TAM 210/211 - Worksheet 4

Objectives: evaluate moments in 2D and 3D problems; obtain resultant forces and moments for equivalent systems.

1. Moment of a force - scalar and vector formulation

a) Figure 1: determine the moment of the force about point O using the scalar formulation.

\[ M = -F \cos \theta (l_2 \sin \alpha) + F \sin \theta (l_2 \cos \alpha + l_1) \]

\[ M = 36.71 \text{ N.m} \]

b) Figure 2: determine the moment of the force \( F = 200i - 300j + 140k \) about point O using the vector formulation.

\[ \mathbf{r}_{OB} = \langle 0.3, 0.4, -0.2 \rangle \]

\[ \mathbf{M} = \mathbf{r}_{OB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.4 & -0.2 \\ 200 & -300 & 140 \end{vmatrix} = (0.4(140) - 0.2(300)) \mathbf{i} - (0.3(140) - (-0.2)(200)) \mathbf{j} + (0.3(-300) - 0.4(200)) \mathbf{k} \]

\[ \mathbf{M} = \langle -4, -82, -170 \rangle \text{ N.m} \]
2. Moment of a force about an axis

c) Figure 2: determine the magnitude of the moment of the force $\mathbf{F} = 200\mathbf{i} - 300\mathbf{j} + 140\mathbf{k}$ N about the $OA$ axis.

\[
\mathbf{\hat{u}}_{OA} = \frac{\mathbf{r}_{OA}}{|\mathbf{r}_{OA}|} = \frac{\langle 0.3, 0.4, 0 \rangle}{\sqrt{0.3^2 + 0.4^2 + 0^2}} = \langle 0.6, 0.8, 0 \rangle
\]

Projection

\[
M_{OA} = \mathbf{\hat{u}}_{OA} \cdot \mathbf{\hat{r}}_{OA} = \langle -4, -8.2, -17 \rangle \cdot \langle 0.6, 0.8, 0 \rangle = -68 \text{ N.m}
\]

Magnitude = 68 N.m

3. Moment of a couple

d) Figure 3: determine the magnitude of $\mathbf{F}$ so that the resultant couple moment is 600 lb-ft counterclockwise. Where on the beam does the resultant couple moment act?

\[
M = 200(1.5) - F\cos 30^\circ (d)
\]

\[
F = \frac{200(1.5) - 600}{1.25 \sin 60^\circ \cos 30^\circ}
\]

\[
F = 320 \text{ lb}
\]

![Figure 3](image-url)
2. Moment of a force about an axis

c) Figure 2: determine the magnitude of the moment of the force \( \mathbf{F} = 300\mathbf{i} - 200\mathbf{j} + 150\mathbf{k} \) N about the \( OA \) axis.

To find the moment about an axis \( \mathbf{M}_{0x} \), we need the moment about a point on that axis \( \mathbf{r} \) and a unit vector pointing in the direction of the axis.

\[
\mathbf{M}_{0x} = 200 \mathbf{e} - 105 \mathbf{s} - 180 \mathbf{r} \quad \text{(from part b)}
\]

\[
\mathbf{U}_{0x} = 0.6 \mathbf{e} + 0.8 \mathbf{s}
\]

\[
\mathbf{M}_{0x} = \mathbf{M}_{0x} \cdot \mathbf{U}_{0x} \quad \Rightarrow \quad \mathbf{M}_{0x} = -72 \text{ N.m}
\]

3. Moment of a couple

d) Figure 3: determine the magnitude of \( \mathbf{F} \) so that the resultant couple moment is 600 lb-ft counterclockwise. Where on the beam does the resultant couple moment act?

\[
\mathbf{M}_{\text{total}} = +600 \mathbf{r} \quad \text{lb-ft} = M_1 + M_2 \quad \text{caused by 200 lb force couple}
\]

\[
\Rightarrow +600 \mathbf{r} = +200(1.5) \mathbf{r} + F(1.25 \cos 30) \mathbf{r}
\]

\[
\Rightarrow \text{solve for } F \quad \Rightarrow |F| = 277.1 \text{ lb}
\]
4. Equivalent systems

e) Figure 4: replace the force system acting on the beam by an equivalent force and couple moment at point \( O \). Sketch your equivalent system on the beam (right) below.

* Need equivalent Force \( (\vec{F}_R) \) & equivalent couple moment

at pt. \( O \) \( (\vec{M}_o) \)

* First, \( \vec{F}_R \) \( \Rightarrow \vec{F}_R = \vec{F}_1 + \vec{F}_2 \)
\[ \vec{F}_R = -4 \hat{\imath} + 8 \left( \frac{3}{5} \right) \hat{j} + 8 \left( \frac{4}{5} \right) \hat{j} \]
\[ \Rightarrow \vec{F}_R = 4.8 \hat{\imath} + 2.4 \hat{j} \text{ KN} \]

* Second, \( \vec{M}_o \), Add the couple \( (15 \text{ KN.m}) \) and the moment contribution of \( \vec{F}_1 \& \vec{F}_2 \) about pt. \( O \)
\[ \vec{M}_o = -15 \]
\[ \Rightarrow \vec{M}_o = -15 - 4(1.5) - 8(\frac{3}{5})(0.5) + 8(\frac{4}{5})(4.5) \]
\[ \Rightarrow \vec{M}_o = 5.4 \hat{k} \text{ KN.m} \]

Figure 4
f) Figure 5: replace the force system acting on the frame by an equivalent force and couple moment at point C. Sketch your equivalent system on the frame (middle) above.

Following the same method as in part (e)

$$\vec{F}_R = -300 \hat{j} - 200 \hat{j} - 400 \hat{j} - 200 \hat{k}$$

$$\Rightarrow \vec{F}_R = -200 \hat{i} - 900 \hat{j} \text{ lb}$$

$$\vec{M}_C = +600 \hat{i} + 300(\hat{x}) + 200(\hat{y}) + 400(\hat{y}) + 200(\hat{x}) \hat{R}$$

$$\Rightarrow \vec{M}_C = 4900 \text{ lb-ft} \hat{R}$$

all equivalent systems :)
g) Figure 5: replace the force system acting on the frame by a single resultant force. Specify where its line of actions intersects member AB, measures from A. Sketch your equivalent system on the frame (right) below.

From part (f), we already know what \( \overrightarrow{F_R} \) is, now place it on AB such that the system remains equivalent but with a single force.

For systems to be equivalent \( \overrightarrow{F_R} \) must be the same and resultant couple moment must be the same.

\[
\overrightarrow{F_R} = -200 \overrightarrow{\hat{r}} - 900 \overrightarrow{\hat{z}} \text{ lb}
\]

\[
M_c = 4900 \text{ lb-ft ccw}
\]

(From part f)

\[
4900 = 200(x) + 900(9)
\]

solve for \( x \) \( \Rightarrow x = 3.44 \text{ ft} \)

distance from A = 3.56 ft

see page 4 for sketch
h) Figure 6: Replace the force system acting on the slab by a single resultant force. Locate its point of application on the slab. Sketch your equivalent system on the slab (right) below.

First, find \( \vec{F}_R \):

\[
\vec{F}_R = -1400 \hat{k} \text{ N}
\]

Now, find \( \vec{M}_o = \sum \vec{r} \times \vec{F} \):

\[
\vec{M}_o = (8 \hat{i} \times -600 \hat{k}) + (10 \hat{j} \times -400 \hat{k})
\]

\[
+ (6 \hat{i} \times 5 \hat{j}) \times 100 \hat{k}
\]

Note that a force that goes through a point does NOT produce a moment about that point.

\[
\vec{M}_o = -3500 \hat{i} + 4200 \hat{j}
\]

Now, place \( \vec{F}_R \) at \( (X, Y, 0) \)

such that the system is equivalent.

(see fig below)

\[
\vec{M}_o = -3500 \hat{i} + 4200 \hat{j}
\]

\[
= (X \hat{i} + Y \hat{j} + 0 \hat{k}) \times \vec{F}_R
\]

\[
\Rightarrow -3500 \hat{i} + 4200 \hat{j} = X (1400) \hat{i} - Y (1400) \hat{j}
\]

i: \[-3500 = -Y (1400)\]

j: \[4200 = X (1400)\]

\(\Rightarrow (X, Y) = (3, 2.5)\) m