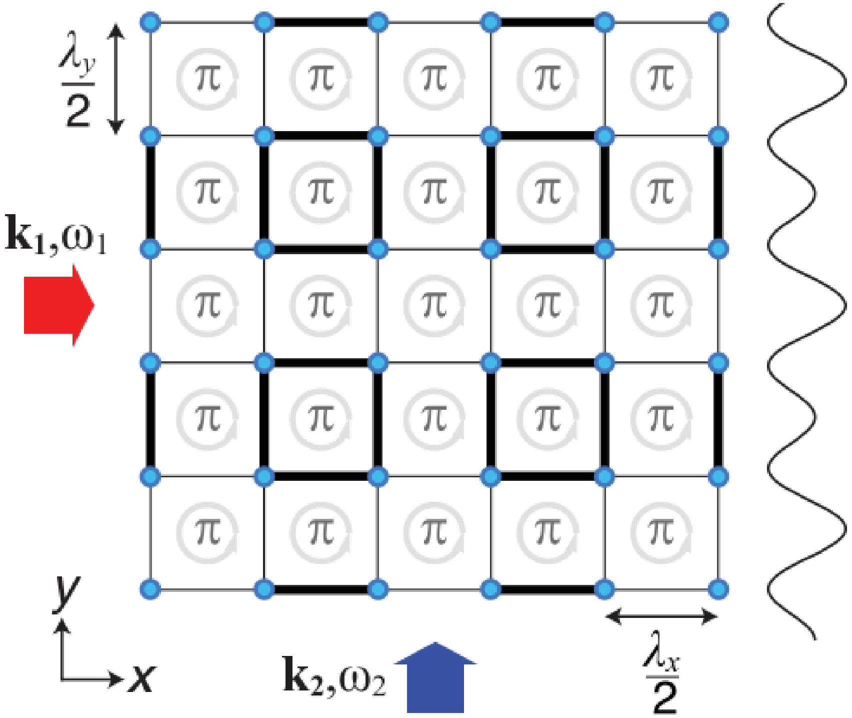
Quantized electric multipole insulators

Benalcazar, W. A., Bernevig, B. A., & Hughes, T. L. (2017). Quantized electric multipole insulators. *Science*, *357*(6346), 61–66.



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Topological phases of matter give rise to quantized physical quantities

- Examples are
  - Charge polarization in crystals (1D) $P_1 = -\frac{e}{2\pi}\int_{\mathrm{BZ}} \mathrm{Tr}[\mathcal{A}]$
  - Hall conductance (2D) $\sigma_{xy} = -\frac{e^2}{2\pi\hbar}\int_{\mathrm{BZ}} \mathrm{Tr}[d\mathcal{A} + i\mathcal{A}\wedge\mathcal{A}]$
  - Magnetoelectric polarizability (3D) $P_3 = -\frac{e^2}{4\pi\hbar}\int_{\mathrm{BZ}} \mathrm{Tr}[\mathcal{A}\wedge d\mathcal{A} + \frac{2i}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}]$

- $\mathcal{A}$ is the Berry phase vector potential
- σ_{xy} and P_3 are natural mathematical extensions of the P_1 Berry phase expression

There is no generalization of the Berry phase expression for quantized polarization to higher electric multipole moments

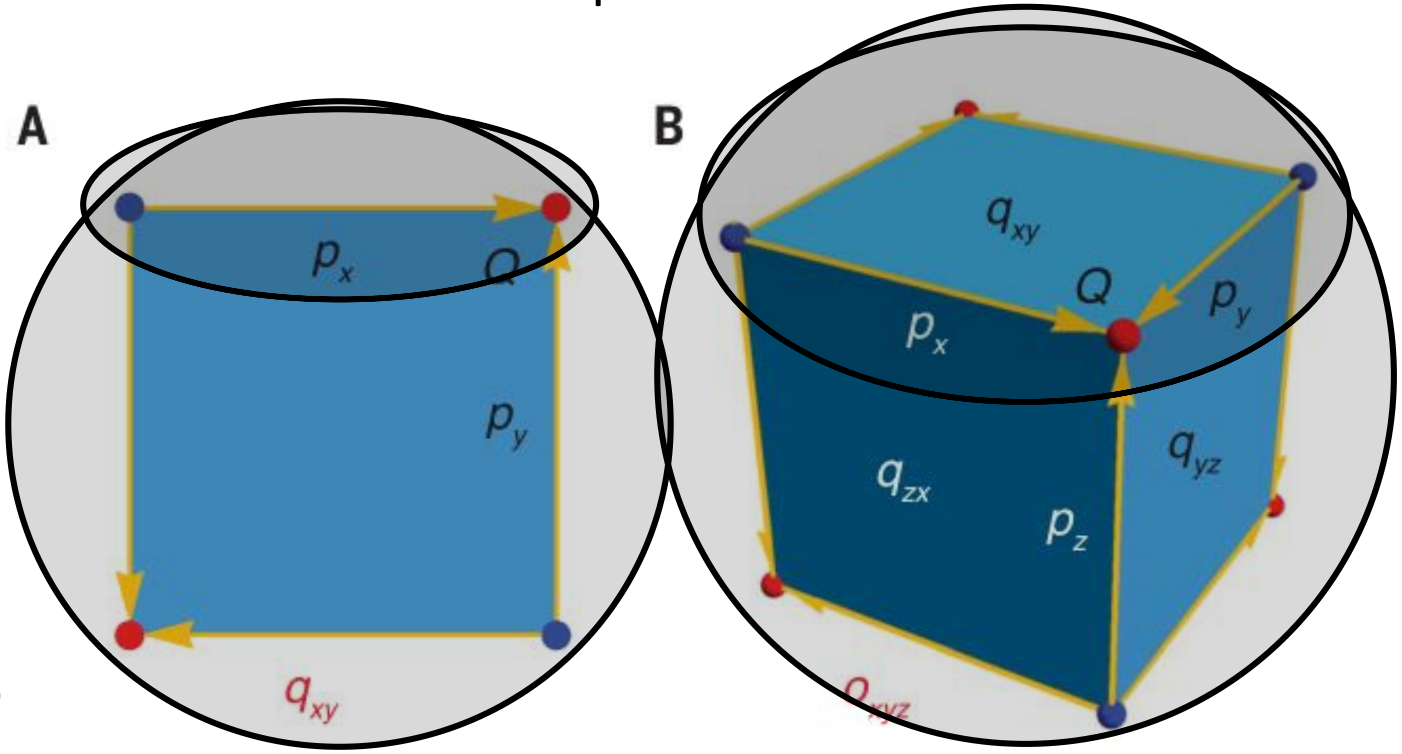
In the classical, continuous limit, multipole moments are

Dipole: $p_i = \int d^3 r \rho(r) r_i$

Quadrupole: $q_{ij} = \int d^3 r \rho(r) r_i r_j$

Octupole: $o_{ijk} = \int d^3 r \rho(r) r_i r_j r_k$

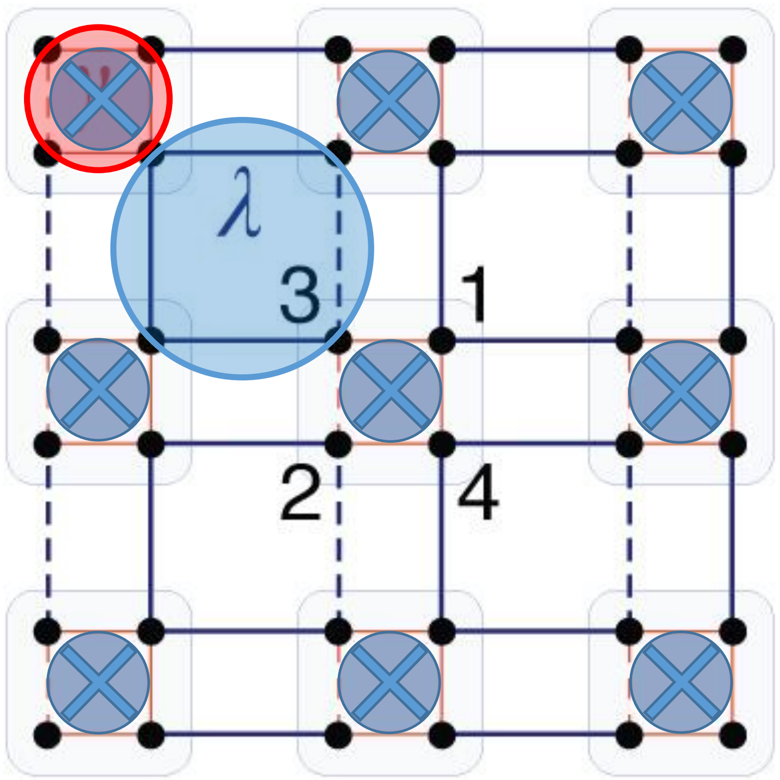
Goal: construct crystalline insulator models exhibiting quantized quadrupole and octupole moments



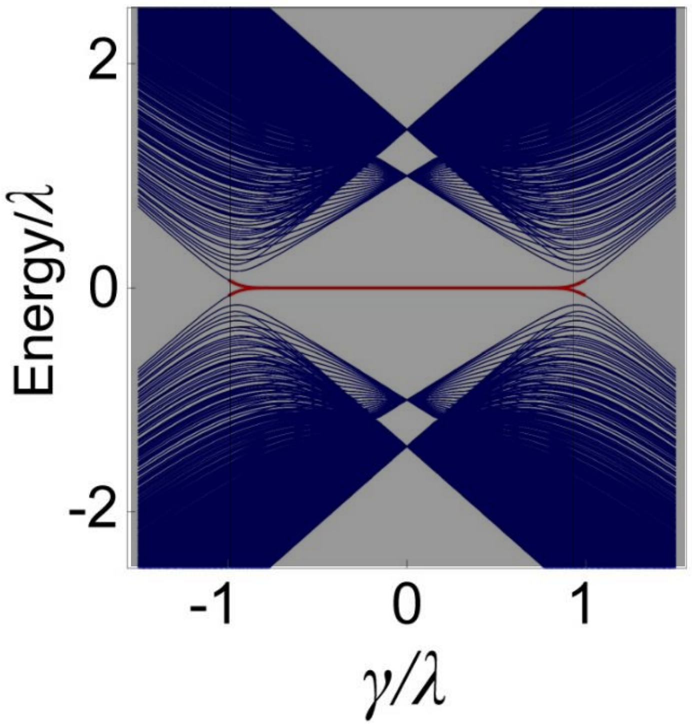
Bulk quadrupole (A) and octupole (B) moments and the induced moments: surface quadrupoles, edge polarization, corner charges

The minimal components for a quadrupole insulator are 4 (2 occupied) bands and reflection symmetries M_x, M_y

- γ, λ are hopping parameters
- Complex phases emulate flux quanta piercing each plaquette

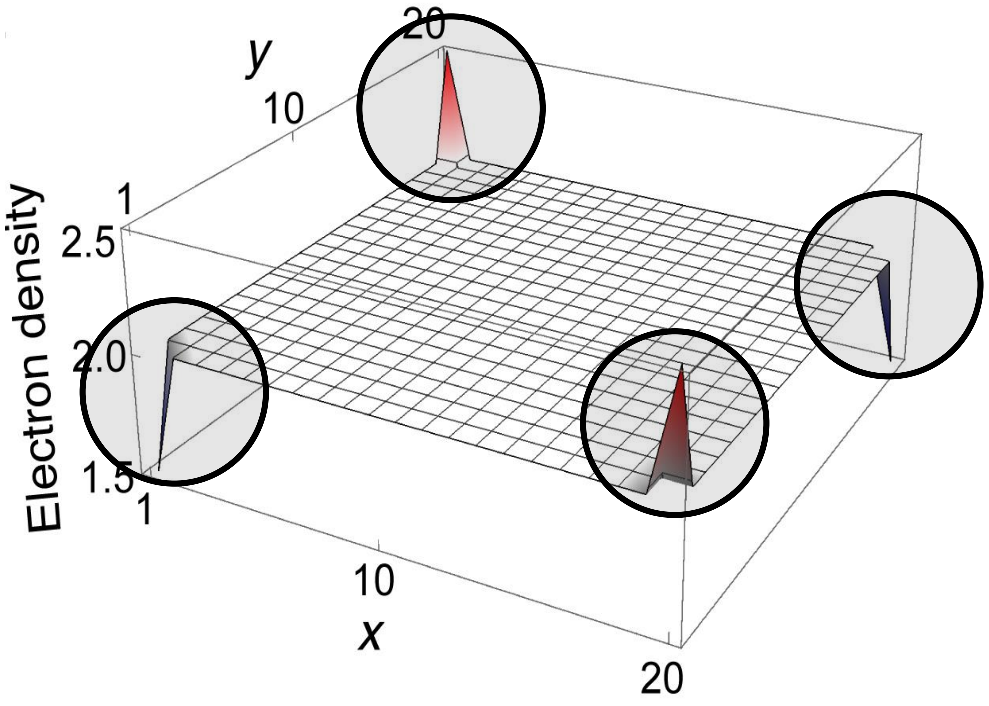


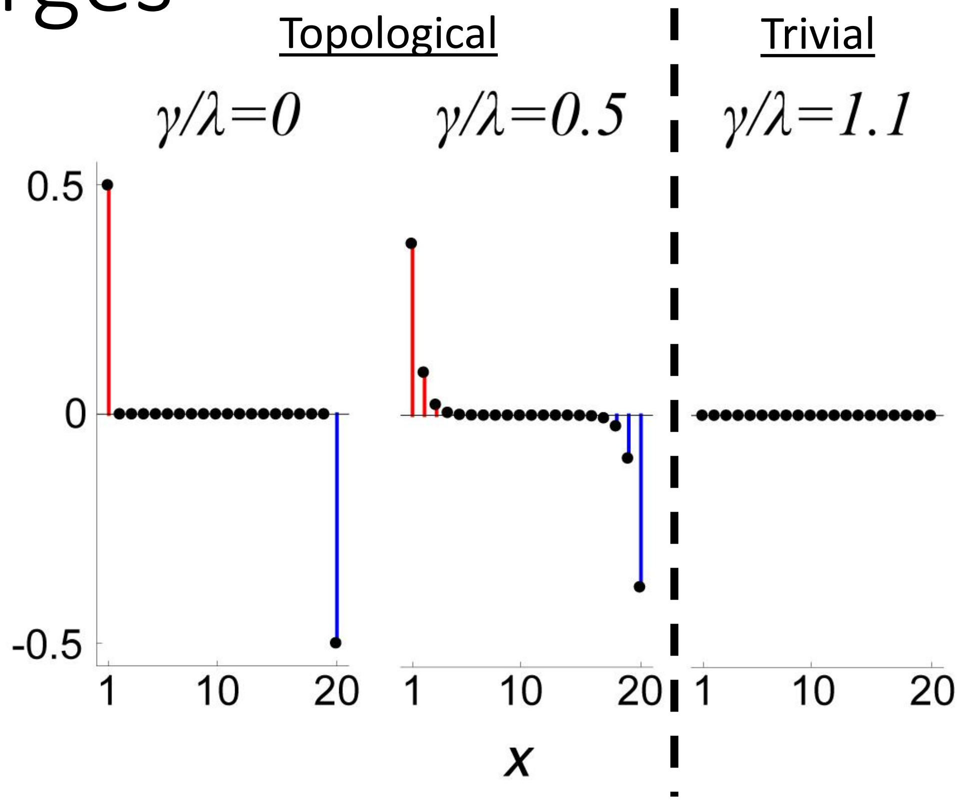
- Topological: $|\gamma/\lambda| < 1$
    - Quantized edge polarization
        - $P = \pm e/2$
    - Quantized corner charge
        - $Q = \pm e/2$
- Trivial: $|\gamma/\lambda| > 1$
    - No P or Q



Benalcazar, Bernevig, Hughes, *Science*, *357*(6346), (2017).

Numerical simulations confirm quantized polarization and corner charges





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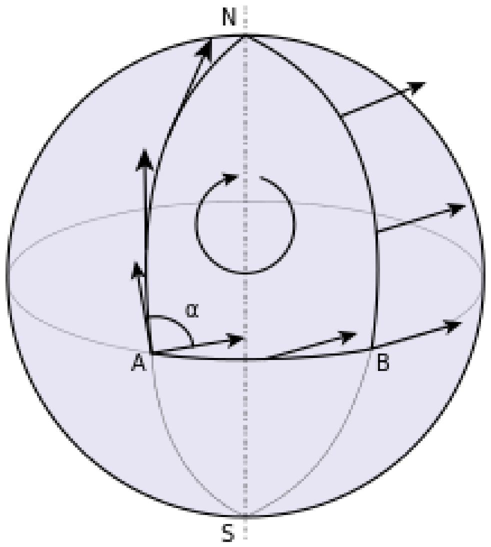
- Corner states located at boundary of the boundary
- Exponential decay and sudden disappearance indicate topological origin
- Edge polarization also quantized, but there is no nice picture

Berry Phases in Quantum Mechanics



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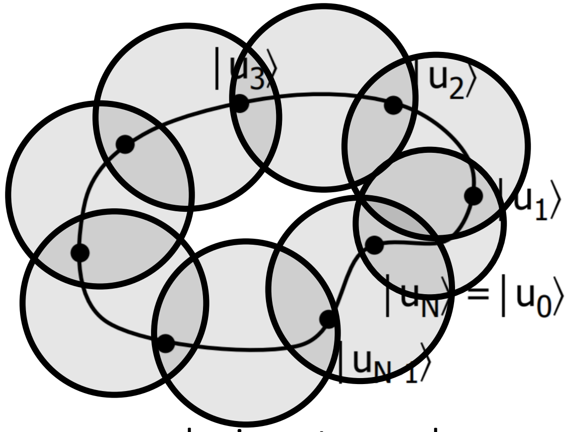

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- Movement along curved paths can result in an acquired (geometric) phase

- Berry Phase θ : QM geometric phase
  - $e^{-i\theta} = \langle u_N | u_{N-1} \rangle \langle u_{N-1} | u_{N-2} \rangle \cdots \langle u_2 | u_1 \rangle \langle u_1 | u_0 \rangle$
  - $|u_N\rangle$ is the orbital wavefunction

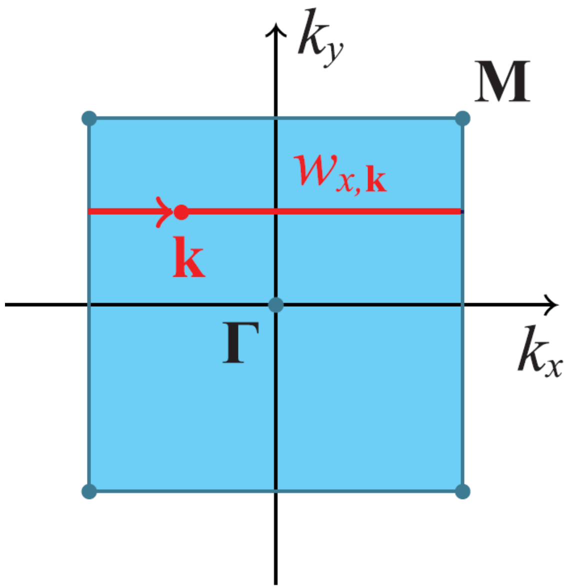
- Crystal momentum space is a torus, allowing nontrivial loops

- Berry phase is equivalent to location of electrons in the unit cell (polarization) Zak (1989)

- How to generalize to multiple bands (quadrupole/octupole moments)?

Wilson Loops are a generalization of the Berry phase integral in multiple band systems

- Wilson loops over 2D energy bands give 1D bands of Wannier centers (electron positions)
- Wilson loops on 1D Wannier bands give polarizations of each Wannier center
- Each electron contributes opposite polarizations
- Quantized as 0 or $\pm e/2$



Left: Wilson loop path in Brillouin zone
Right: resulting gapped Wannier bands

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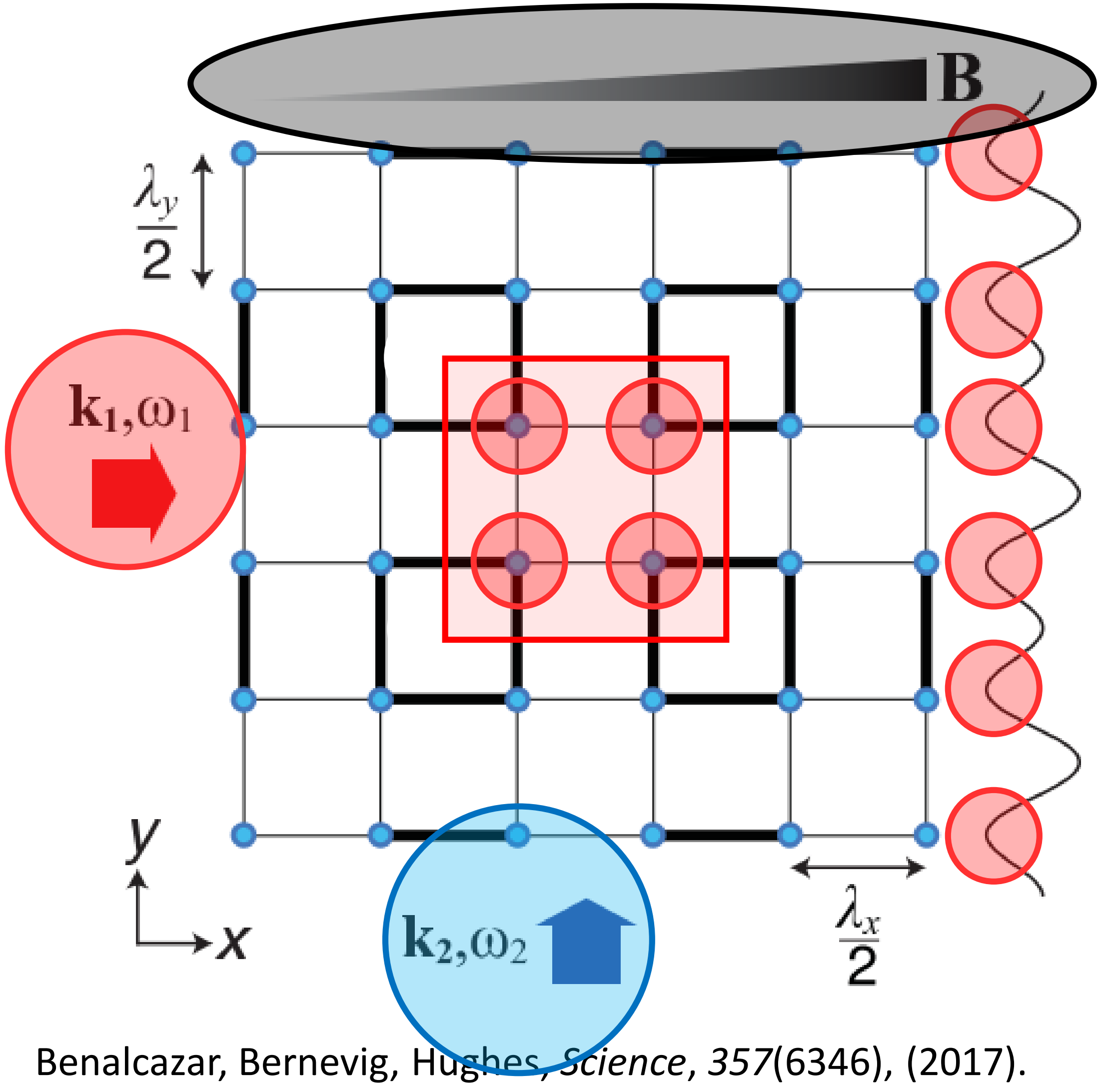
Cold atoms in optical lattices could realize a quantized quadrupole moment

- A 2D superlattice is created using orthogonal standing optical waves
- X-hopping inhibited with a magnetic gradient
- X-hopping is restored <u>with a complex phase</u> via laser beams
- This phase mimics a π flux per plaquette



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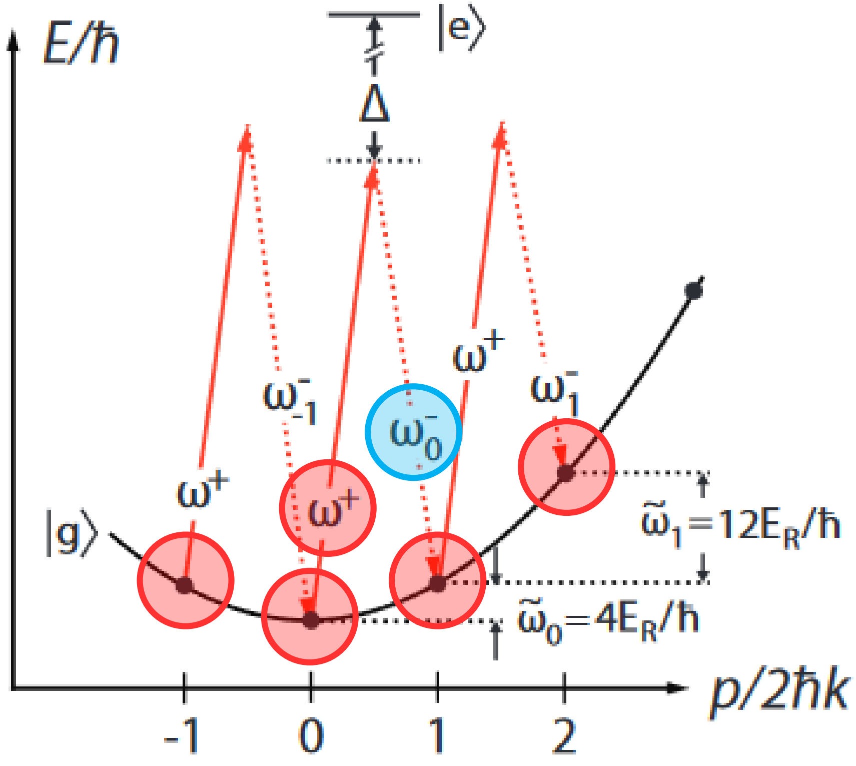
Bragg transitions between plane-wave BEC states can also model the quadrupole

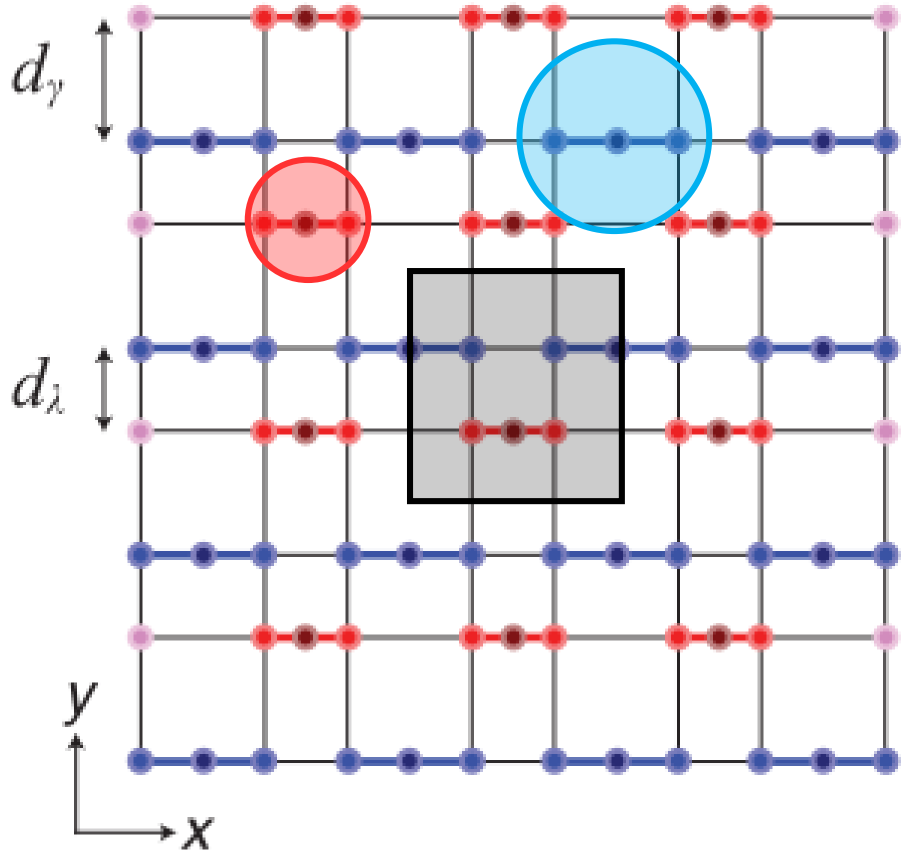
- Local atomic orbitals -> BEC planewaves
- Hopping -> 2-photon transitions
- Acousto-optic modulators control hopping amplitude and phase
  - Allows effective flux per plaquette
- Has only been achieved in 1D so far



B. Gadway, Phys. Rev. A 92, 043606 (2015).

Recent advancements in photonics allows this model to be realized with laser etched waveguides

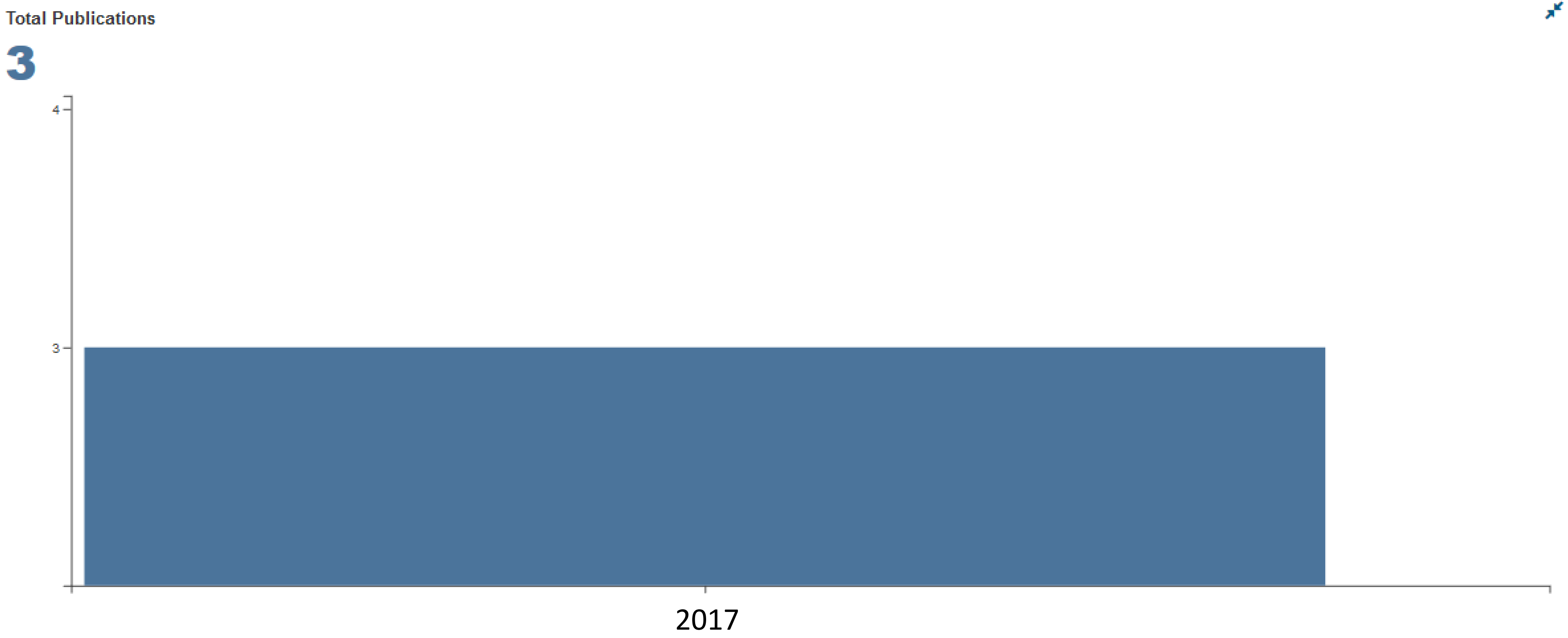
- Model can be replicated with arrays of parallel waveguides
- Orbitals -> Waveguides
- Hopping -> Evanescent Tunneling
- New negative couplings allow complex hopping
- Topology can be confirmed by illuminating a corner of the lattice



This paper is of extremely high quality overall

- Good:
  - The paper is reasonably accessible
  - The figures are very illustrative and aid in understanding
  - The work represents a significant advancement in understanding of topology and provides a new framework for calculating invariants (nested Wilson loops)
  - The predictions have been verified in multiple experiments
    - arXiv:1708.03647 (topoelectrical circuit)
    - arXiv:1710.03231 (microwave circuit)
- Bad
  - The supplement is enormous compared to the core paper, but that is nearly unavoidable

Citation Analysis



Summary

- Authors wanted to extend the quantum theory of polarization to higher multiple moments
- Designed Hamiltonians demonstrating quantized quadrupole and octupole moments
- Discovered new topological paradigm (nested Wilson loops)
- Provided experimental proposals for physical realizations of quantized quadrupole insulators