To do ...

- WA12 due Sunday April 24

- Quiz 6 (last quiz!)

Happy Earth Day!!
Conservative forces

- Work done is independent of path traveled.
- Depend only on initial and final positions.
- Examples are: Gravitational
  Spring / Elastic

Consider the ball:

\[ U_1 = mgh \]
\[ U_2 = mgh \]

For weight:

\[ du = \vec{W} \cdot dr = W (dr \cos \theta) = -Wdy \]

\[ u = -\int_0^h Wdy = -Wh \]
Potential energy

A conservative force can give the body the capacity to do work.

Gravitational potential energy $\rightarrow V_g = mg \cdot y$

Elastic/Spring potential energy $\rightarrow V_e = \frac{1}{2} k s^2$

Potential function: $-V = V_g + V_e$

displacement from equilibrium
Criterion for equilibrium

- valid for a frictionless connected system with one degree of freedom.

Consider the displacement, the work done

\[ du = V_1 - V_2 \implies du = -dv \]

for a virtual displacement

\[ \delta u = \delta v = 0 \]

At equilibrium,

\[ \frac{dv}{dq} = 0 \]

The first derivative of the potential function is zero.

\( x \) also assumes the function and derivatives are continuous.
Stability of equilibrium configuration

- The potential function can be used to investigate the stability of the equilibrium configuration.

Consider the ball:

**Case I**

Consider the displacement $\Delta x$,

- **Case I**
  - Stable
  - returns to original position

- **Case II**
  - Unstable
  - displaces farther away from original position

- **Case III**
  - Neutral
  - Remains in $EM$, potential energy constant.

\[ \Delta x \]
\[
\frac{d^2 v}{dq^2} > 0 \\
\frac{d^2 v}{dq^2} < 0 \\
\frac{dv}{dq^2} = 0
\]
Criterion for equilibrium
The uniform link has a mass of 10 kg. If the spring is unstretched when \( \theta = 0^\circ \), determine the angle for equilibrium and investigate the stability.

- **Draw FBD**
- **Set** \( y = 0 \) / coord. system

**Potential function**

\[
V = V_g + V_e
\]

\[
V_g = W y = W \left( \frac{l}{2} \cos \theta \right)
\]

\[
V_e = \frac{1}{2} k s^2
\]

\[
l = s + lc \cos \theta \quad \Rightarrow \quad s = l \left( 1 - \cos \theta \right)
\]

\[
V_e = \frac{1}{2} kl^2 \left( 1 - \cos \theta \right)^2
\]

\[
V = \frac{1}{2} kl^2 \left( 1 - \cos \theta \right)^2 + W \left( \frac{l}{2} \cos \theta \right)
\]
\[ \frac{dv}{d\theta} = \frac{1}{2} kl^2 \left( 2(1 - \cos \theta) \sin \theta \right) - \frac{w}{2} l \sin \theta = 0 \]

\[ l \left( kl (1 - \cos \theta) - \frac{w}{2} \right) \sin \theta = 0 \]

1. \( \sin \theta = 0 \) \hspace{1cm} \text{At} \hspace{1cm} \theta = 0

2. \( kl (1 - \cos \theta) - \frac{w}{2} = 0 \)

\[ \theta = \cos^{-1} \left( 1 - \frac{w}{2kl} \right) \]

\[ \frac{d^2v}{d\theta^2} = kl^2 (\cos \theta - \cos 2\theta) - \frac{w}{2} \cos \theta \]

Evaluate \( \frac{d^2v}{d\theta^2} \) at \( \theta = 0 \) and \( \theta = \cos^{-1} \left( 1 - \frac{w}{2kl} \right) \)
\[
\frac{d^2 V}{d\theta^2} \bigg|_{\theta = 0} < 0 \quad \text{:: unstable equilibrium}
\]

\[
\frac{d^2 V}{d\theta^2} \bigg|_{\theta = \cos^{-1}(\cdots)} > 0 \quad \text{:: stable equilibrium}
\]
Determine the angle for equilibrium and investigate the stability at this position. The bars each have a mass of 3 kg and the suspended block \( D \) has a mass of 7 kg. Cord \( DC \) has a total length of 1 m.

- **Draw** \( FB\) of force.

**Potential function**

\[
V = V_{g_1} + V_{g_2} + V_{D}
\]

\[
V = 2W_{y_1} - W_{D}y_2
\]

\[
y_1 = \frac{l}{2} \sin \theta
\]

\[
y_2 + x' = y_2 + (2l \cos \theta - l)
\]

\[
y_2 = 1 + l - 2l \cos \theta
\]

\[
V = 2W_{y_1} - W_{D}y_2
\]
\[ V = 2W \left( \frac{l}{2} \sin \theta \right) - W_b (1 + l - 2l \cos \theta) \]

\[ V = W_l \sin \theta - W_b (1 + l - 2l \cos \theta) \]

**Equilibrium Position**

\[ \frac{dV}{d\theta} = W_l \cos \theta - 2W_b l \sin \theta = 0 \]

\[ W_l \cos \theta = 2W_b l \sin \theta \]

\[ \tan \theta = \frac{W}{2W_b} \quad \therefore \theta = \tan^{-1} \left( \frac{W}{2W_b} \right) \]

**Stability**

\[ \frac{d^2V}{d\theta^2} = l \left( -W \sin \theta - 2W_0 \cos \theta \right) \]

\[ \left. \frac{d^2V}{d\theta^2} \right|_{\theta = \tan^{-1} \left( \frac{W}{2W_b} \right)} < 0 \quad \therefore \text{unstable} \]

\[ \text{Equilibrium} \]
The cylinder is made of two materials such that it has a mass of $m$ and a center of gravity at point $G$. Show that when $G$ lies above the centroid of the cylinder, the equilibrium is unstable.

Potential function

$$V = V_g = W_y = mg y$$

$$y = r + a \cos \theta$$

$$V = mg(r + a \cos \theta)$$

Equilibrium position

$$\frac{dV}{d\theta} = -mg \sin \theta = 0$$

1. $\sin \theta = 0$ @ $\theta = 0$ and $\theta = \pi$

Stability
\[ \frac{d^2V}{d\theta^2} = -mg\cos\theta \]

\[ \left. \frac{d^2V}{dq^2} \right|_{\theta=0} = -mg\cos(0) = -mg(1) < 0 \quad \text{Unstable Equilibrium} \]

\[ \left. \frac{d^2V}{dq^2} \right|_{\theta=\pi} = -mg\cos(\pi) = -mg(-1) > 0 \quad \text{Stable Equilibrium} \]