To do ...

- **Quiz 1** sign up now!
  - Tues – Fri of next week (Sept 12 – 15)
  - "Practice" quiz available

- HW4 due **Tues**
- HW 5 due **Thurs**
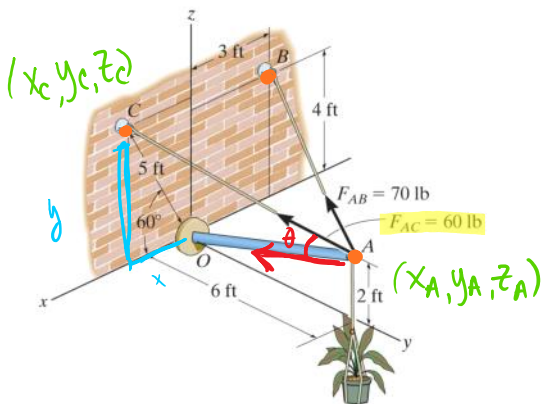
Example

Determine the projected component of the force vector F_{AC} along the axis of strut AO. Express your result as a Cartesian vector

(x_c, y_c, z_c)

$F_{AB} = 70$ lb
$F_{AC} = 60$ lb

(x_A, y_A, z_A)

given: $|F_{AB}|$, $|F_{AC}|$, x,y,z of A, B, C

PLAN: — use Dot product
— find unit vectors $\vec{u}_{AC}$ and $\vec{u}_{AO}$
— write force vector $\vec{F}_{AC}$ as cartesian vector
— take dot product of $\vec{F}_{AC}$ and $\vec{u}_{AO}$

$$\vec{u}_{AC} = \frac{(x_c - x_A)\hat{\imath} + (y_c - y_A)\hat{\jmath} + (z_c - z_A)\hat{k}}{|\vec{u}_{AC}|}$$

$$\vec{u}_{AC} = \frac{(5\cos(60)-0)\hat{\imath} + (0-6)\hat{\jmath} + (5\sin(60)-2)\hat{k}}{|\vec{u}_{AC}|}$$

$$\vec{u}_{AC} = 0.362\hat{\imath} - 0.869\hat{\jmath} + 0.338\hat{k} \qquad *\left(\begin{array}{c}\text{unit}\\\text{vector}\end{array}\right)$$

$$\vec{u}_{AO} = \frac{(x_o - x_A)\hat{\imath} + (y_o - y_A)\hat{\jmath} + (z_o - z_A)\hat{k}}{|\vec{u}_{AO}|} = \frac{(0-0)\hat{\imath} + (0-6)\hat{\jmath} + (0-2)\hat{k}}{|\vec{u}_{AO}|}$$

$$\vec{u}_{AO} = -0.949\hat{\jmath} - 0.316\hat{k} \qquad *\left(\begin{array}{c}\text{unit}\\\text{vector}\end{array}\right)$$

$$\vec{F}_{AC} = F_{AC}\,\vec{u}_{AC} = 60\left(0.362\hat{\imath} - 0.869\hat{\jmath} + 0.338\hat{k}\right) \text{lb} \qquad *(\text{cartesian vector})$$

$$\left(F_{AC}\right)_{AO} = \vec{F}_{AC} \cdot \vec{u}_{AO} = F_{AC}\,\vec{u}_{AC} \cdot \vec{u}_{AO} = F_{AC} \cdots = 43.057 \text{ lb}$$

✓ unit of lb because

$$\left(T_{AC} \right)_{AO} = T_{AC} \cdot U_{AO} = \quad T_{AC} \quad U_{AC} \quad U_{AO} \quad \cdots$$

* unit of lb because

* unit of lb because
unit vector $\vec{u}_{AO}$ dimensionless

$$\left(\vec{F}_{AC} \right)_{AO} = \left(F_{AC} \right)_{AO} \vec{u}_{AO} = 43 \left(0\hat{\imath} - 0.9487\hat{\jmath} - 0.3162\hat{k} \right) \text{ * } \begin{pmatrix} \text{cartesian} \\ \text{vector} \end{pmatrix}$$

Chapter 3: Equilibrium of a particle
Main goals and learning objectives

- Introduce the concept of a free-body diagram for an object modelled as a particle

- Solve particle equilibrium problems using the equations of equilibrium

General procedure for analysis

1. Read the problem carefully; write it down carefully.

2. Model the problem: Draw given diagrams neatly and construct additional figures as necessary.

3. Apply principles needed.

4. Solve problem symbolically. Make sure equations are dimensionally homogeneous

5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).

6. See if answer is reasonable.

Equilibrium of a particle

According to Newton's first law of motion , a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

$$\sum F = 0$$

where $\sum F = 0$ is the resultant force vector of all forces acting on a particle.

In three dimensions, equilibrium requires:

STATIC EQUILIBRIUM
if $\vec{v} = 0$

$$\sum \vec{F} = \sum f_x \hat{\imath} + \sum f_y \hat{\jmath} + \sum f_z \hat{k}$$

necessary And sufficient !

$\sum f_x = 0$
$\sum f_y = 0$
$\sum f_z = 0$

Account for All Known And unKnown forces!

What happens if the forces Are coplanar?

then $\sum f_x \hat{\imath} + \sum f_y \hat{\jmath} = 0$
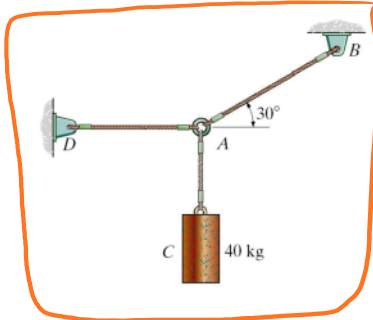
Free body diagram

* CONSIDER this coplanar system

Q: find the tension in the cables
for a given weight.

forces

system of objects

└ DRAWING that shows all external forces
Acting on the body

→ key to writing the equations of equilibrium

→ can draw for any object/subsystem of system,
pick the most appropriate object

1) DRAW outlined shape — imagine object free
of its surroundings
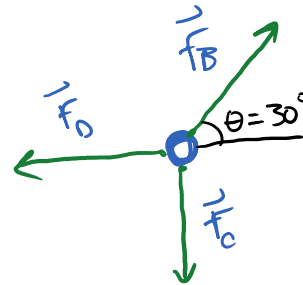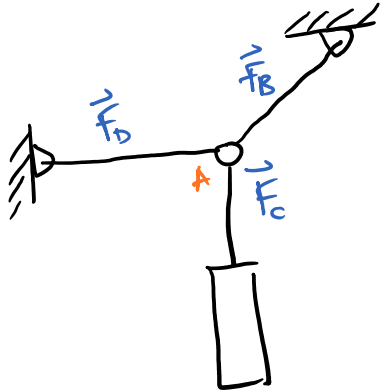
2) Show and identify all forces acting on
the object

NOTE: force represents interaction between two bodies,
vectors with:

— point of Application

— ~~Direction~~

Sense
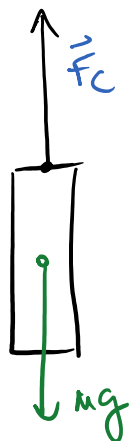
Magnitude

for the system ABOVE:

DRAW FBD of object A:

now, use equations of equilibrium:

$$\Sigma F_x: \qquad F_B \cos\theta - F_D = 0$$

$$\Sigma F_y: \qquad F_B \sin\theta - F_C = 0$$
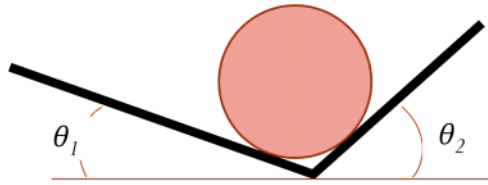
two equations with 3 unknowns. cannot solve!

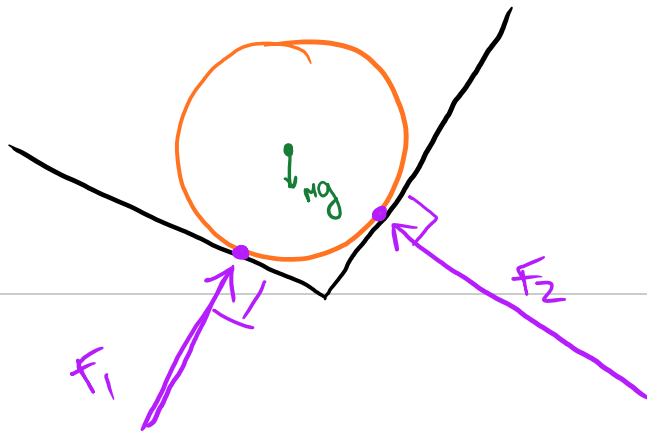unless, we draw another FBD of the mass:

$$\Sigma F_y: \qquad F_C - mg = 0$$

now we solved for F_C,

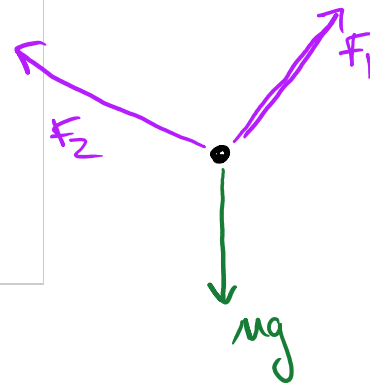we can solve for $\vec{F_D}$ and $\vec{F_B}$

Idealization

Contact force in smooth surface: (frictionLESS)

CONSIDER A uniform sphERE of weight W
BECAUSE the SURFACE IS SMOOTH, force
ACTS perpendicular to SURFACE AT the
point of contAct.

mg

F_2

F_1

this object
is simplifiED
to A particle →
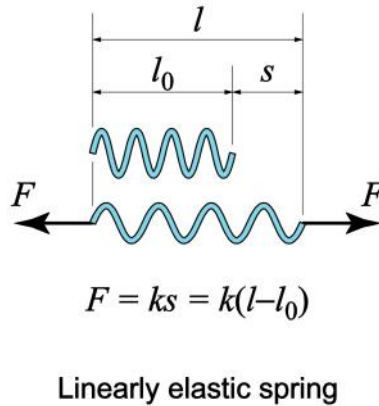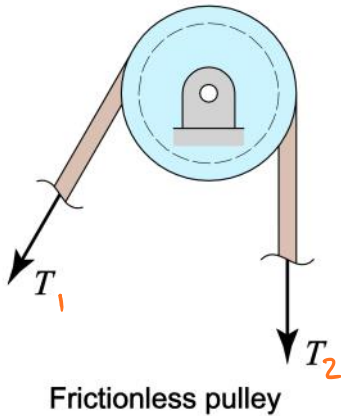b/c of line
of Action!

F_1

F_2

mg

Idealizations

+ MASSLESS

Pulleys are regarded as *frictionless*; then the tension in a rope or cord around the pulley is the same on either side.

Springs are regarded as *linearly elastic*; then the tension is proportional to the *change* in length s.

$$F = ks = k(l - l_0)$$

Frictionless pulley

Linearly elastic spring

$s = l - l_0$

if $s > 0$ then elongation

if $s < 0$ then compression

* cable is rigid no stretching, And massless

$\therefore \quad T_1 = T_2$

* Springs stretch And compress but At $\equiv$M $\Delta s = 0$, no deformation

Equilibrium of a system of particles

Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton's first law
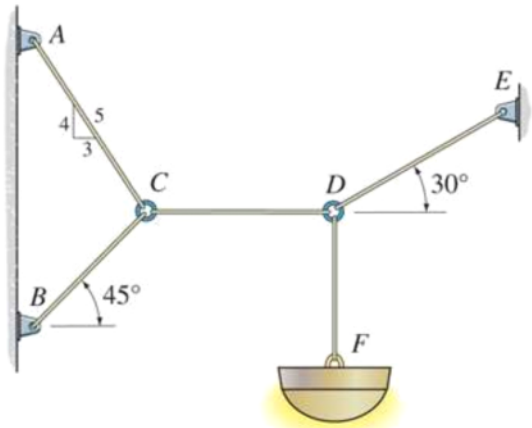
$$\Sigma \mathbf{F} \ = \ \mathbf{0}$$

on selected multiple free-body diagrams of particles or groups of particles.

Applications

For a spool of given weight, how would you find the forces in cables AB and AC? If designing a spreader bar (BC) like this one, you need to know the forces to make sure the rigging (A) doesn't fail.
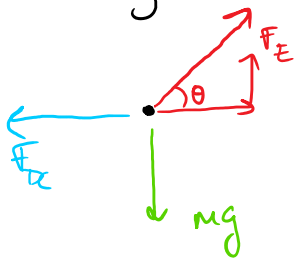
Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400N.

given: constraint of max tension

unknown: 6 unknowns!

Plan: find which cord will have the greatest force for given mass.
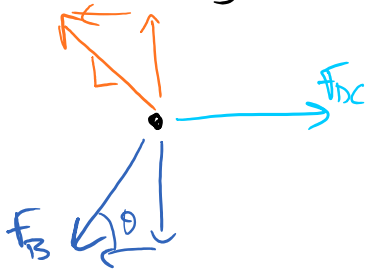
Starting with D:

$\sum f_x:$ $F_E \cos(30) - F_{DC} = 0$

$\sum f_y:$ $F_E \sin(30) - Mg = 0$

$F_E = \dfrac{Mg}{\sin(30)} = \underline{19.62\ m}$

$F_{DC} = F_E \cos(30) = \underline{16.99\ m}$

now moving to C:

$\sum F_x:$ $F_B \cos(45) + \dfrac{3}{5} F_A - F_{DC} = 0$

$\sum f_y:$ $\dfrac{4}{5} F_A - F_B \sin(45) = 0$

taking $\sum F_y$ gives $F_A = \dfrac{5}{4} F_B \sin(45)$

put in to $\sum f_x:$

$F_B \cos(45) + \dfrac{3}{4} F_B \sin(45) - F_{DC} = 0$

$$F_B = \frac{F_{DC}}{\cos(45) + \frac{3}{4}\sin(45)} = \underline{13.73 \ M}$$

$$F_A = \frac{5}{4} F_B \sin(45) = \underline{12.14 M}$$

* therefore cord DE is subject to the greatest force, then the maximum allowable mass is

$$F_M = 400 = 19.62 \ M$$

$$M = \frac{400 \ N}{19.62} = \boxed{20.4 \ Kg}$$