To do ...

- Quiz 1 – This week!
  - CBTF instructions on website

- i>clickers

- HW 4PL due Tues
- HW 5ME due Thurs

Good Luck!
Recap

- Equilibrium of a particle
- General procedure for analysis
- Free body diagram
- Equation of equilibrium
- Idealizations (pulleys, springs, smooth surfaces)
Idealizations

**Pulleys** are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side. **Springs** are (usually) regarded as linearly elastic; then the tension is proportional to the change in length $s$.

![Diagram](image)

- Frictionless pulley
  - $T_1 = T_2$
  - Normal force at point of contact perpendicular to surface!
- Linearly elastic spring
  - $F = ks = k(l-l_0)$
  - Spring force opposes external force
  - Spring constant $F = k\Delta s$
  - Force $= [\,?] \cdot$ length
  - Dimension of $k \rightarrow \frac{\text{force}}{\text{length}}$
Units $\rightarrow \frac{N}{m}$ or $\frac{lb}{ft}$
Determine the required length of cord AC so that the 8-kg lamp can be suspended in the position shown. The undeformed spring length is 0.4 m and has a stiffness of 300 N/m.

\[ \begin{align*}
given: & \quad m = 8 \text{ kg} \quad \theta = 30^\circ \\
given: & \quad l_0 = 0.4 \text{ m} \\
given: & \quad k = 300 \text{ N/m} \\
find: & \quad l_{AC} 
\end{align*} \]

Start w/FBD at A

\[ \begin{align*}
T_{AC} \\
F_S \\
mg
\end{align*} \]

Equations of equilibrium:

\[ \begin{align*}
\Sigma F_x: & \quad F_S - T_{AC} \cos \theta = 0 \quad (eq. 1) \\
\Sigma F_y: & \quad T_{AC} \sin \theta - mg = 0 \quad (eq. 2)
\end{align*} \]
the spring force is \( F_s = k\Delta s = k(l_f - l_0) \)

\[
k(l_f - l_0) = \overline{A_C} \cos \theta = \left( \frac{m g}{\sin \theta} \right) \cos \theta
\]

\( \rightarrow \) (eq. 2) \[

l_f = \left( \frac{m g}{k} \right) \frac{\cos \theta}{\sin \theta} + l_0 = 0.853 \text{ m}
\]

using geometrical constraint:

\[
2\mu = l_f + \overline{A_C} \cos \theta
\]

\[
\overline{A_C} = \frac{2\mu - l_f}{\cos \theta} = \frac{2 - 0.853}{\cos \theta} = 1.32 \text{ m}
\]
Cable ABC has a length of 5 m. Determine the position \( x \) and the tension developed in ABC required for equilibrium of the 100-kg sack.

\[ \text{given:} \quad M = 100 \text{ kg} \]
\[ \text{\( l_{ABC} = 5 \text{ m} \)} \]

\[ \text{find:} \quad x, \ T_{ABC} \]

**Eqns. of equilibrium**

\[ \Sigma F_x: \quad T_{BC} \cos \theta_2 - T_{BA} \cos \theta_1 = 0 \]

\[ \Sigma F_y: \quad T_{BA} \sin \theta_1 + T_{BC} \sin \theta_1 - Mg = 0 \]

But for a pulley, we know

\[ T_{BA} = T_{BC} = T \]

\[ \therefore \quad T \cos \theta_1 = T \cos \theta_2 \quad \therefore \quad \theta_1 = \theta_2 \]
\[ \Sigma F_y: \quad 2T \sin \theta = mg \]

\[
T = \frac{mg}{2 \sin \theta}
\]

*almost there, need to solve for \( \theta \)*

**Using geometric constraints**

**Total length:**

\[ 5M = l_{AB} + \ell_{BC} \]

Express using \( \theta \)

\[ x = l_{AB} \cos \theta \]
\[ (3.5 - x) = \ell_{BC} \cos \theta \]

\[ 5M = \frac{x}{\cos \theta} + \frac{3.5 - x}{\cos \theta} = \frac{3.5}{\cos \theta} \]

\[ \theta = \cos^{-1} \left( \frac{3.5}{5} \right) = 45.57^\circ \]
This allows you to solve for $T$, but still need to find $x$...

Again using geometry:

$$x \tan \theta + 0.75 = (3.5 - x) \tan \theta$$

$$2x \tan \theta = 3.5 \tan \theta - 0.75$$

$$x = \frac{3.5 \tan \theta - 0.75}{2 \tan \theta} = 1.38 \text{ m}$$

$$T = \frac{mg}{2 \sin \theta} = 687 \text{ N}$$
3D force systems

Find the tension developed in each cable

given: mag dir of applied force
unknown: \( T_1 \), \( T_2 \), \( T_3 \)
plan: -draw FBD of A
- find direction & magnitude forces
- use equation of equilibrium

\[ \sum \mathbf{F} = (\Sigma F_x) \mathbf{i} + (\Sigma F_y) \mathbf{j} + (\Sigma F_z) \mathbf{k} \]

Determining direction vectors and use

\[ \mathbf{\hat{u}}_1 = \frac{1}{\sqrt{5}} \mathbf{i} + \frac{1}{\sqrt{5}} \mathbf{j} + \frac{1}{\sqrt{5}} \mathbf{k} \]

\[ \mathbf{\hat{u}}_2 = \left[ \frac{-3}{5}, \frac{4}{5}, 0 \right] \]

\[ \mathbf{\hat{u}}_3 = \left[ 0, \frac{-3}{5}, \frac{4}{5} \right] \]
<table>
<thead>
<tr>
<th>\vec{F}</th>
<th>T</th>
<th>\vec{C}</th>
<th>\vec{F}</th>
</tr>
</thead>
<tbody>
<tr>
<td>\frac{1}{T_1}</td>
<td>T_1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\frac{3}{5} T_2</td>
<td>\frac{4}{5} T_2</td>
<td>\frac{4}{5} T_3</td>
<td>0</td>
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<tr>
<td>0</td>
<td>\frac{3}{5} T_3</td>
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<tr>
<td>\vec{F}</td>
<td>0</td>
<td>0</td>
<td>\vec{F} = 0</td>
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</tbody>
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\[ \sum F_x = 0 \]
\[ \sum F_y = 0 \]
\[ \sum F_z = 0 \]

\[ \sum F_x: \quad T_1 - \frac{3}{5} T_2 = 0 \]
\[ \sum F_y: \quad -\frac{3}{5} T_3 + \frac{4}{5} T_2 = 0 \]
\[ \sum F_z: \quad \frac{4}{5} T_3 - 900 = 0 \]

\[ \Rightarrow \quad T_3 = \frac{5}{4} (900) N \]

\[ \therefore \quad T_3 = 1125 N \]

Now using \( \sum F_y \)

\[ T_2 = \frac{3}{4} T_3 = \frac{844}{3} N \]
$$T_2 = \frac{3}{4} T_3 = 844 \text{ N}$$

finally,

$$T_1 = \frac{2}{5} T_2 = \frac{2}{5} (844) = 506 \text{ N}$$