To do ...

- HW 31 due Today
- HW 32 – last Mastering Eng – due Friday
- HW 33(PL) – last PL – due Friday

- WA13 – last WA – due Sunday May 1
Equilibrium of a particle

$2D$ vs. $3D$

\[ \sum f_x = 0 \]
\[ \sum f_y = 0 \]
\[ \sum f_z = 0 \]
**Given:** The four forces and geometry shown.

**Find:** The tension developed in cables AB, AC, and AD.

- Draw the FBD
- Write forces in Cartesian vector form

\[
\begin{align*}
\mathbf{T}_B & \\
\mathbf{T}_C & = -T \cos 60 \sin 30 \\
\mathbf{T}_D & = T \cos 120 \\
\mathbf{T}_E & = 0 \\
\Sigma F_x & = T_B - 0.25T_C - 0.5T_D = 0 \quad (1) \\
\Sigma F_y & = 0.48T_C - 0.5T_B = 0 \quad (2)
\end{align*}
\]
\[ \Sigma f_y = 0.43T_c - 0.5b = 0 \]
\[ \Sigma F_2 = 0.87T_c + 0.71T_D - 360 = 0 \] (3)

\[ T_c = 203 \text{ lb} \]
\[ T_D = 176 \text{ lb} \]
\[ T_b = 139 \text{ lb} \]
**Given:** The force and geometry shown.

**Find:** Moment of $F$ about point $A$

\[ \mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.55 & 0.4 & -0.2 \\ 49.5 & 53.1 & -40 \end{vmatrix} = (-5.4 \hat{i} + 13.1 \hat{j} + 11.4 \hat{k}) \text{ N}\cdot\text{m} \]

\[ \mathbf{r}_{AC} = \langle 0.55, 0.4, -0.2 \rangle \text{ m} \]

\[ \mathbf{F} = \langle 80 \cos 30 \sin 40, 80 \cos 30 \cos 40, -80 \sin 30 \rangle \]
Our goal is to unscrew the tapped faucet A. As such, we use two pipe wrenches as indicated below.

Mark the correct statement:

a) The tapped faucet cannot be removed since there is no torque about the z-axis when two wrenches are used

b) The tapped faucet is removed due to the torque provided by the wrench at A about the z-axis equal to 400 lb.in

c) The elbow joint at C is subjected to a torque equal to 800 lb.in about the z-axis and therefore is detached from pipe CD before the faucet at A can be removed
Our goal is to unscrew the tapped faucet A. As such, we use two pipe wrenches as indicated below.

Obtain the resultant couple moment at the elbow C:

\[\alpha \hat{f} (P_a - 2P_a \cos \theta) f\]
\[\beta \hat{f} 2P_a \sin \theta \hat{i} - 2P_a \cos \theta \hat{j}\]
\[\gamma 2P_a \sin \theta \hat{i} + (P_a - 2P_a \cos \theta) \hat{j}\]
\[\delta P_a \sin \theta \hat{i} + (P_a - P_a \cos \theta) \hat{j}\]

**Compute the moment about C.**

\[\vec{N}_C = \vec{r}_{CG} \times \vec{f}_G + \vec{r}_{CH} \times \vec{f}_H\]

\[\vec{r}_{CG} = \langle 0, L, a \rangle\]
\[\vec{r}_{CH} = \langle -L \sin \theta, L \cos \theta, 2a \rangle\]
\[\vec{f}_G = \langle P, 0, 0 \rangle\]
\[\vec{f}_H = \langle -P \cos \theta, -P \sin \theta, 0 \rangle\]

\[\vec{N}_G = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & L & a \\ P & 0 & 0 \end{vmatrix} = OR \hat{i} + Pa \hat{j} - PL \hat{k}\]

\[\vec{N}_H = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -L \sin \theta & L \cos \theta & 2a \\ -P \cos \theta & -P \sin \theta & 0 \end{vmatrix} = 2Pa \sin \theta \hat{c} - 2Pa \cos \theta \hat{j} + (P \sin \theta + P \cos \theta) \hat{f} + PL \hat{k}\]

\[\vec{N}_C = \vec{N}_G + \vec{N}_H = 2Pa \sin \theta \hat{c} + Pa(1 - 2 \cos \theta) \hat{j}\]
Our goal is to unscrew the tapped faucet \( A \). As such, we use two pipe wrenches as indicated below.

\[
\vec{\mathbf{N}}_c = 2 \rho a \sin \theta \mathbf{t} + \rho a (1 - 2 \cos \theta) \mathbf{t}
\]

What should be the angle \( \theta \) if elbow \( C \) is not allowed to rotate about the \( y \)-axis?

\[
\begin{align*}
\text{\( \uparrow \) component} &= 0 \quad \therefore \quad 1 - 2 \cos \theta &= 0 \\
\theta &= \cos^{-1} \left( \frac{1}{2} \right) = 60^\circ
\end{align*}
\]

What would be the corresponding resultant force-couple system at \( C \)?

\[
\begin{align*}
\vec{\mathbf{F}}_E &= \vec{\mathbf{F}}_G + \vec{\mathbf{F}}_A \\
&= \langle \rho (1 - \cos \theta), -\rho a \sin \theta, 0 \rangle \\
&= \langle \frac{1}{2} \rho, \frac{1}{2} \rho, 0 \rangle
\end{align*}
\]
Equilibrium of a rigid body

\[ \sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0 \quad \sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0 \]

And proper constraints!
**TABLE 5-2  Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems**

<table>
<thead>
<tr>
<th>Types of Connection</th>
<th>Reaction</th>
<th>Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) cable</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td><strong>One unknown.</strong> The reaction is a force which acts away from the member in the known direction of the cable.</td>
</tr>
<tr>
<td>(2) smooth surface support</td>
<td><img src="image2.png" alt="Diagram" /></td>
<td><strong>One unknown.</strong> The reaction is a force which acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>(3) roller</td>
<td><img src="image3.png" alt="Diagram" /></td>
<td><strong>One unknown.</strong> The reaction is a force which acts perpendicular to the surface at the point of contact.</td>
</tr>
</tbody>
</table>
### TABLE 5-2  Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

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<tr>
<td>(4)</td>
<td><img src="image" alt="Diagram" /></td>
<td>Three unknowns. The reactions are three rectangular force components.</td>
</tr>
<tr>
<td>ball and socket</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| (5)                 | <img src="image" alt="Diagram" /> | Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are <em>generally not applied</em> if the body is supported elsewhere. See the examples. |
| single journal bearing |          |                    |</p>
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<tr>
<th>Types of Connection</th>
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<tr>
<td>(6)</td>
<td><img src="image" alt="Diagram" /></td>
<td>Five unknowns. The reactions are two force and three couple-moment components. <em>Note:</em> The couple moments <em>are generally not applied</em> if the body is supported elsewhere. See the examples.</td>
</tr>
<tr>
<td>single journal bearing with square shaft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td><img src="image" alt="Diagram" /></td>
<td>Five unknowns. The reactions are three force and two couple-moment components. <em>Note:</em> The couple moments <em>are generally not applied</em> if the body is supported elsewhere. See the examples.</td>
</tr>
<tr>
<td>single thrust bearing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td><img src="image" alt="Diagram" /></td>
<td>Five unknowns. The reactions are three force and two couple-moment components. <em>Note:</em> The couple moments <em>are generally not applied</em> if the body is supported elsewhere. See the examples.</td>
</tr>
<tr>
<td>single smooth pin</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 5–2  Continued

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<th>Types of Connection</th>
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<tr>
<td>(9) single hinge</td>
<td>![Diagram of single hinge]</td>
<td><strong>Five unknowns.</strong> The reactions are three force and two couple-moment components. Note: The couple moments are generally not applied if the body is supported elsewhere. See the examples.</td>
</tr>
<tr>
<td>(10) fixed support</td>
<td>![Diagram of fixed support]</td>
<td><strong>Six unknowns.</strong> The reactions are three force and three couple-moment components.</td>
</tr>
</tbody>
</table>
1. A plate is supported by a ball-and-socket joint at A, a roller joint at B, and a cable at C. How many unknown support reactions are there in this problem?

A) Four forces and two moments
B) Six forces
C) Five forces
D) Four forces and one moment
2. What will be the easiest way to determine the force reaction $B_Z$?

A) Scalar equation $\sum F_Z = 0$

B) Vector equation $\sum \mathbf{M}_A = 0$

C) Scalar equation $\sum M_Z = 0$

D) Scalar equation $\sum M_Y = 0$
1. The rod AB is supported using two cables at B and a ball-and-socket joint at A. How many unknown support reactions exist in this problem?

A) Five force and one moment reaction

B) Five force reactions

C) Three force and three moment reactions

D) Four force and two moment reactions