Transport in Out-of-Equilibrium XXZ Chains: Exact Profiles of Charges and Currents

Bertini et al., PRL **117**, 207201 (2016)

Julian May-Mann, Yutao Ma, Rong Nie, Kevin Ly

Solving Many-Body Systems

- For systems with small parameters perturbation theory can be used (QED)
- However there are many systems that do not even have a perturbative solution (QCD, Hubbard Model, XXZ model)
- However in 1 spatial dimension there are actually a large class of solvable models called integrable models (XXZ model, Heisenberg spin ½ chain)

$$\hat{H} = -t \sum_{\langle i,j \rangle,\sigma} (\hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma} + \hat{c}^{\dagger}_{j,\sigma} \hat{c}_{i,\sigma}) + U \sum_{i=1}^{N} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

Integrable Models

- Actual definition is pretty complicated, but here are some key properties of the scattering processes of these systems:
  - Particle number conservation
  - Momentum conservation conserved during scattering
  - Scattering can be factored into products of 2 particle scattering
- We can solve integrable systems using the Bethe Ansatz

Bethe Ansatz

- Basic process:
- A given theory has an S matrix which characterizes all scattering processes
- The Integrability of the theory places constraints on the S matrix
- Using these constraints you can construct equations “Bethe Equations”
- The solutions to the “Bethe Equations” give the eigenvalues of the theory

$$S_{fi} \equiv \lim_{t \to +\infty} \langle \Phi_f | \Psi(t) \rangle \equiv \langle \Phi_f | \, S \, | \Phi_i \rangle \,,$$

$$\left(\frac{\sinh\left(\lambda_j + i\frac{\gamma}{2}\right)}{\sinh\left(\lambda_j - i\frac{\gamma}{2}\right)} \right)^L = \prod_{l \neq j}^{N} \left(\frac{\sinh\left(\lambda_j - \lambda_l + i\gamma\right)}{\sinh\left(\lambda_j - \lambda_l - i\gamma\right)} \right)$$

Problems with Bethe Ansatz

- Because they are exactly solvable Integrable Models have high theoretical value, and can be used to understand more realistic systems
- However, a large number of experimental systems we would like to study are incompatible with the Bethe Ansatz
- For example, spatially inhomogeneous systems in 1 dimension
    - These systems can be realized in quasi 1d cold atom/AMO systems
- As a result to study 1d inhomogeneous systems, we need other tools

Model: XXZ spin-½ chain

Features:

$$H = J\sum_{\ell=1}^{L}(s_\ell^x s_{\ell+1}^x + s_\ell^y s_{\ell+1}^y + \Delta s_\ell^z s_{\ell+1}^z),$$

- Diagonalizable by the Bethe ansatz
- In the thermodynamic limit the states can be characterized by “root densities,” which are distributions of quasiparticles
- Local observables in long times can be described by a locally quasistationary state (LQSS), via dephasing

Goal: Put together two globally different chains and try and get analytical* result for charges and currents

Kinetic theory

$$\langle \boldsymbol{q} \rangle_{x,t+\delta t} - \langle \boldsymbol{q} \rangle_{x,t} = \int d\tilde{x} \left(\underset{\tilde{x}\to x,t}{\Delta^{q}} - \underset{x\to\tilde{x},t}{\Delta^{q}} \right),$$

Steps:

- Observe how charge density changes at a given point
- Use what is known about excitations and LQSS to rewrite RHS
- Relate the motion of charges to root densities

Equations of continuity

Final result:

$$\boxed{\partial_t \rho_{\zeta,k}(\lambda) + \partial_x (v_{\zeta,k}(\lambda)\rho_{\zeta,k}(\lambda)) = 0.}$$

$$\langle \rho | \boldsymbol{j}_\ell[\boldsymbol{Q}] | \rho \rangle \sim \sum_k \int d\lambda q_k(\lambda) v_k(\lambda) \rho_k(\lambda),$$

To include inhomogeneity:

$$\vartheta_{\zeta,k}(\lambda) = \theta_H(v_{\zeta,k}(\lambda) - \zeta)(\vartheta_k^L(\lambda) - \vartheta_k^R(\lambda)) + \vartheta_k^R(\lambda).$$

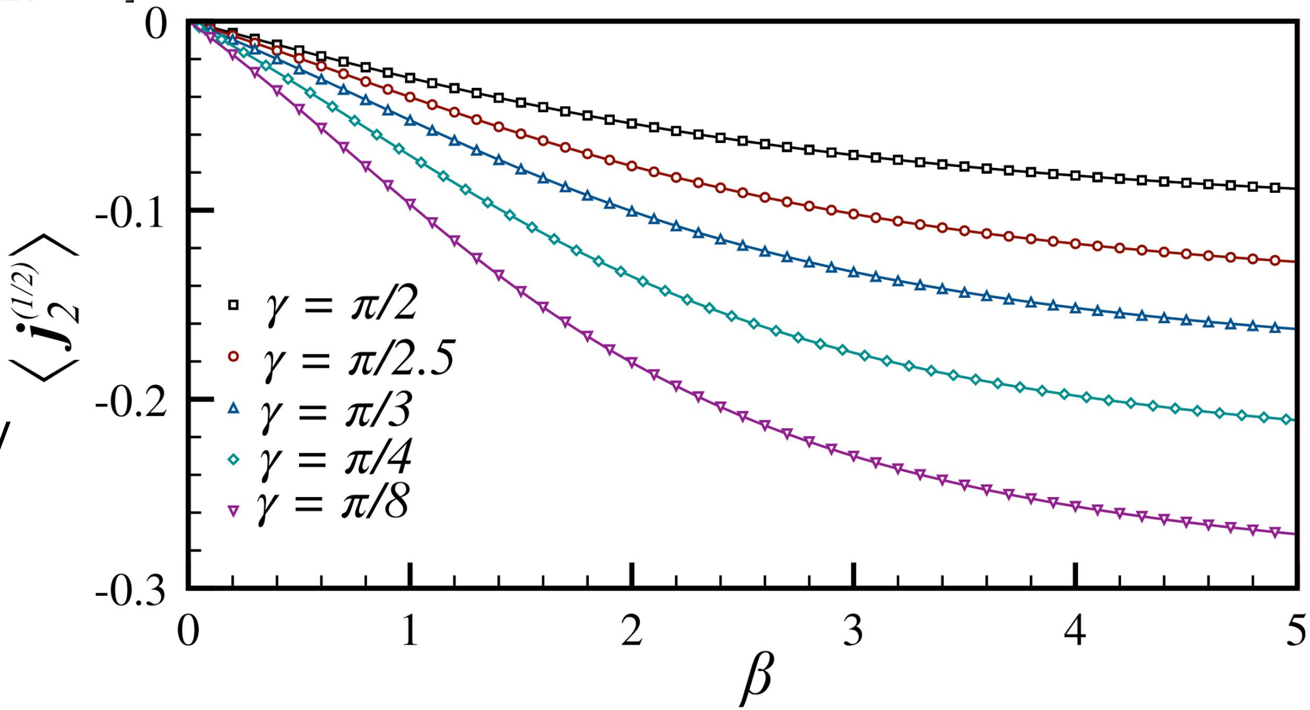
□ Charge currents defined by

$$j_{\ell+1}[Q] - j_{\ell}[Q] = i[q_{\ell}, H]$$

□ Symbols are predictions

□ Full lines are numerical data from matrix product density operator (MPDO)

➡Unquestionably perfect agreement : the discrepancies are smaller than the MPDO



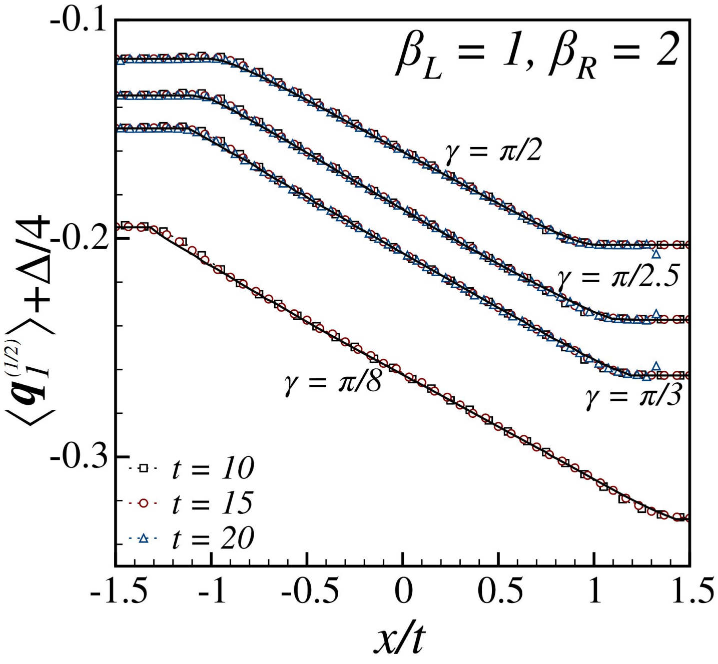
➨Numerical data are in excellent agreement with the analytical predictions.

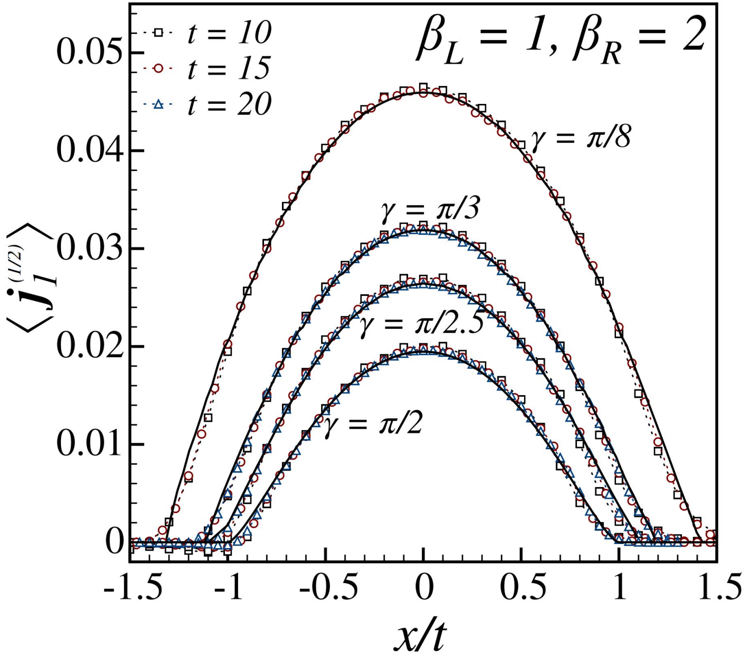
➨Strongly suggests that the solution characterizes the state of the system at late times.

- Two chains prepared at different temperatures an joined together

- Full lines are predictions

- Symbols are MPDO data





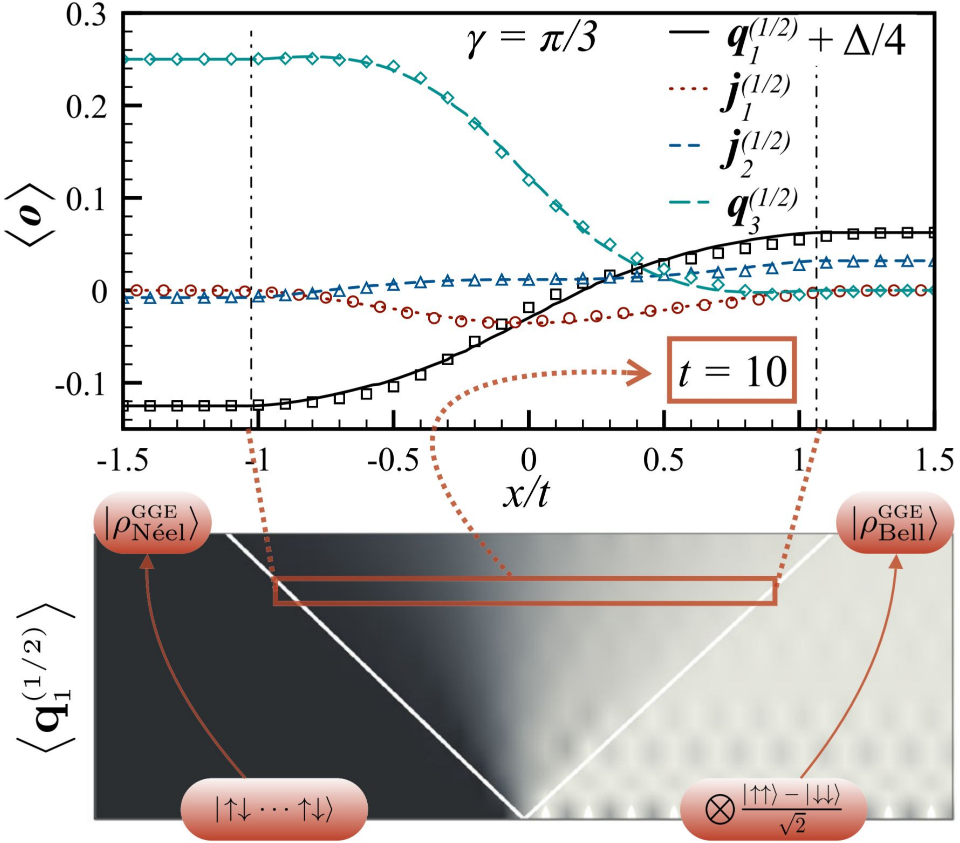
- Joining together two globally different pure states

- Lines are predictions

- Symbols are time-evolving block decimation (TEBD) data

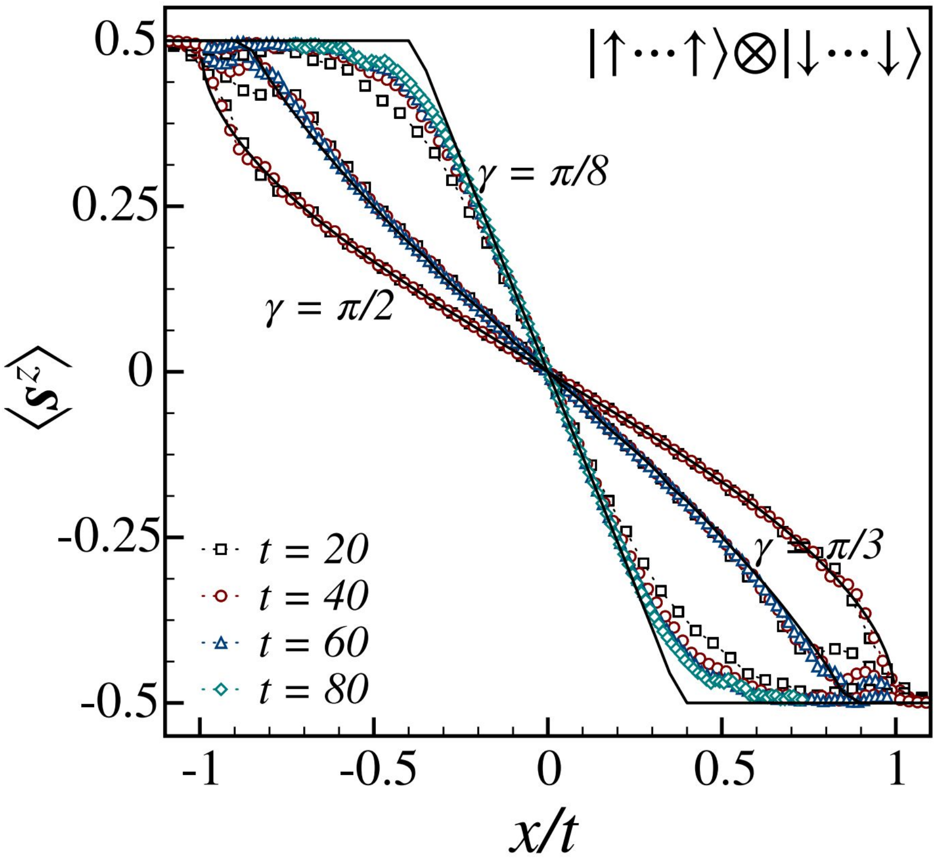
- The vertical dotted-dashed lines denote the light-cone edges

➡The agreement is fairly good



➨Predictions are fairly good

- Domain-wall initial state $|\uparrow\cdots\uparrow\rangle \otimes |\downarrow\cdots\downarrow\rangle$
- Lines are predictions
- Symbols are numerical data



Critiques

Pros:

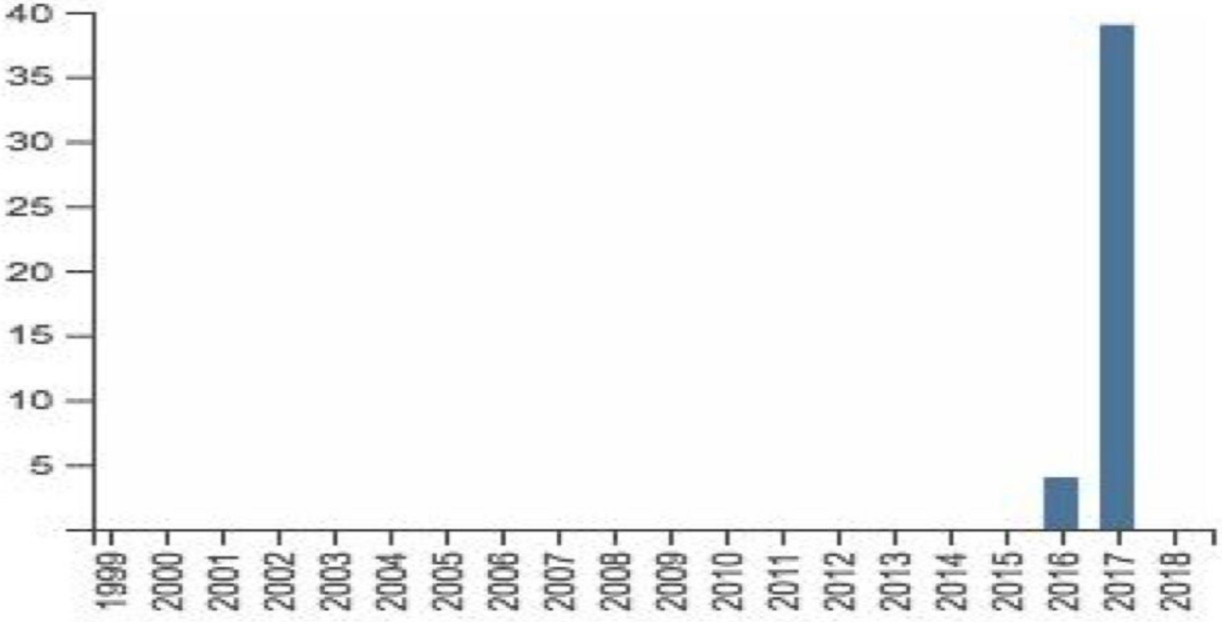
- From the continuity equation, the paper obtains analytic result characterizing the dynamics of integrable XXZ chain
- The solution agrees well with numerical simulations within numerical accuracy
- The approach can in principle be applied to other integrable models

Cons:

- The paper does not describe the steps of their construction in detail.
- There is no new physics; the paper just proposes a new perspective on existing model

Citation Analysis

Total number of times cited : 43



Citation Analysis

An example of papers that cite the current paper

- Ilievski, Enej, and Jacopo De Nardis. "Microscopic origin of ideal conductivity in integrable quantum models." Physical Review Letters 119.2 (2017): 020602.
  APA
- It has been Cited 11 times since it was published
- It uses hydrodynamic approach to efficiently compute Drude weights

Summary

- A continuity equation describing the dynamics of integrable XXZ chain is developed
- The validity of the equation is confirmed by comparing the analytic solutions with numerical results
- The construction can be applied to other integrable models