Housekeeping

- Excused absence form (see Policies on website) can be used for issues due to late registration, lack of access (send a screen shot)
- Mastering Engineering
  - HW1 was due last night
  - HW2 due **Wednesday** (tonight)
- HW3 Prairie Learn due **Friday** (Jan 29)
  - Prairie Learn guide on website schedule!
- WA1 due **Friday** (Jan 29 – Compass)
- CATME is coming
- Quiz 1 (next week)
- i>clicker time…
Recap

- Pay attention to units!
- Solve problem symbolically
- Equations must be dimensionally homogenous
- 3 significant figures
- Scalar – defined by magnitude (negative/positive)
- Vector – defined by magnitude and direction
- Resolve vector into components
- Vector operations – addition/subtraction
i>clicker time

1. Which one of the following is a scalar quantity?
   A) Force  B) Position  C) Mass  D) Velocity

2. For vector addition, you have to use ______ law.
   A) Newton’s Second
   B) the arithmetic
   C) Pascal’s
   D) the parallelogram
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3. Vector algebra, as we are going to use it, is based on a __________ coordinate system.

   A) Euclidean   B) Left-handed
   C) Greek       D) Right-handed   E) Egyptian

4. The symbols $\alpha$, $\beta$, and $\gamma$ designate the __________ of a 3-D Cartesian vector.

   A) Unit vectors   B) Coordinate direction angles
   C) Greek societies D) X, Y and Z components
Addition of a system of coplanar vectors

Resolve into x-y components

\[ \vec{F}_r = \sum F_x \hat{i} + \sum F_y \hat{j} \]

\[ \vec{F}_r = (F_{rx} + F_{ex} + F_{sx}) \hat{i} + (F_{ry} + F_{ey} + F_{sy}) \hat{j} \]

Magnitude of resultant vector is

\[ |\vec{F}_r| = \sqrt{F_{rx}^2 + F_{ry}^2} \]

Direction of the resultant vector is

\[ \theta = \tan^{-1} \left( \frac{F_{ry}}{F_{rx}} \right) \]
Example

a) **Resolve** the forces into their x-y components.

b) **Add** the respective **components** to get the resultant vector.

c) Find **magnitude** and **angle** from the resultant components.

\[
\begin{align*}
\vec{F}_1 &= 300 \left[ 0\hat{i} + 1\hat{j} \right] \text{N} \\
\vec{F}_2 &= 450 \left[ \cos(135)\hat{i} + \cos(45)\hat{j} \right] \text{N} \\
\vec{F}_3 &= 600 \left[ \left( \frac{3}{5} \right)\hat{i} + \left( \frac{4}{5} \right)\hat{j} \right] \\
\frac{F_{x}}{F_{3}} &= \frac{3}{5} \rightarrow F_{x} = \frac{3}{5} F_{3} \\
\frac{F_{y}}{F_{3}} &= \frac{4}{5} \rightarrow F_{y} = \frac{4}{5} F_{3}
\end{align*}
\]

b) \[ \vec{F}_R = \sum F_{x}\hat{i} + \sum F_{y}\hat{j} = (F_{x} + F_{x} + F_{x})\hat{i} + (F_{y} + F_{y} + F_{y})\hat{j} \]

\[ F_{x}\hat{i} + F_{y}\hat{j} \]

C) **Magnitude:**
\[ |\vec{F}_R| = \sqrt{F_{x}^2 + F_{y}^2} \]

**Direction:**
\[ \theta = \tan^{-1} \left( \frac{F_{y}}{F_{x}} \right) \]
**Magnitude of Cartesian vectors**

\[ A = |A| = \sqrt{A_x^2 + A_y^2 + A_z^2} \]

**Direction of Cartesian vectors**

Expressing the direction using a unit vector:

\[ \hat{a} = \frac{\vec{A}}{|A|} = \frac{A_x}{A} \hat{i} + \frac{A_y}{A} \hat{j} + \frac{A_z}{A} \hat{k} \]

Rewrite \( \vec{A} \) as function of cosine:

\[ \vec{A} = A \cos(\alpha) \hat{i} + A \cos(\beta) \hat{j} + A \cos(\gamma) \hat{k} \]

**Addition of Cartesian vectors**

\[ \vec{R} = \vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} + (A_z + B_z) \hat{k} \]
Example

The cables attached to the screw eye are subjected to the three forces shown.

(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector.

(c) Determine the direction cosines of the resultant force vector.

\[
\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}
\]

\[
\hat{i}, \hat{j}, \hat{k}
\]

\[
\vec{A} = A \cos(\omega) \hat{i} + A \cos(\beta) \hat{j} + A \cos(\gamma) \hat{k}
\]

\[
A_x = \cos(\omega)
\]

\[
A_y = \cos(\beta)
\]

\[
A_z = \cos(\gamma)
\]

\[
\hat{i}, \hat{j}, \hat{k}
\]

\[
\vec{F}_1 = 350 \begin{bmatrix} 0 & \cos(50) & \cos(40) \end{bmatrix} \text{N}
\]

\[
\vec{F}_2 = 100 \begin{bmatrix} \cos(45) & \cos(60) & \cos(120) \end{bmatrix} \text{N}
\]

\[
\vec{F}_3 = 250 \begin{bmatrix} \cos(60) & \cos(135) & \cos(60) \end{bmatrix} \text{N}
\]

\[
b) \quad |\vec{F}_2| = \sqrt{\sum F_{x}^2 + \sum F_{y}^2 + \sum F_{z}^2}
\]

\[
c) \quad \cos(\alpha) = \frac{F_{x}}{|\vec{F}_2|} \quad \cos(\beta) = \frac{F_{y}}{|\vec{F}_2|} \quad \cos(\gamma) = \frac{F_{z}}{|\vec{F}_2|}
\]
Why do we care?

X Build & DESIGN stuff that works!
Position vectors

A position vector \( \mathbf{r} \) is defined as a fixed vector which locates a point in space relative to another point. For example, 
\[
\mathbf{r} = x \hat{i} + y \hat{j} + z \hat{k}
\]
expresses the position of point \( P(x, y, z) \) with respect to the origin \( O \).

The position vector \( \mathbf{r} \) of point \( B \) with respect to point \( A \) is obtained from
\[
\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A
\]
Hence,
\[
\mathbf{r} = (x_B, y_B, z_B) - (x_A, y_A, z_A)
\]
\[
\mathbf{r} = (x_B - x_A) \hat{i} + (y_B - y_A) \hat{j} + (z_B - z_A) \hat{k}
\]
Thus, the \((i, j, k)\) components of the position vector \( \mathbf{r} \) may be formed by taking the coordinates of the tail (point \( A \)) and subtracting them from the corresponding coordinates of the head (point \( B \)).
Example

The ring at D is midway between points A and B. Determine the lengths of wires AD, BD and CD.

\[ \text{first find } \mathbf{x}, \mathbf{y}, \mathbf{z}. \]

\( (x_A, y_A, z_A) = (2, 0, 1.5) \, \text{m} \)
\( (x_B, y_B, z_B) = (0, 2, 0.5) \, \text{m} \)
\( (x_C, y_C, z_C) = (0, 0, 2) \, \text{m} \)

\[ x_D = \frac{(x_A + x_B)}{2} = \frac{2+0}{2} = 1 \, \text{m} \]
\[ y_D = \frac{y_A + y_B}{2} = \frac{0+2}{2} = 1 \, \text{m} \]
\[ z_D = \frac{z_A + z_B}{2} = \frac{1.5+0.5}{2} = 1 \, \text{m} \]

\[ \mathbf{r}_{AD} = (x_D-x_A) \mathbf{i} + (y_D-y_A) \mathbf{j} + (z_D-z_A) \mathbf{k} = (-1) \mathbf{i} + (1-0) \mathbf{j} + (1-1.5) \mathbf{k} \]

\[ \mathbf{r}_{AD} = -1 \mathbf{i} + 1 \mathbf{j} - 0.5 \mathbf{k} \]

\[ |\mathbf{r}_{AD}| = \sqrt{1+1+0.25} = 1.5 \, \text{m} \]
Force vector directed along a line

The force vector $\mathbf{F}$ acting along the rope can be defined by the unit vector $\mathbf{u}$ (defined the direction of the rope) and the magnitude of the force.

$$\mathbf{F} = F \mathbf{u}$$

The unit vector $\mathbf{u}$ is specified by the position vector:

$$\mathbf{r} = (x_B - x_A) \mathbf{i} + (y_B - y_A) \mathbf{j} + (z_B - z_A) \mathbf{k}$$

$$\mathbf{u} = \frac{\mathbf{r}}{|\mathbf{r}|}$$

The man pulls on the cord with a force of 70 lb. Represent the force $\mathbf{F}$ as a Cartesian vector.

$$\mathbf{r} = (12 - 0) \mathbf{i} + (-8 - 0) \mathbf{j} + (6 - 30) \mathbf{k}$$

$$\mathbf{r}_{AB} = [12 \mathbf{i} - 8 \mathbf{j} - 24 \mathbf{k}] \text{ ft}$$

$$|\mathbf{r}_{AB}| = 28 \text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{12}{28} \mathbf{i} - \frac{8}{28} \mathbf{j} + \frac{24}{28} \mathbf{k}$$

$$\mathbf{F}_{AB} = 70 \mathbf{u}_{AB} = 70 \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|}$$
i>clacker time

5. If you know only $u_A$, you can determine the ________ of $A$ uniquely.
   A) magnitude  
   B) angles ($\alpha$, $\beta$ and $\gamma$)
   C) components ($A_x$, $A_y$, & $A_z$)  
   D) All of the above.

6. What is not true about an unit vector, e.g., $u_A$?
   A) It is dimensionless. ✓
   B) Its magnitude is one. ✓
   C) It always points in the direction of positive X-axis. ✗
   D) It always points in the direction of vector $A$. ✓