To do …

- **Quiz 1** sign up now!
  - Tues – Fri of next week (Sept 12 – 15)
  - "Practice" quiz available

- HW3 due **Thurs**
- HW4 PL due **Tues**
- **Quiz 2** (Sept 19 – 22)
- Written assignment coming soon

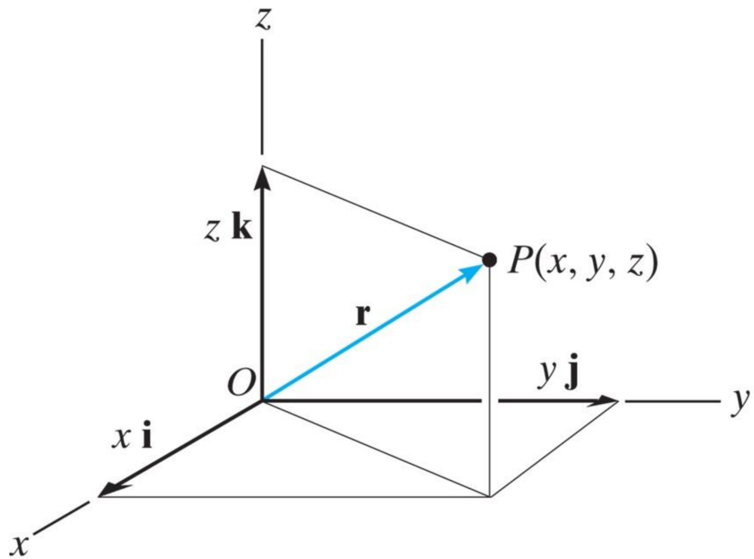
Recap

- A force can be treated as a vector since forces obey all the rules that vectors do

- Vector representations
  - Rectangular components
  - Cartesian components
  - Unit vector
  - Directional cosines

- Position vectors

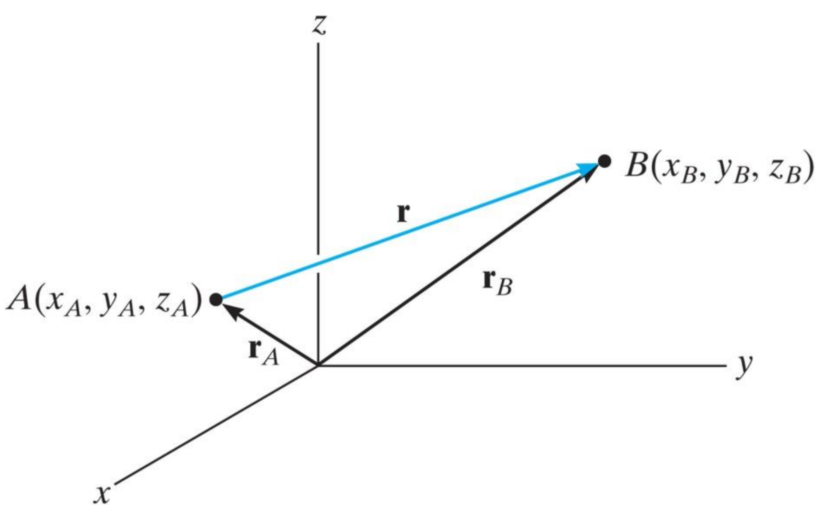
Position vectors



A position vector $\boldsymbol{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example,

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$

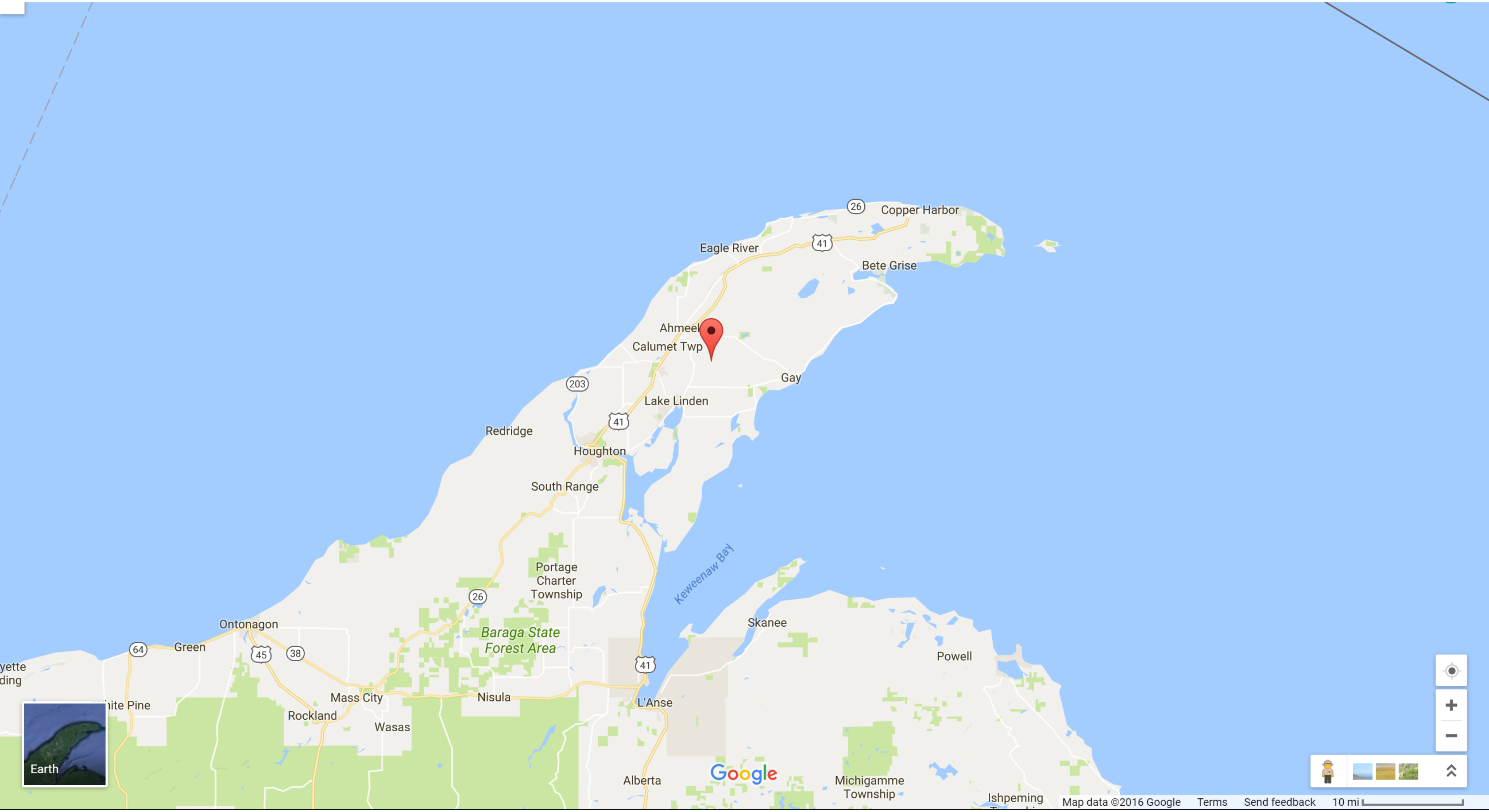
expresses the position of point $P(x,y,z)$ with respect to the origin O.



The position vector $\boldsymbol{r}$ of point $\boldsymbol{B}$ with respect to point $\boldsymbol{A}$ is obtained from

Hence,

Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).



Copper Harbor
Eagle River
Bete Grise
Ahmeek
Calumet Twp
Gay
Lake Linden
Redridge
Houghton
South Range
Portage
Charter
Township
Keweenaw Bay
Baraga State
Forest Area
Skanee
Powell
Ontonagon
Green
Mass City
Nisula
Rockland
Wasas
L'Anse
Alberta
Michigamme
Township
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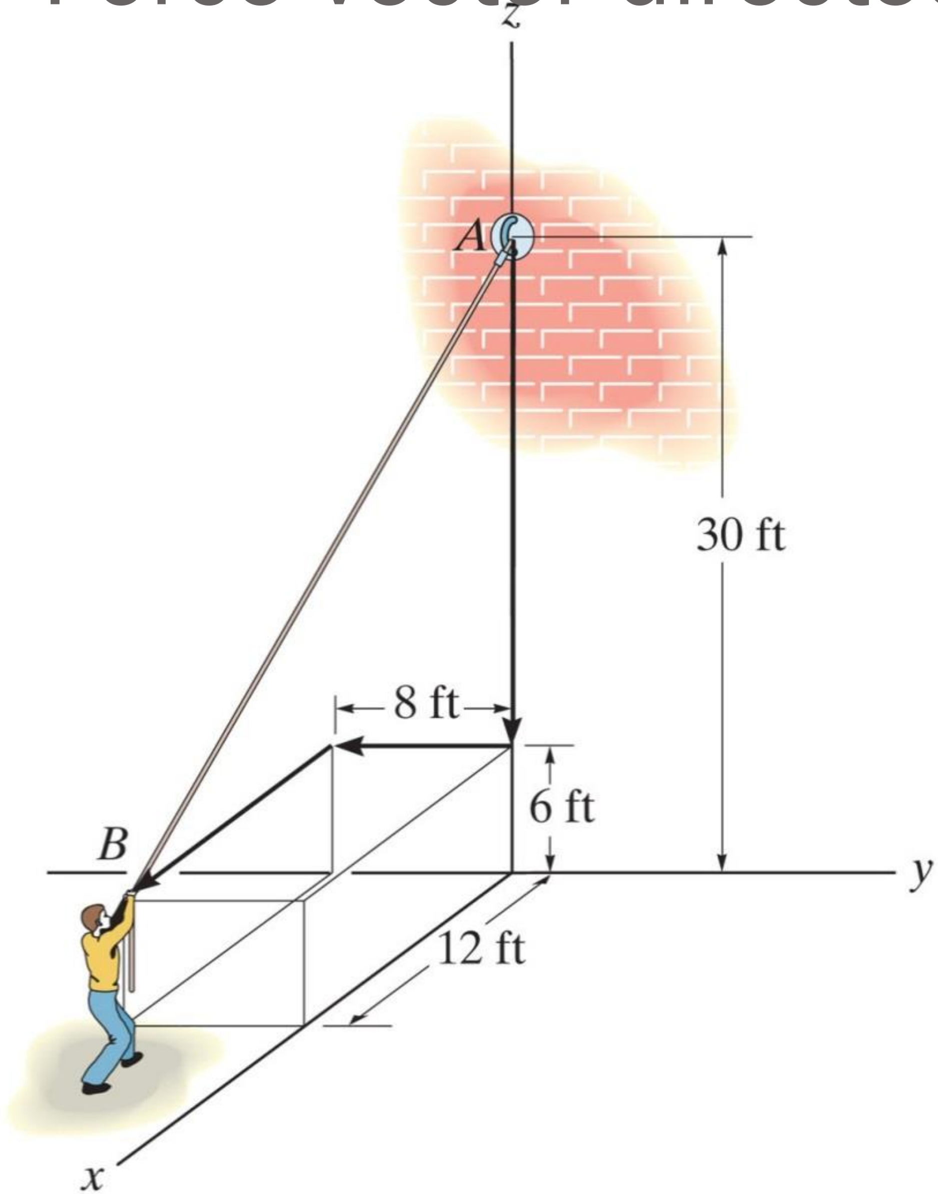
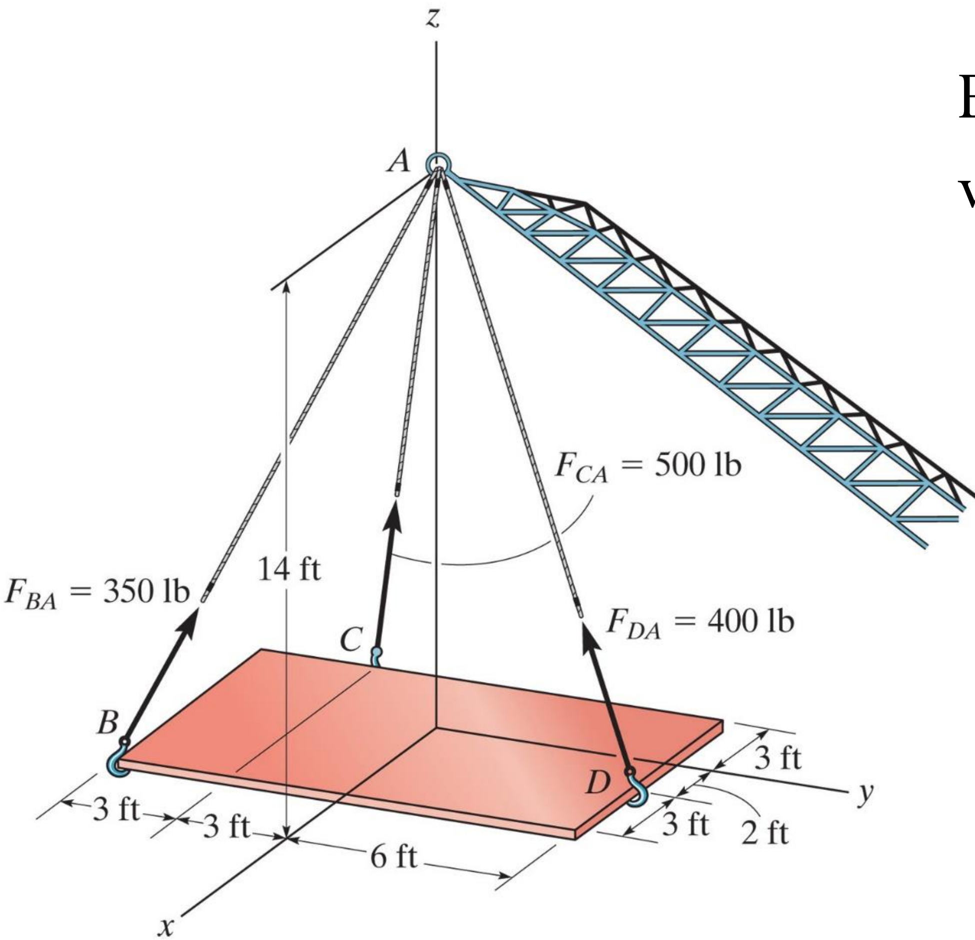
Force vector directed along a line

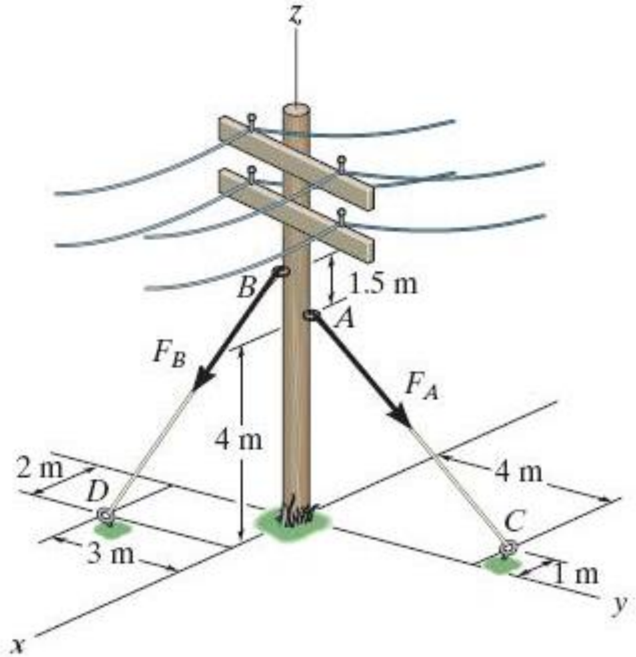


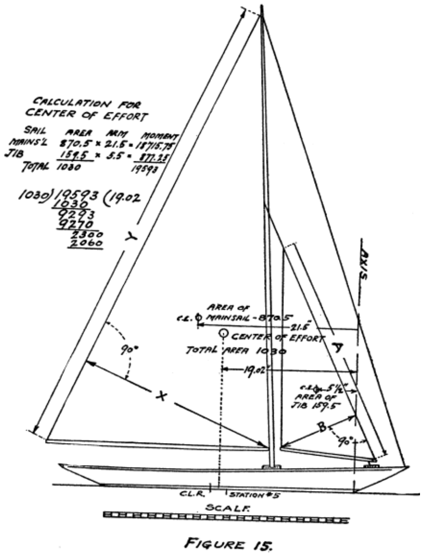
The man pulls on the cord with a force of 70 lb. Represent the force F as a Cartesian vector.



Express each force as a Cartesian vector

Why do we care?





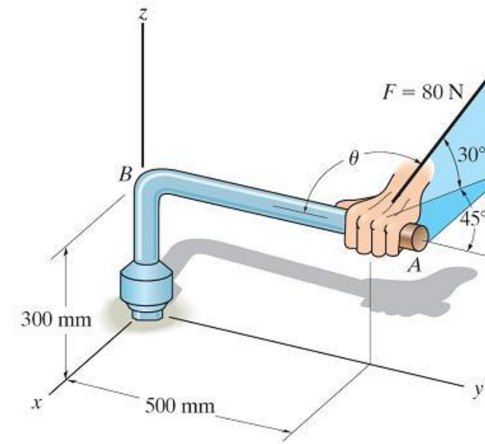
Force vector directed along a line

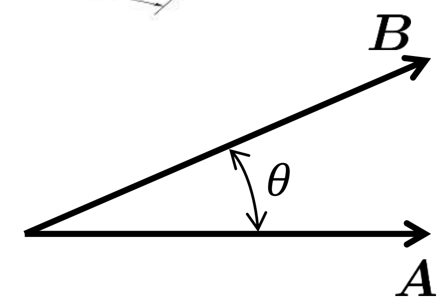
Don’t look up!

Dot (or scalar) product

The dot product of vectors **A** and **B** is defined as such

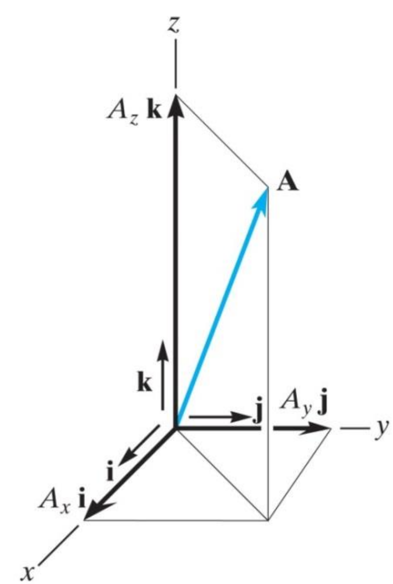
$$\boldsymbol{A} \cdot \boldsymbol{B} =$$



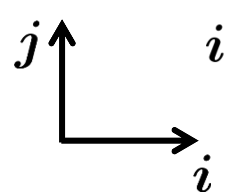


Cartesian vector formulation:

$$\boldsymbol{A} \cdot \boldsymbol{B} \quad =$$



Note that:

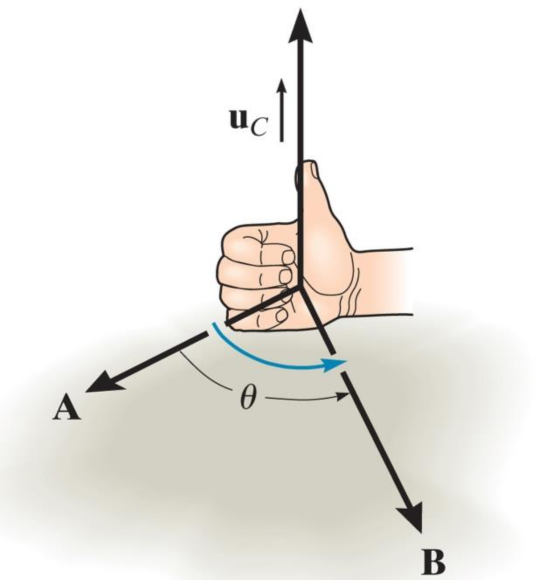


$$\boldsymbol{i} \cdot \boldsymbol{j} = 0$$

$$\boldsymbol{i} \cdot \boldsymbol{i} = 1$$

Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written



$$\boxed{\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}}$$

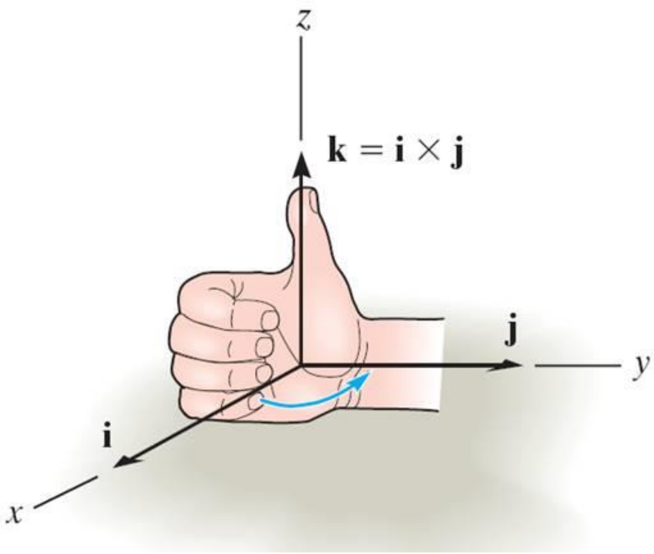
The magnitude of vector **C** is given by:

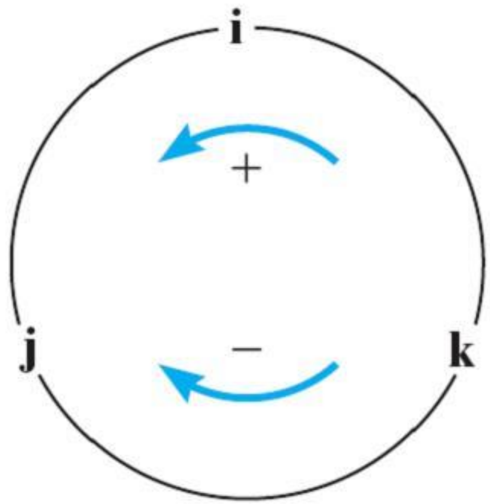
The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

$$\boldsymbol{C} =$$

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$





Considering the cross product in Cartesian coordinates

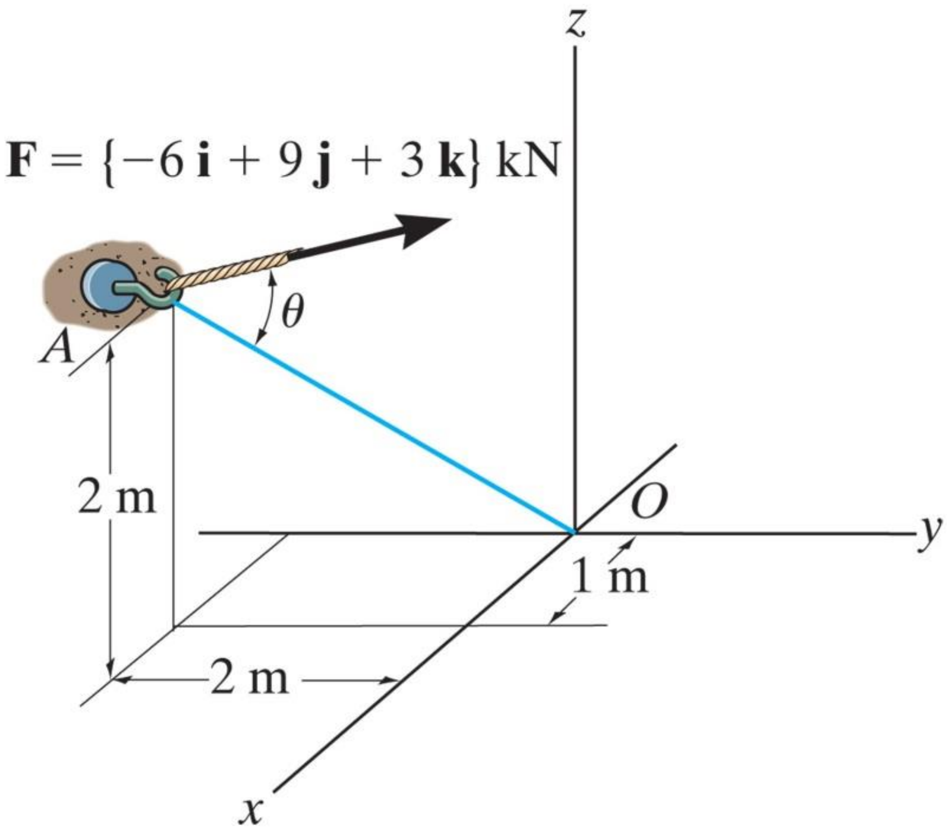
$$\boldsymbol{A} \times \boldsymbol{B} = (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}) \times (B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k})$$

Cross (or vector) product

Also, the cross product can be written as a determinant.

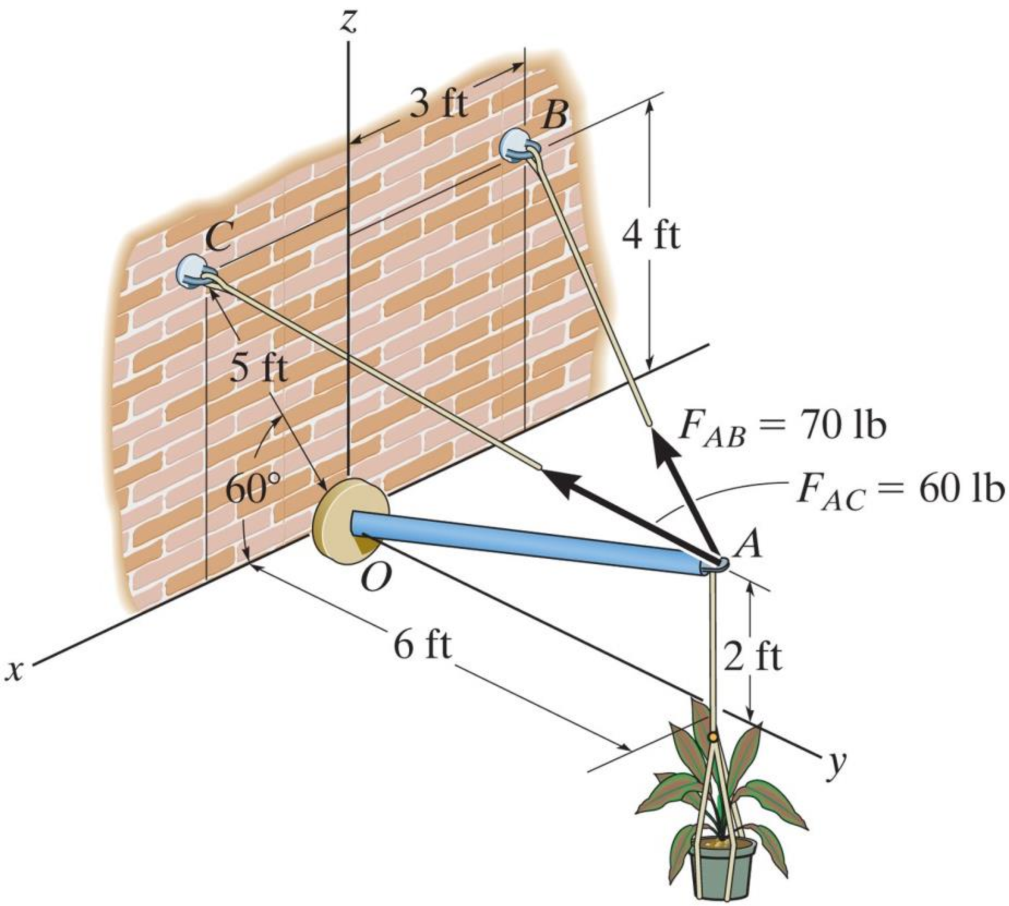
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2×2 determinants.



Given: The force acting on the hook at point A.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.



Determine the projected component of the force vector F_{AC} along the axis of strut AO. Express your result as a Cartesian vector

Chapter 3: Equilibrium of a particle
Main goals and learning objectives

- Introduce the concept of a free-body diagram for an object modelled as a particle

- Solve particle equilibrium problems using the equations of equilibrium

Applications

For a spool of given weight, how would you find the forces in cables AB and AC? If designing a spreader bar (BC) like this one, you need to know the forces to make sure the rigging (A) doesn't fail.

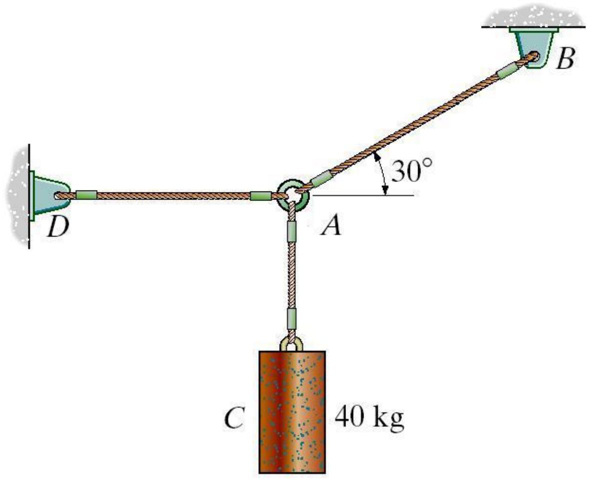
Equilibrium of a particle

According to Newton's first law of motion , a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

$$\sum \boldsymbol{F} = \mathbf{0}$$

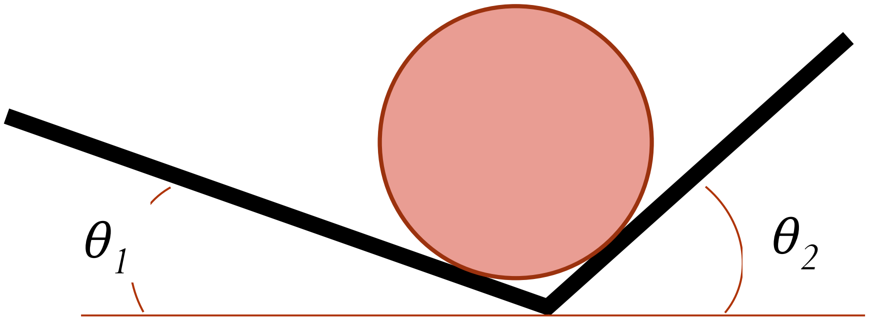
where $\sum \boldsymbol{F} = \mathbf{0}$ is the resultant force vector of all forces acting on a particle.

In three dimensions, equilibrium requires:



Free body diagram

Equilibrium of a particle (cont.)



Contact force in smooth surface: