
\]

**Where is the resultant force located?**

\[
\overrightarrow{M_r} = \overline{x} \overrightarrow{F_r}
\]
Line of action passes through centroid.

\[ M_R = x \cdot I_R \]

\[ \int_{0}^{L} x \omega(x) \, dx = \bar{x} \int_{0}^{L} \omega(x) \, dx \]

\[ \bar{x} = \frac{\int_{0}^{L} x \omega(x) \, dx}{\int_{0}^{L} \omega(x) \, dx} = \text{geometric center, centroid} \]
Triangular loading

\[ \omega(x) = 0 \]
\[ w(x) = w_0 \frac{[\text{force}]}{[\text{length}]} \]
\[ y(x) = mx + b \quad \text{y-intercept} \]
\[ \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{w_0}{L} \]
\[ w(x) = \frac{w_0}{L} x \]
\[ \vec{F}_p = \int_0^L w(x) \, dx = \int_0^L \frac{w_0}{L} x \, dx = \frac{w_0}{L} \int_0^L x \, dx = \frac{w_0}{L} \left[ \frac{x^2}{2} \right]_0^L \]
\[ \vec{F}_p = \frac{1}{2} w_0 L \quad *\text{(Area of triangle)} * \]
\[ \vec{M}_p = \int_0^L x w(x) \, dx = \int_0^L \frac{w_0}{L} x^2 \, dx = \frac{w_0}{L} \int_0^L x^2 \, dx = \frac{w_0}{L} \left[ \frac{x^3}{3} \right]_0^L \]
\[ \vec{M}_p = \frac{1}{3} w_0 L^2 \]
\[ \vec{M}_p = \bar{x} \vec{F}_p = 0 \quad \bar{x} = \frac{\vec{M}_p}{\vec{F}_p} = \frac{\frac{1}{3} w_0 L^2}{\frac{1}{2} w_0 L} = \frac{2}{3} L \]
\[ N_R = \bar{x} T_R \]

\[ T_R = \frac{1}{2} L u_0 \]

Geometric center \((\bar{x}, \bar{y})\)

*Line of action passes through \((\bar{x}, \bar{y})\)\n
\[ \vec{F}_R = \text{Area of triangle} \]

\[ \vec{F}_R = \frac{1}{2} L u_0 \text{ (magnitude)} \]
**Triangular loading**

\[ w(L) = w_0 \]

- find \( \overrightarrow{F}_R \)
- find \( \bar{x} \)

for triangular loading

\[ \overrightarrow{F}_R = (\text{Area of triangle}) = \frac{1}{2} L w_0 \]

Geometric center - \( (\bar{x}, \bar{y}) \)

the line of action passes through \( (\bar{x}, \bar{y}) \)
for a triangle in this configuration,

\[ \bar{x} = \frac{1}{3} L \]

To summarize:

* Magnitude is equal to the area under the curve \( w(x) \)

\[ \text{Ld triangle is } F_r = \int w \, dx = \frac{1}{2} bh \]

\[ F_r = \frac{1}{2} bh \]

* The force acts at the geometric center

\( \text{Ld for a triangle...} \)
\[(\bar{x}, \bar{y}) = (\frac{1}{3}b, \frac{1}{3}h)\]  \[(\bar{x}, \bar{y}) = (\frac{2}{3}b, \frac{1}{3}h)\]

Q: What about this one?

\[(\bar{x}, \bar{y}) = ?\]  \[(\bar{x}, \bar{y}) = ?\]
Rectangular loading \( w(x) = w_0 \)

1. Find \( F_R = \int w(x) \, dx \)
2. Find \( M_R = \int x \, dF = \int x \, w(x) \, dx \)
3. Find \( \bar{x} = \frac{M_R}{F_R} \)

\[
\omega(x) = mx + b = 0 + w_0 = w(x) = w_0
\]

\[
\overline{F_R} = \int_0^L w_0 \, dx = w_0 \left[ x \right]_0^L = w_0 L \quad \text{\(\text{Area of Rectangle!}\)}
\]

\[
\overline{M_R} = \int_0^L x \, dF = \int_0^L w_0 x \, dx = w_0 \left[ \frac{x^2}{2} \right]_0^L = \frac{1}{2} w_0 L^2
\]

\[
\bar{x} = \frac{\overline{M_R}}{\overline{F_R}} = \frac{\frac{1}{2} w_0 L^2}{w_0 L} = \frac{L}{2} \quad \text{Always!}
\]

Geometric Center \( \left( \bar{x}, \bar{y} \right) \)

\[
\bar{x} = \frac{L}{2}, \quad \bar{y} = \frac{w_0}{2}
\]

\[
\overline{F_R} = (\text{Area of Rectangle} - w_0 L)
\]