Housekeeping

- HW 10 due **Tonight**
- WA4 posted – due **Sunday Feb 21st**
- Quiz 2 (**Tues Feb 16 – Sat Feb 20**) – **DO IT!**

- Prof Kersh – 126 MEB
  - Fridays 9:00-10:00

- Prof Juarez – 4415 MEL
  - Mondays 15:00-16:00
Rapid Refresh ...
1. The resultant force ($F_R$) due to a distributed load is equivalent to the _____ under the distributed loading curve, $w = w(x)$.

A) Centroid  B) Arc length  
C) Area  D) Volume

\[
F_R = \sum dF = \int_0^L dF = \int_0^L w(x) dx = \text{Area}
\]

Q: What are the units of $w(x)$?

\[
\text{force/length} \rightarrow \frac{N}{m}
\]

\[
w(x) = \frac{dF}{dx}
\]
2. The line of action of the distributed load’s equivalent force passes through the _____ of the distributed load.

A) Centroid  B) Mid-point  C) Left edge  D) Right edge

\[ M_e = \sum x dF = \int_0^l x dF = \int_0^l x w(x) dx \]

\[ \mu_e = \bar{x} \frac{F_e}{F} \quad \Rightarrow \quad \bar{x} = \frac{\mu_e}{F_e} = \frac{\int_0^l x w(x) dx}{\int_0^l w(x) dx} = \text{Geometric Center} \]
3. What is the location of $F_R$, i.e., the distance $d$?

A) 2 m  
B) 3 m  
C) 4 m  
D) 5 m  
E) 6 m

$F_R = \text{Area} = \frac{1}{2} L \omega_0$.

$F_R = \text{Area} = \frac{1}{2} \omega_0 L$.
What is the equivalent system?
iQ>clicker

1. For this force system, the equivalent system at P is __________.

   A) $F_{RP} = 40 \text{ lb (along } +x\text{-dir.) and } M_{RP} = +60 \text{ ft} \cdot \text{lb}$
   B) $F_{RP} = 0 \text{ lb and } M_{RP} = +30 \text{ ft} \cdot \text{lb}$
   C) $F_{RP} = 30 \text{ lb (along } +y\text{-dir.) and } M_{RP} = -30 \text{ ft} \cdot \text{lb}$
   D) $F_{RP} = 40 \text{ lb (along } +x\text{-dir.) and } M_{RP} = +30 \text{ ft} \cdot \text{lb}$
Reduction of a simple distributed load

\[ \Sigma F_y : F_e = \left( \frac{L}{2}, 0 \right) F_0 = \frac{w_0 L}{2} \]

\[ \vec{M}_e = \vec{r} \cdot F_e = \left( \frac{L}{2} + \frac{L}{2} \frac{w_0 L}{2} \right) F_0 = \left( \frac{5}{6} \right) \frac{w_0 L}{2} \left( \frac{w_0 L}{2} \right) = \frac{5w_0^2 L^2}{24} \cdot C_0 \]

Q: units? Direction?
Replace the loading by an equivalent resultant force and specify its location on the beam measured from point B.

1. Sum the forces
2. Sum the moments.

\[ \sum F_y: \quad (q \cdot 5000) + \left( \frac{1}{2} \cdot q \cdot 300 \right) + \left( \frac{1}{2} \cdot 12 \cdot 800 \right) = \]

\[ \bar{N}_R = \left( \frac{1}{3} \cdot 12 \right) \left( \frac{1}{2} \cdot 12 \cdot 800 \right) - \left( \frac{1}{3} \cdot 9 \cdot 5000 \right) - \left( \frac{1}{3} \cdot q \cdot \frac{1}{2} \cdot q \cdot 300 \right) = \ldots \]
iQ>clicker

Determine the length “a” such that the resultant force and couple moment acting on the beam are zero.

A) 7.5 ft
B) 3.75 ft
C) 4.5 ft
D) 9.75 ft

1. Sum forces
   \[ \sum F_y = -40b + \frac{1}{2} \cdot 6 \cdot 60 \Rightarrow b = \frac{180}{40} \]

2. Sum moments.
   \[ M_R = \left( 10 + \frac{1}{3} \cdot 6 \right) \left( \frac{1}{2} \cdot 6 \cdot 60 \right) - \left( a + \frac{b}{2} \right) \left( 40 \cdot b \right) = 0 \]
FTW!!

\[
\Sigma F_x: \quad f_b \cos(75)
\]

\[
\Sigma F_y: \quad f_b \sin(75) - W_B - \frac{1}{2} a \omega_o
\]

\[
\vec{N}_r = (2b)f_b \sin(75) - (2b+a)W_B - (2b+\frac{2}{3}a)(\frac{1}{2} a \omega_o)
\]
Chapter 5: Equilibrium of Rigid Bodies
Equilibrium of a Rigid Body

In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body.

We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point O.

\[ \sum \mathbf{F}_o + \sum \mathbf{M}_o = 0 \]

\[ R = \sum \mathbf{F}_o - \sum \mathbf{M}_o \]

\[ \mathbf{N}_r = \sum \mathbf{F}_o + \sum \mathbf{M}_c \] (free vectors)

\[ \mathbf{M}_o = \sum (\mathbf{r} \times \mathbf{F}) \]

\[ \mathbf{N}_c = \mathbf{N}_i + \mathbf{M}_z \]
Equilibrium of a Rigid Body

Static equilibrium:
\[ \sum F = 0 \quad \text{no/constant translation!} \]
\[ \sum M = 0 \quad \text{no rotation!} \]

Maintained by reaction forces and moments
- forces from supports/constraints are exactly enough to produce zero forces and moments.

Assumption of rigid body
- shape/dimensions unchanged due to applied loads
- internal forces are never shown. To cancel out not create external effect on body

Necessary and sufficient conditions
Process of solving rigid body equilibrium problems

1. Create idealized model (modeling and assumptions)

2. Draw free body diagram showing ALL the external (applied loads and supports)

3. Apply equations of equilibrium

\[ \Sigma F_x: T_x - R_x = 0 \]
\[ \Sigma F_y: -T_y - W + R_y = 0 \]
\[ \Sigma \tau: \tau = 0 \]

Constraint forces/couples outlined shape of the body, free of surroundings.
Equilibrium in two-dimensional bodies

Support reactions

- Support prevents translation by exerting a force in opposite direction.
- Support prevents rotation by exerting a couple moment in opposite direction.
The uniform truck ramp has weight 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the reaction forces at the pins and the tension in the cables.

\[ \sum F_x: \quad T \cos(\theta) - A_x = 0 \quad (1) \]

\[ \sum F_y: \quad A_y - W - T \sin(\theta) = 0 \quad (2) \]