Housekeeping

- WA2 due **Sunday Feb 7th**
- Quiz 1 (**Tues Feb 2 – Sat Feb 6**)  
  - CBTF instructions on website
  - No calculator, use matlab!
  - Good luck! or Congrats!

Καλημέρα!  
(Kaliméra) → Good morning!

*Check compass for grades on WA1 and i>clcker.*
Recap ...

• Forces
  - Obey the rules of vectors... magnitude and direction
  - Components added/subtracted
    \[ -F_x, F_y, F_z \] for \( \mathbf{j}, \mathbf{k} \)
  - \( \mathbf{i}, \mathbf{j}, \mathbf{k} \)

• Idealizations
  - Forces transmitted through... cables (\( \mu=0 \) rigid)
  - Smooth contact (frictionless/normal)
  - Pulleys (\( \mu=0 \) frictionless)
  - Springs (\( m=0 \)) for \( \Delta \mathbf{S} \)

• Static equilibrium of a particle

\[
\sum \mathbf{F} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k}
\]

\[= 0 \]

\[= 0 \]

\[= 0 \]
Chapter 4: Force System Resultants
Applications

Beams are often used to bridge gaps in walls. We have to know what the effect of the force on the beam will have on the supports of the beam.

\[ \sum \vec{F} = m \vec{a} = 0 \]

tendency for a body to move, or translate
Applications

Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force $F_h$ at the handle pull the nail? How can you mathematically model the effect of force $F_h$ at point O?

**Moment** 1. a very brief period of time. An Exact point in time. 2. importance. 3. A turning Effect produced by a force acting at a distance on An object.
Applications

Using moments

If suspended, a body will come to rest with its centre of mass directly below the point of suspension.

Moment 1. a very brief period of time. An exact point in time. 2. importance. 3. A turning effect produced by a force acting at a distance on an object.
Moment of a force  \((i \geq \text{clicker})\)

1) Which force(s) have NO turning effect?

\[ A, D, \quad \checkmark \quad E \]
Moment of a force

1) Which force(s) yields a “tighty” effect?

E AND C
Moment of a force

1) Which force(s) yields a “loosey” effect?

B and A

obot!!
Moment of a force – scalar formulation

The moment of a force about a point provides a measure of the tendency for rotation (sometimes called torque).

\[ M = Fd \]

**Magnitude of a Moment:**

\[ M_o = Fd \]

**Direction?** = perpendicular to the plane that contains \( F \) \& moment arm \( d \)

* Use right hand rule!

- Counter clockwise \( \rightarrow G + M \) \( \frac{1}{2} \)
- Clockwise \( \rightarrow C - M \) \( -\frac{1}{2} \)

**Units** = combination of a physical quantity and a distance
And a distance (meter) \cdot (Newton) \uparrow kg \cdot m/s^2

Consider the following:

Q: What is the moment due to $F$ about $O$?

$$N_0 = (r_x F_y - r_y F_x) \hat{k}$$
Moment of a force – vector formulation

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).

The moment of a force $\mathbf{F}$ about point $\mathbf{O}$, or actually about the moment axis passing through $\mathbf{O}$ and perpendicular to the plane containing $\mathbf{O}$ and $\mathbf{F}$, can be expressed using the cross (vector) product, namely:

\[
\vec{M}_o = \vec{r} \times \vec{F}
\]

(position vector directed from $\mathbf{O}$ to any point along the line of action $\mathbf{F}$)

\[
\vec{M} = \begin{pmatrix} r \cos \theta, -r \sin \theta \end{pmatrix} \times \begin{pmatrix} F, 0, 0 \end{pmatrix}
\]

\[
\vec{M} = |\vec{r}| |\vec{F}| \sin \theta \, \hat{z}
\]

*Cross product – interactions b/n different dimensions*
Moment of a force – vector formulation

\[ \vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \]

\[ \vec{M} = (r_y F_x - r_z F_y) \hat{i} - (r_x F_z - r_z F_x) \hat{j} + (r_x F_y - r_y F_x) \hat{k} \]

Q: What components cause rotation about the x axis?

Consider the following.
\[ \vec{M} = (r_y \vec{F}_2 - \vec{F}_3 \times \vec{F}_2) \hat{i} + \ldots \]
Resultant moment of a system of forces

**Given:**
\[ F_1 = \{100 \, i - 120 \, j + 75 \, k\} \, \text{lb} \]
\[ F_2 = \{-200 \, i + 250 \, j + 100 \, k\} \, \text{lb} \]

**Find:** Resultant moment by the forces about point O.

**Approach:**
1. Find resultant \( \vec{F}_R \)
2. Find moment about \( \vec{r}_{AO} \)

\[
\vec{F}_R = F_1 + F_2 = (100 - 200)\hat{i} + (-120 + 250)\hat{j} + (75 + 100)\hat{k} \, \text{lb}
\]
\[
= (-100)\hat{i} + (130)\hat{j} + (175)\hat{k} \, \text{lb}
\]

Now the moment about \( \vec{r}_{AO} \):

\[
\vec{r}_{AO} = 4\hat{i} + 5\hat{j} + 3\hat{k}
\]

\[
\vec{M} = \vec{r}_{AO} \times \vec{F} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
4 & 5 & 3 \\
-100 & 130 & 175
\end{vmatrix} = (5(175) - 3(130))\hat{i}
\]
\[
- (4(175) - 3(-100))\hat{j}
\]
\[
+ (4(175) - 5(-100))\hat{k} \, \text{lb}\cdot\hat{r}
\]

\[
\vec{M} = 485\hat{i} - 1000\hat{j} + 1020\hat{k} \, \text{lb}\cdot\hat{r}
\]
i>clicker question

1. If a force of magnitude F can be applied in four different 2-D configurations (P, Q, R, & S), select the cases resulting in the maximum and minimum torque values on the nut. (Max, Min).

   A) (Q, P)           B) (R, S)  
   C) (P, R)           D) (Q, S)
i>clicker question

2. If \( \mathbf{M} = \mathbf{r} \times \mathbf{F} \), then what will be the value of \( \mathbf{M} \cdot \mathbf{r} \)?

A) 0  
B) 1  
C) \( r^2 F \)  
D) None of the above.

we know that \( \mathbf{M} \) is perpendicular to \( \mathbf{r} \) and \( \mathbf{F} \)

\[ \mathbf{M} \cdot \mathbf{r} = 0 \]  
and  
\[ \mathbf{M} \cdot \mathbf{F} = 0 \]