Housekeeping

- WA4 posted – due **Sunday Feb 21st**
- Quiz 2 (**Tues Feb 16 – Sat Feb 20**) – **DO IT!**
Rapid Refresh ...

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1. If a support prevents translation of a body, then the support exerts a
   __________ on the body.

   A) Couple moment
   B) Force
   C) Both A and B.
   D) None of the above
Equilibrium of a Rigid Body

In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body.

We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point \( O \).

\[
\begin{align*}
\sum F_x &= 0 \\
\sum F_y &= 0 \\
\sum M_o &= 0
\end{align*}
\]
Equilibrium in two-dimensional bodies

Support reactions

1. normal to surface
   F

2. smooth pin or hinge
   prevent translation
   not rotation
   Fy

3. fixed support
   prevent translation
   and rotation
   Fx
### Types of Connectors

**TABLE 5-1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems**

<table>
<thead>
<tr>
<th>Types of Connection</th>
<th>Reaction</th>
<th>Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) cable</td>
<td><img src="cable_diagram" alt="Diagram" /></td>
<td>One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.</td>
</tr>
<tr>
<td>(2) weightless link</td>
<td><img src="weightless_diagram" alt="Diagram" /></td>
<td>One unknown. The reaction is a force which acts along the axis of the link.</td>
</tr>
<tr>
<td>(3) roller</td>
<td><img src="roller_diagram" alt="Diagram" /></td>
<td>One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>(4) rocker</td>
<td><img src="rocker_diagram" alt="Diagram" /></td>
<td>One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.</td>
</tr>
</tbody>
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Types of connectors

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<th>Types of Connection</th>
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<tr>
<td>smooth contacting surface</td>
<td>$\theta_1$, $\theta_2$, $\theta_3$</td>
<td>One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>roller or pin in confined smooth slot</td>
<td>$\theta_1$, $\theta_2$, $\theta_3$</td>
<td>One unknown. The reaction is a force which acts perpendicular to the slot.</td>
</tr>
<tr>
<td>member pin connected to collar on smooth rod</td>
<td>$\theta_1$, $\theta_2$, $\theta_3$</td>
<td>One unknown. The reaction is a force which acts perpendicular to the rod.</td>
</tr>
</tbody>
</table>

continued
## Types of connectors

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<tr>
<th>Types of Connection</th>
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<tr>
<td>smooth pin or hinge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two unknowns. The reactions are two components of force, or the magnitude and direction $\theta$ of the resultant force. Note that $\phi$ and $\theta$ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].</td>
<td></td>
<td></td>
</tr>
<tr>
<td>member fixed connected to collar on smooth rod</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fixed support</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction $\phi$ of the resultant force.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The uniform truck ramp has weight 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the reaction forces at the pins and the tension in the cables.

\[ \Sigma F_x: \quad T \cos(20) - A_x = 0 \]  
\[ \Sigma F_y: \quad A_y - W - T \sin(20) = 0 \]

\[ T \left( 7 \cos(20) \sin(20) - 7 \sin(20) \cos(20) \right) = 5 \cos(30) \ W \]

\[ T = \frac{5(400) \cos(30)}{7(\cos(30) \sin(20) - \sin(30) \cos(20))} = 1425 \text{ lb} \]

\[ A_x = T \cos(20) = 1339 \text{ lb} \]

\[ A_y = W + T \sin(20) = 887 \text{ lb} \]
The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at B is 20 lb. Determine the vertical force applied to the pedal.

\[ \Sigma F_x: \quad 20 + 30 - A_x = 0 \quad (1) \]

\[ \Sigma F_y: \quad A_y - F = 0 \quad (2) \]

\[ \hat{M}_x = 5F - (1 \times 30) - (2.5 \times 20) = 0 \quad (3) \]

\[ F = \frac{30 + 50}{5} = 16 \text{ lb} \]
iQ>clicker

2. The beam and the cable (with a frictionless pulley at D) support an 80 kg load at C. In a FBD of only the beam, there are how many unknowns?

A) Two forces and one couple moment
B) Three forces and one couple moment
C) Three forces
D) Four forces

\[
\sum F_x: \quad A_x - T \frac{3}{5} = 0
\]

\[
\sum F_y: \quad A_y + T + \frac{4}{5}T - U = 0
\]

\[
\sum M = 0
\]

\[
2T + 4 \times \frac{4}{5}T - 5.5U = 0
\]
3. How many unknown support reactions are there in this problem?

A) Two forces and two couple moments
B) One force and two couple moments
C) Three forces
D) Three forces and one couple moment
Two-force members

In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.

This fact simplifies the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).

\[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M = 0 \]
**Given:** The 4kN load at B of the beam is supported by pins at A and C.

**Find:** The support reactions at A and C.

**Approach:** Draw x-y axis and FBD

- Any 2-force member?

\[ \sum F_x: \quad A_x + F_c \cos(45) = 0 \quad (2) \]

\[ \sum F_y: \quad A_y + F_c \sin(45) - 4kN = 0 \quad (3) \]

\[ (M_x)_{A} = (1.5)(F_c \sin(45)) - (4kN)(3) = 0 \quad (4) \]

\[ F_c = \frac{12kN}{1.5 \sin(45)} = \boxed{11.3 \text{ kN}} \]

\[ A_x - F_c \cos(45) = -8 \text{ kN} \]

\[ A_y = 4kN - F_c \sin(45) = -4 \text{ kN} \]
The lever ABC is pin supported at A and connected to a short link BD. If the weight of the members is negligible, determine the reaction forces at pins D and A.

\[ M_R = (0.1)(F_0 \sin 45\degree) + (0.2)(F_0 \cos 45\degree) - (400)(0.7) = 0 \]

\[ F = \frac{(0.7)(400)}{(0.1) \sin 45\degree + (0.2) \cos 45\degree} = 1.32 \text{ kN} \]

\[ \sum F_x: \quad 400 + A_x - F_0 \cos 45\degree = 0 \quad (1) \]

\[ \sum F_y: \quad A_y - F_0 \sin 45\degree = 0 \quad (2) \]

\[ A_x = F_0 \cos 45\degree - 400 \]

\[ A_y = F_0 \sin 45\degree \]
How many “two-force” members in this system?

A) 0
B) 1
C) 2
D) 3
E) 4
The woman exercises on the rowing machine. If she exerts a holding force of \( F = 200 \text{ lb} \) on the handle ABC, determine the reaction force at pin C and the force developed along the hydraulic cylinder BD on the handle.

### Idealized Model

\[
\begin{align*}
F &= 200 \text{ N} \\
F_{\text{AC}} &= (\langle -9, 0.5 \rangle \mu) \text{ N} \\
F_{\text{BC}} &= (\langle 2, 0.25 \rangle \mu) \text{ N} \\
F_{\text{BD}} &= (\langle -9, 0.22 \rangle \mu) \\
\end{align*}
\]

\[
(M_{\text{C}})_C = F_{\text{BC}} \times F_{\text{B}} - F_{\text{AC}} \times F = 0
\]

\[
(M_{\text{C}})_C = \begin{bmatrix} x & y & z \\ F_{\text{xB}} & F_{\text{yB}} & 0 \\ F_{\text{xB}} & F_{\text{yB}} & 0 \\ \end{bmatrix} - \begin{bmatrix} F_{\text{AC}} & F_{\text{ay}} & 0 \\ F_{\text{xB}} & F_{\text{yB}} & 0 \\ \end{bmatrix} = 0
\]

\[
\overline{F} = \overline{v}_B \overline{F}_B = F_{\text{B}} \overline{v}_B + F_{\text{yB}}
\]

\[
(M_{\text{C}})_C = (F_{\text{xB}} F_{\text{yB}} - F_{\text{yB}} F_{\text{xB}}) \hat{k} - (F_{\text{xB}} F_{\text{yB}} - F_{\text{yB}} F_{\text{xB}}) \hat{k} = 0
\]

\[
F_{\text{B}} \overline{v}_B = F_{\text{B}} \overline{v}_B + F_{\text{yB}}
\]

\[
\begin{align*}
F_{\text{x}} &= F_{\text{cos}(30)} - F_{\text{Bx}} + A_x = 0 \\
F_{\text{y}} &= F_{\text{sin}(30)} - F_{\text{By}} + A_y = 0
\end{align*}
\]

\[ Q: \text{What is direction of } \overline{v}_B? \]
Constraints

To ensure equilibrium of a rigid body, it is not only necessary to satisfy equations of equilibrium, but the body must also be properly constrained by its supports.

- **Redundant constraints**: the body has more supports than necessary to hold it in equilibrium; the problem is **STATICALLY INDETERMINATE** and cannot be solved with statics alone.

- **Improper constraints**: In some cases, there may be as many unknown reactions as there are equations of equilibrium. However, if the supports are not properly constrained, the body may become unstable for some loading cases.
**Given:** The beam is supported by the roller at A and a pin at B.

**Find:** The reactions at points A and B on the beam.

\[ \Sigma F_x: \quad N_A \cos(30) - A_x = 0 \quad (1) \]

\[ \Sigma F_y: \quad A_y - N_A \sin(30) - F_R = 0 \quad (2) \]

\[ (M_R)_A = 2F_R - (4 + 3\cos(30))N_A \sin(30) - 3\sin(30)(N_A \cos(30)) = 0 \quad (3) \]

\[ N_A = \frac{2 \left( \frac{4}{\sin(30)} \right)}{(4 + 3\cos(30)) - 3\cos(30)} = 3.71 \text{ kN} \]

\[ A_x = N_A \cos(30) = 1.86 \text{ kN} \]

\[ A_y = F_R + N_A \sin(30) = 8.78 \text{ kN} \]