13.1 Starting with the equations of isotropic linear thermo-elasticity

\[ \sigma_{ij} = \lambda u_{i,k} \delta_{ij} + \mu (u_i,_j + u_j,_i) - \eta (T - T_{ref}) \delta_{ij} \]

\[ \rho C_V \dot{T} = \kappa \nabla^2 T + Q - \eta T_{ref} \dot{u}_{k,k} \]

\[ \partial_i \sigma_{ij} = \rho \ddot{u}_j \]

where \( \lambda, \mu \) are the isothermal moduli.

Find the speed of longitudinal adiabatic waves (ie, the speed when \( \kappa \) goes to zero.)

What is the ratio of displacement amplitude to temperature amplitude for such waves?

(are the temperature and displacement in phase or out of phase with each other?)

13.2 Homogeneous medium thermo-elastodynamics has a material-dependent characteristic time scale \( \tau = D/c_L^2 \). Where D is heat diffusivity (units of m^2/sec) \( D = \kappa / \rho C_V \).

What is that time scale for aluminum? (You'll need to look up the properties of aluminum. Be careful about units/check your dimensions. Thermal properties are reported in many different guises.) (Recall \( \eta \) is a simply related to the thermal expansion coefficient.)

For low frequencies \( \omega << 1/\tau \), show that the longitudinal wave (the one with speed \( \sqrt{\lambda + 2\mu / \rho} \)) has almost the adiabatic speed. Is this the relevant regime for MHz ultrasonics?

In this regime \( \omega << 1/\tau \), what is the dimensionless attenuation/wavenumber = -Im k/Re k as a function of \( \omega \)? Evaluate this in aluminum at \( f = 1 \) MHz, \( \omega = 2\pi f \).

13.3 Given a Maxwell–model material and using units such that speed is unity at high frequency, and intrinsic time scale \( \tau = 1 \), plot the real part of the speed, and the attenuation (equal to -Im(\( \omega \)/speed)) versus frequency.

13.4 It is desired to construct a filter \( f(t) \) that attenuates a signal by 90% over a frequency band from -1 to +1, and to leave it unchanged for all other frequencies (a lo-pass-filter). The magnitude of its FT is plotted here.

Assuming that \( f(\omega) \) has no zeros in the LHP, (so that its log is analytic there) find the required phase function \( \phi(\omega) \) such that \( f = \text{Fexp}(i\phi(\omega)) \) is the FT of a function \( f(t) \) that is causal.