Starvation Dynamics of a Greedy Forager

Bhat U, Redner S and Bénichou O 2017 J. Stat. Mech. 073213

Zejun Liu, Chunyu Lu, Siddharth Mansingh, Robert McKay

Marshall, J. (2017, July 12). Does greed help a forager survive? Retrieved from https://phys.org/news/2017-07-greed-forager-survive.html

Overview

1. Introduction to model
2. Formalisms of model
3. 1D results
4. 2D simulation
5. Analysis of previously existing work
6. Citation evaluation
7. Future outlook

Motivation

- Should one continue to exploit a rich localized cache in a “desert”? Or is it better to move to a region where resources are more plentiful overall?
- Learn about how organisms gain nourishment
- What does greed do to one’s species and ecosystem?

Investigating the Model Used For the Greedy Forager

Model Components:

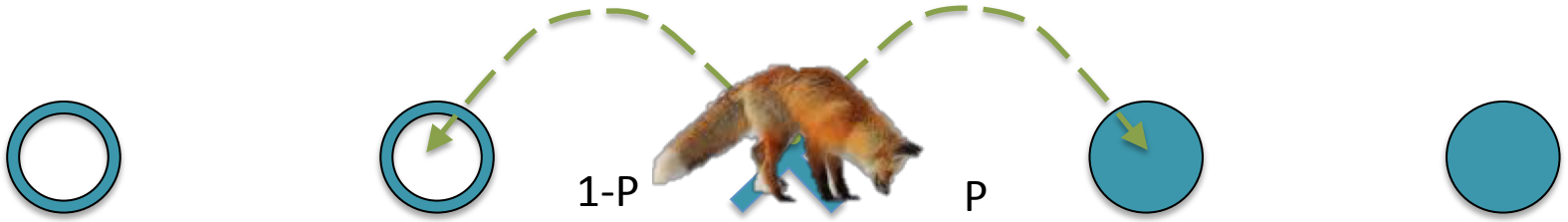
- Random Walk Modeling
- Parameterized by greed $-1 \leq G \leq 1$
- $\mathcal{S}$ metabolic capacity

Measures of Model:

- $\mathcal{N}$ amount of food consumed
- $\mathcal{T}$ average lifetime of forager

Basic Probability Model

Consuming food generates a “desert”:



1D: $$p = \frac{1+G}{2}$$ ← Greed

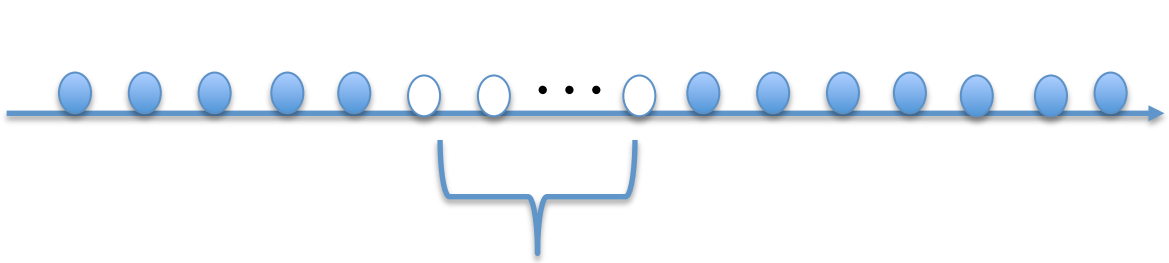
≥2D: $$p = \frac{1+G}{(z-k)(1-G)+k(1+G)}$$

Sites devoid of food

Sites with food

Formalism Used in this Work

We assume a forager starts at 1 or L-1 thus it is on the boundary of desert(area has no food) and paradise (area filled with food).

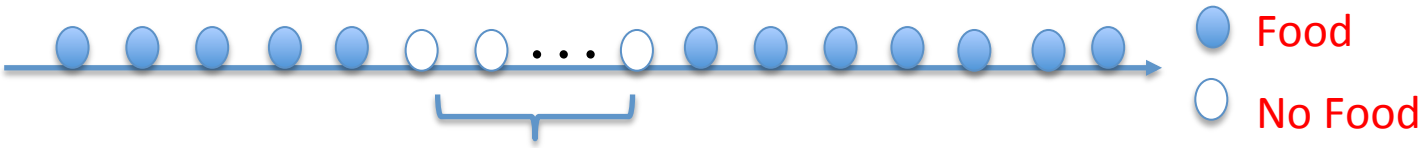


Point 1 to Point L-1

$$F_L(t) = p\,\delta_{t,1} + (1-p)\sum_{t' \leqslant t-1} f_{L-2}(t')\,F_L(t-t'-1).$$

- $f_L(t)$: Probability that an isotropic random walk *first* reaches either edge of the interval at time step t with this initial condition.
- F(t): Probability that the forager starts at either $x = 1$ or $x = L - 1$, first reaches either $x = 0$ or $x = L$ at step t

Formalism Used in this Work



$$V_k = [1 - \mathcal{F}_k(\mathcal{S})] \prod_{j=1}^{k-1} \mathcal{F}_j(\mathcal{S}).$$

$$\mathcal{F}_k(\mathcal{S}) = \sum_{t=0}^{\mathcal{S}} F_k(t),$$

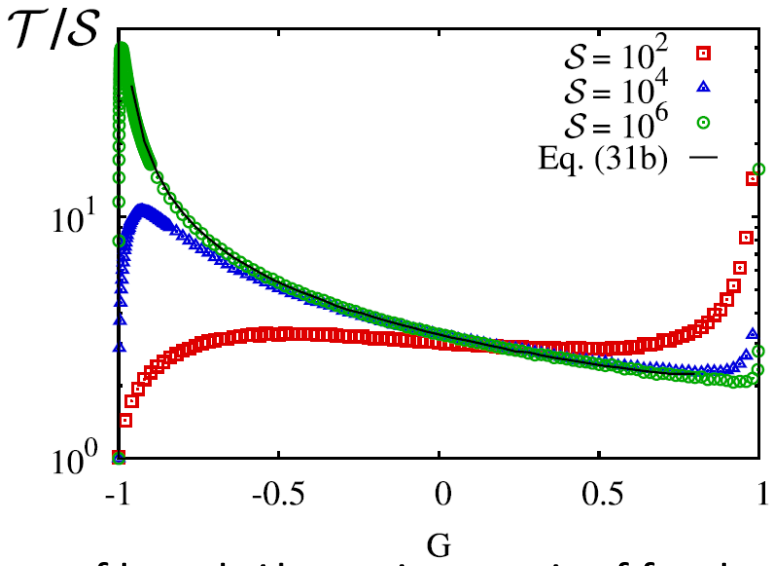
V_k : probability that the forager has visited k distinct sites at the instant of starvation;

$F_k(S)$:probability that the forager eats before it starves in a desert of k sites;

$$\mathcal{N} = \sum_{k \geqslant 0} k V_k,$$

$$\mathcal{T} = \sum_{k \geqslant 0} \left[\sum_{j=1}^{k-1} \tau_j \right] V_k + \mathcal{S},$$

$$\tau_j = \frac{\sum_{t=0}^{\mathcal{S}} t\, F_j(t)}{\sum_{t=0}^{\mathcal{S}} F_j(t)}.$$



τ_j : average time for a forager to escape a desert of length j by eating a unit of food at the desert edge before starvation

1D: semi-infinite desert geometry

Initial Condition: $X \leqslant 0$: Food $X > 0$: Desert

https://www.youtube.com/watch?v=u3zVSiDYWeo

1D: semi-infinite desert geometry

Initial Condition: $X \leqslant 0$: Food $X > 0$: Desert

Asymptotic solution:

1. First passage formalism

2. Generating function for isotropic random walk in SI geometry

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1D: semi-infinite desert geometry

Initial Condition: $X \leqslant 0$: Food $\quad X > 0$: Desert

Asymptotic solution:

1. First passage formalism
2. Generating function for isotropic random walk in SI geometry

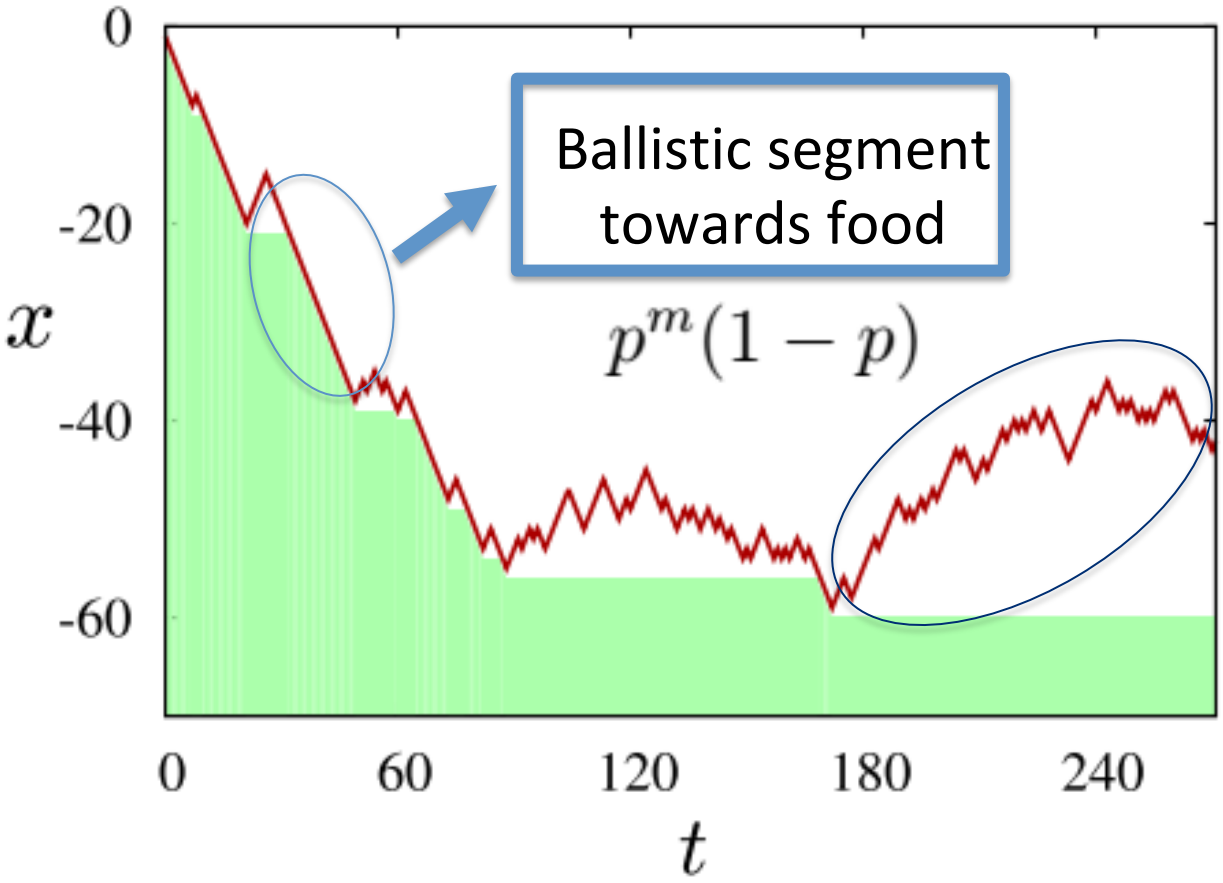
→

For large S:

- Consumed food scale as $\sqrt{S}$.
- Lifetime is proportional to S for strong greedy, S^2 for weak greedy.

https://www.youtube.com/watch?v=u3zVSiDYWeo

Heuristic approach for 1D semi-infinite desert geometry

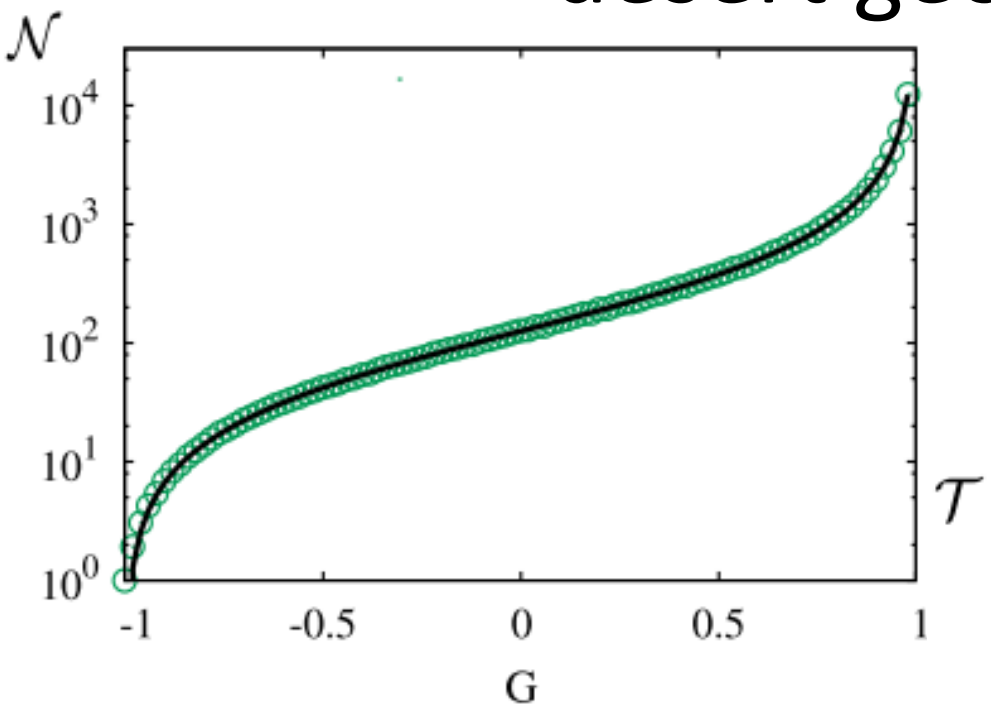


Diffusive segment in the desert

Continuum limit:

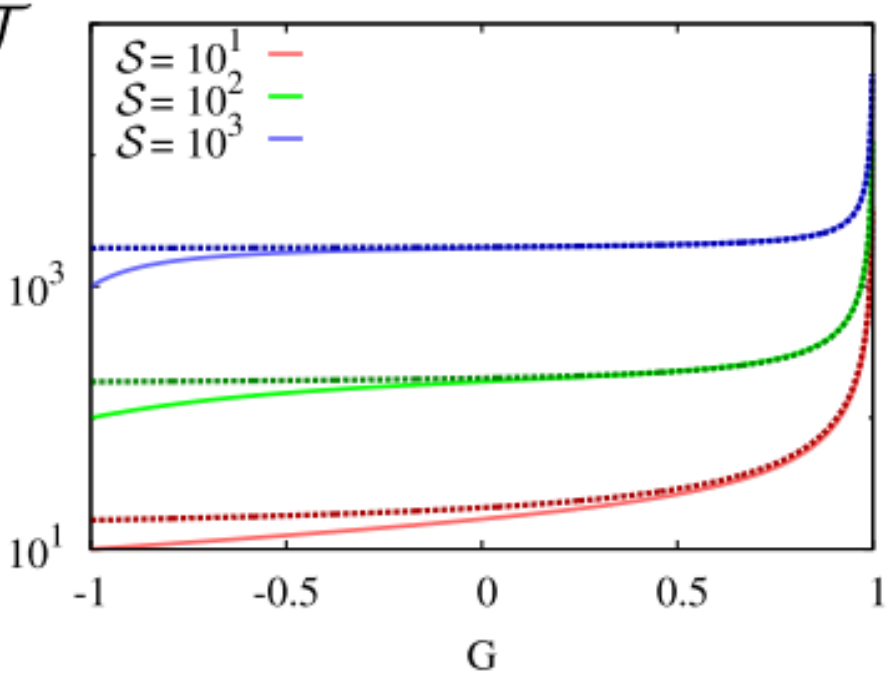
$$\mathcal{R} = \int_0^{\mathcal{S}} \mathrm{d}t \, \frac{\mathrm{e}^{-1/4Dt}}{\sqrt{4\pi D t^3}} = \mathrm{erfc}(1/\sqrt{4\mathrm{D}\mathcal{S}})$$

Heuristic Results for 1D semi-infinite desert geometry

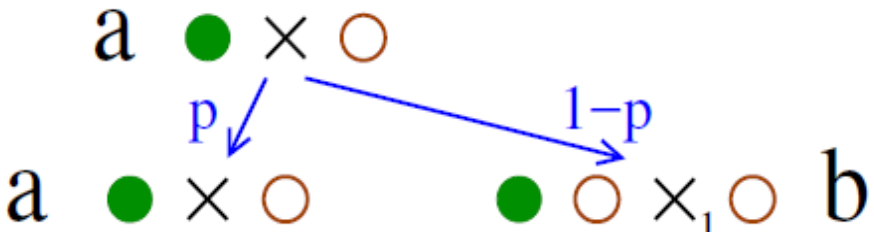


$$\mathcal{N}_{\rm SI} \simeq \frac{p}{1-p}\sqrt{\frac{\pi\mathcal{S}}{2}},$$

$$\mathcal{T}_{\rm SI} \simeq \frac{2p-1}{1-p}\sqrt{\frac{\pi\mathcal{S}}{2}} + 2\mathcal{S}.$$



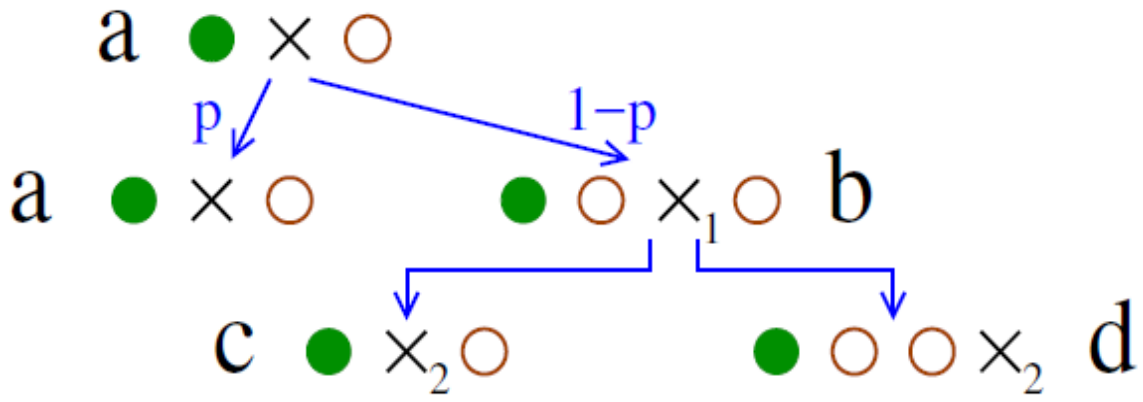
Exact Discrete Solution of Survival Time



Forager with capacity 2

- Let t_a be the average lifetime of the forager
- $t_a = p(1 + t_a) + 2(1 - p)$

Exact Discrete Solution of Survival Time



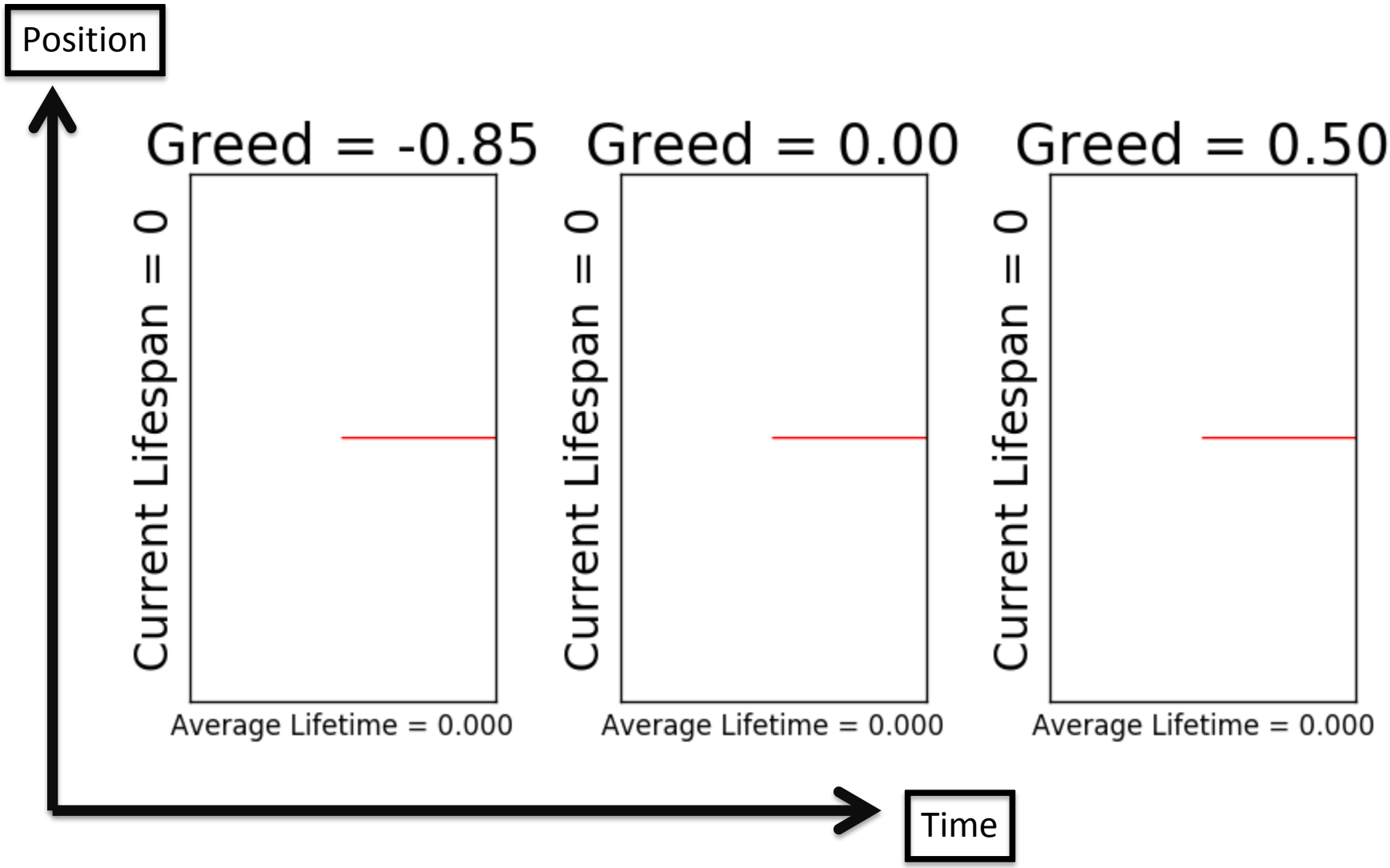
Forager with capacity S

- For General capacity S, split all trajectories into two categories:
- Set **P,** (all paths that return to food)
- Set **Q**,(all paths where the forager starves)

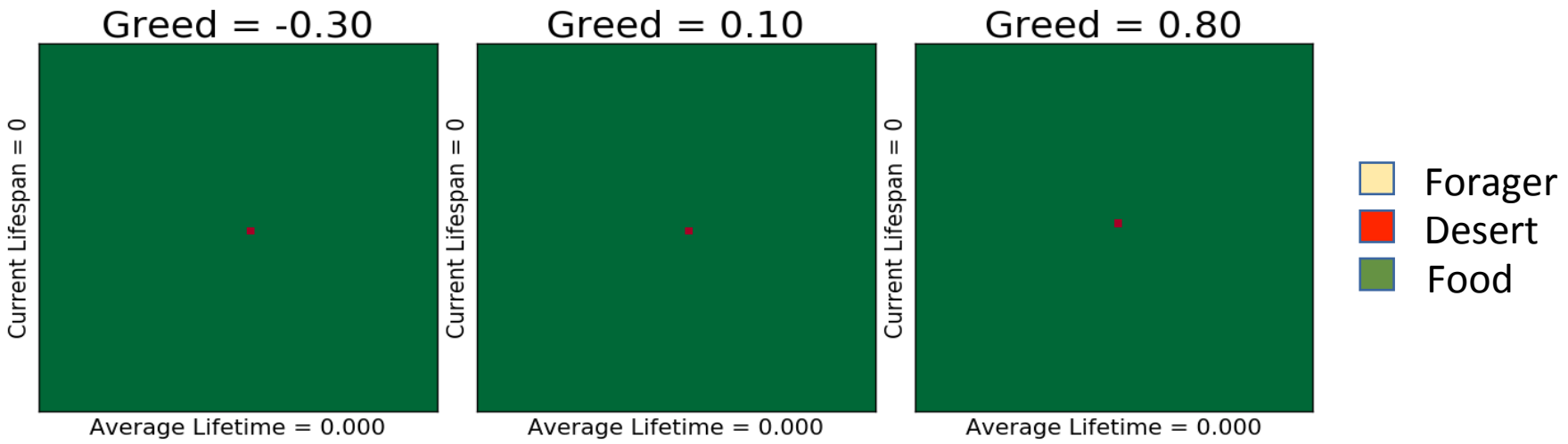
$$\mathcal{T} = \sum_{\mathbf{p}\in\mathbf{P}} \mathcal{P}_{\mathbf{p}} \left(t_{\mathbf{p}} + \mathcal{T}\right) + \sum_{\mathbf{q}\in\mathbf{Q}} \mathcal{P}_{\mathbf{q}}\, \mathcal{S}$$

- How does one calculate $\mathbf{P}_p$?

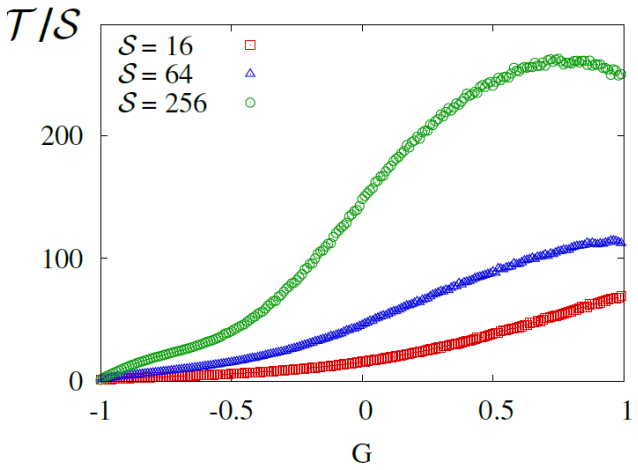
1D Forager Simulation



2D Results



If you are a forager with a higher capacity, absolute greed might not help you to live longer.



Comparisons with Previous Work

PRL 113, 238101 (2014) PHYSICAL REVIEW LETTERS week ending 5 DECEMBER 2014

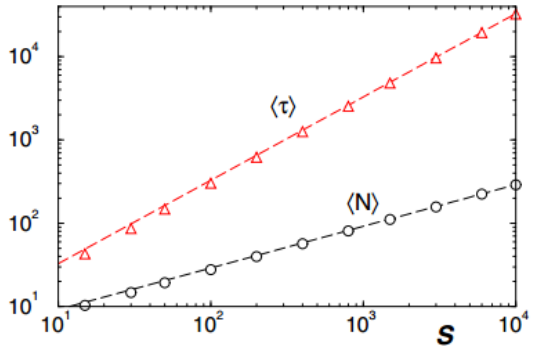
Depletion-Controlled Starvation of a Diffusing Forager

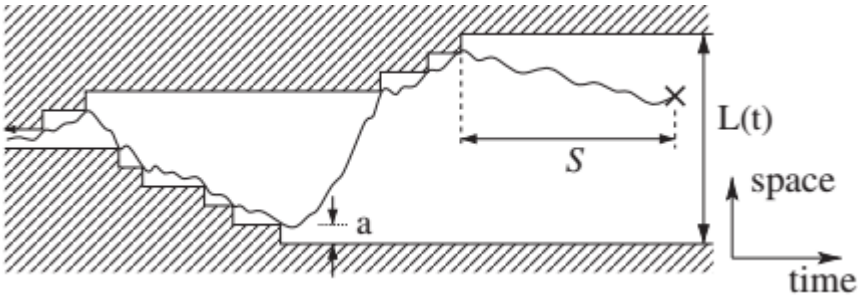
O. Bénichou[1] and S. Redner[2]

- Random walk (p=1/D)
- 1D finite desert:

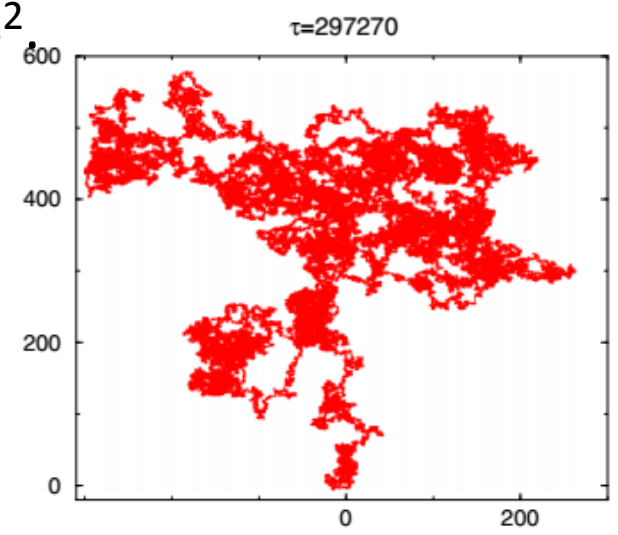
$$\langle \mathcal{N} \rangle \sim 2.90222\sqrt{\mathcal{S}}$$

$$\langle \tau \rangle \sim 3.26786\mathcal{S}.$$



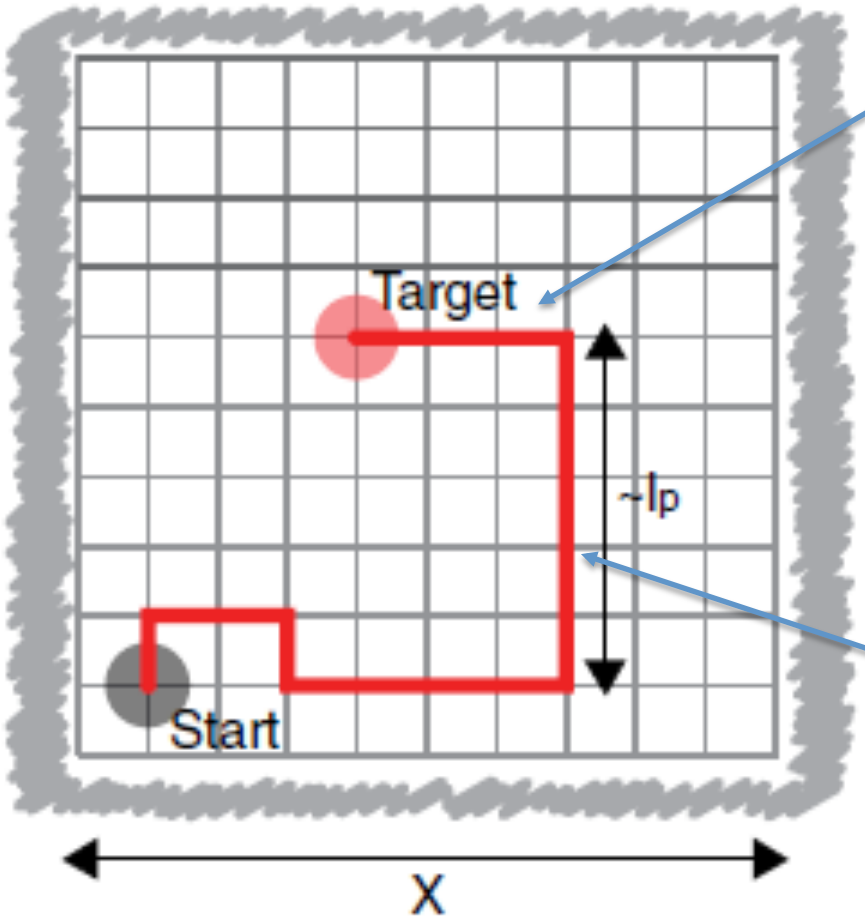


- 2D simulation:
  Both average lifetime and consumed food scale as S^2.



Comparisons with Previous Work

Aim: minimize the search time.



Tejedor V, Voituriez R, Bénichou O. Optimizing persistent random searches[J]. Physical review letters, 2012, 108(8): 088103.

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Article | OPEN | Published: 13 February 2018

Related work

Dynamics of starvation and recovery predict extinction risk and both Damuth's law and Cope's rule

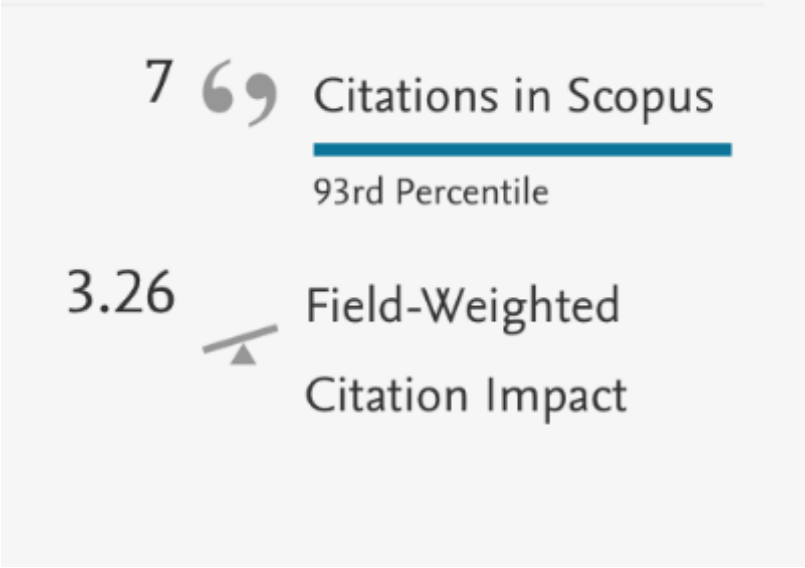
Justin D. Yeakel, Christopher P. Kempes & Sidney Redner

- Introduce a model with both reproducing and nonreproducing consumers. Extension of starving random walk model of foraging (only consumption)

- Provides a framework to predict population abundances:
  - Damuth's law (larger animals have fewer individuals)
  - Cope's rule (larger body size tends to be more distinct)

Citation Evaluation and Progress Since Paper

Since July 2017:



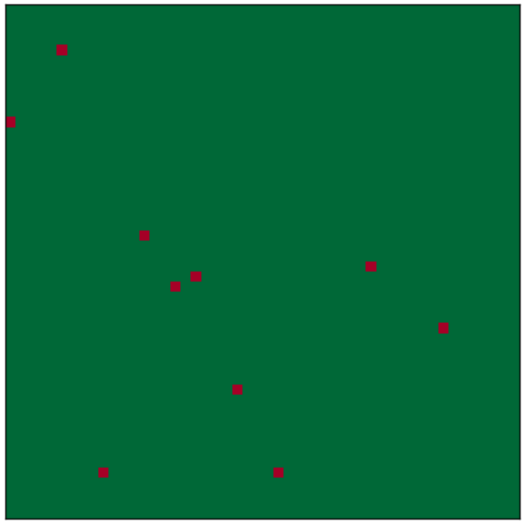
Advancements since paper:

- Applied to chemotaxis in a porous medium (maze of resources without greed)[5]
- Diffusion of a population (many foragers without greed)[6]
- Extension to 2D models (with greed)[7]

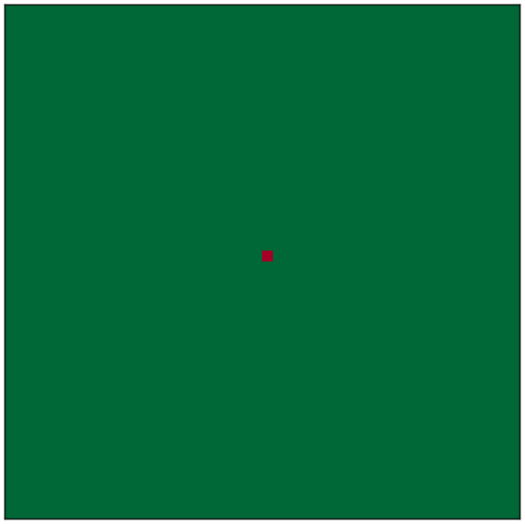
[5] Schilling, T., & Voigtmann, T. (2018, November). Clearing out a maze: A model of chemotactic motion in porous media.
[6] Berbert, J., & Lewis, M. (2018, January). Superdiffusivity due to resource depletion in random searches.
[7] Rager, C. L., Bhat, U., Bénichou, O., & Redner, S. (2018, July 16). The advantage of foraging myopically.

Critical Analysis

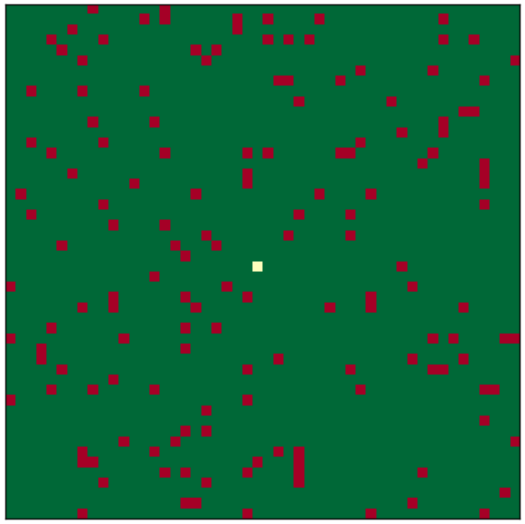
- Number of realizations to be averaged over was not mentioned.
- Too ideal a model. Improvements?



Multiple Foragers



Resource Replenishment



Sparse Resources

Conclusion

- More greediness doesn't imply longer lifetime in 1D
- Lifespan tends to be longer for higher, positive greed values in 2D
- Many variations to this basic model yet to be explored