Measuring Quantum Teleportation

Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

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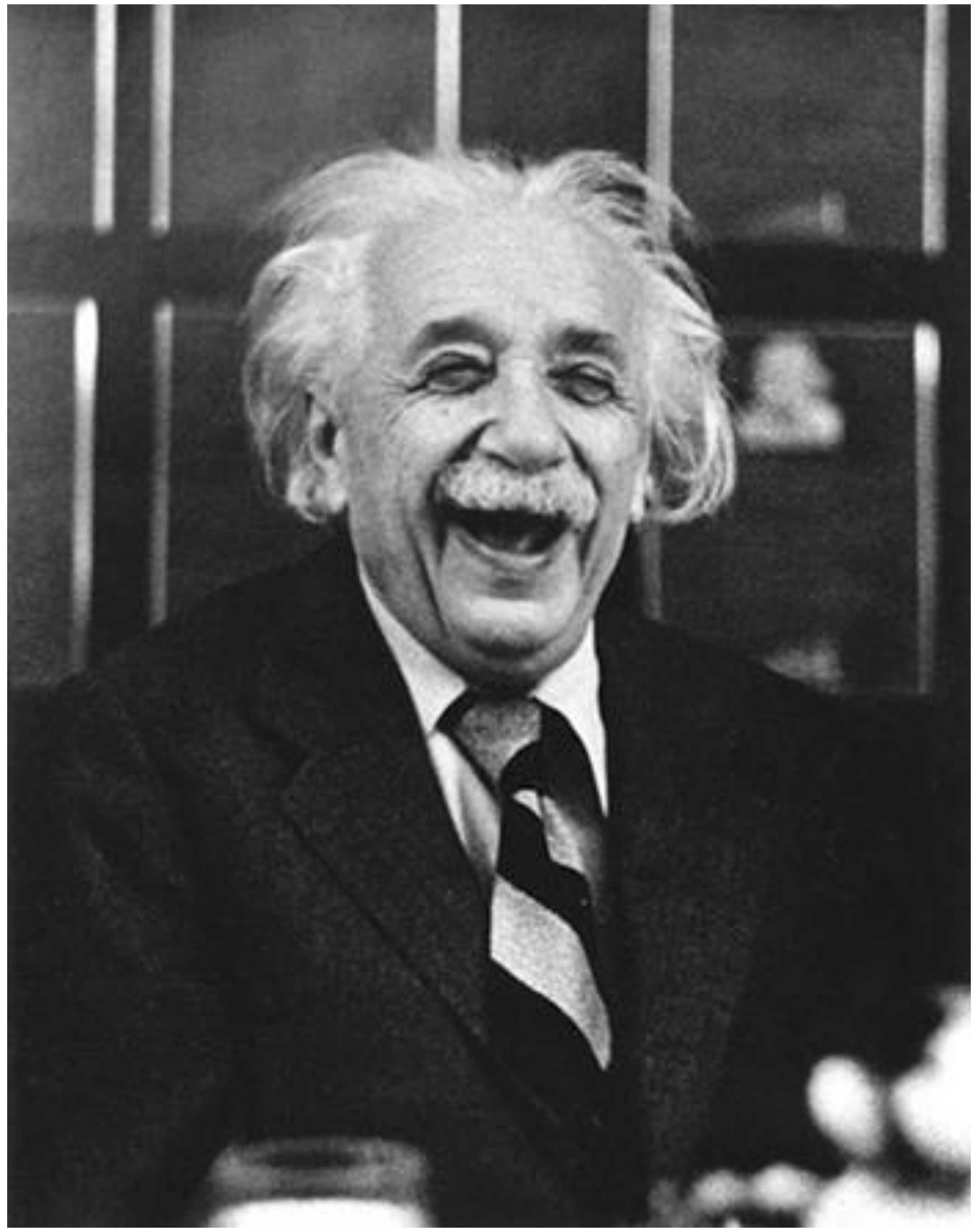
What does Quantum Mechanics have to do with Teleportation ?

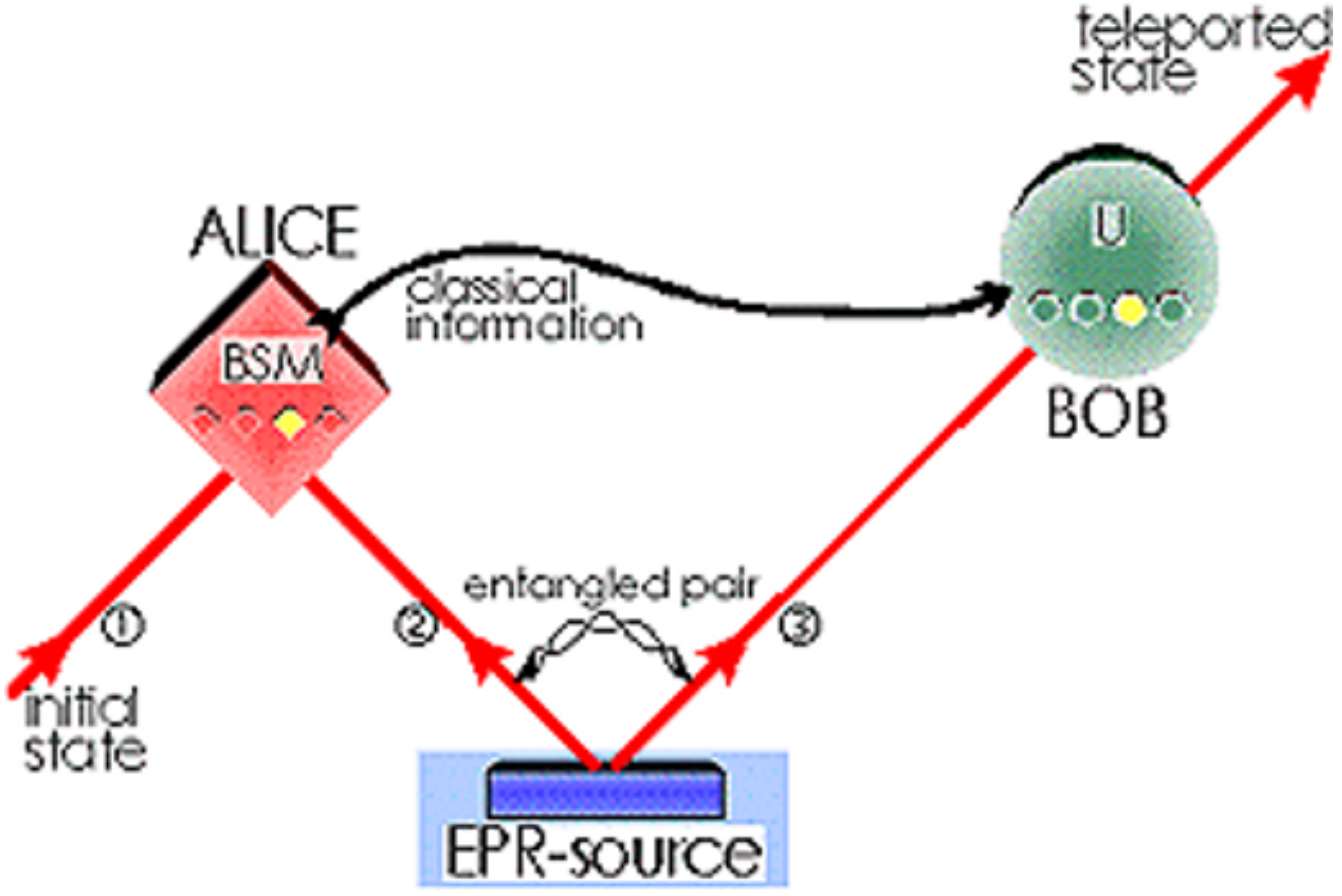
- QM exhibits *non-locality*
- What is locality? *Einstein's Locality Principle*: interactions mediated by fields whose disturbance cannot exceed the speed of light
- Einstein-Podolsky Rosen (EPR) Paradox
  - Consequence of uncertainty state of system prior to measurement



https://imgflip.com/memegenerator/21866701/Einstein-laugh

Quantum Teleportation Setup

- Alice and Bob each receive an EPR state
  - Basically an entangled state
  - We'll say a singlet configuration of two electrons
- Alice receives an additional unknown state
  - For now an electron in some spin state
  - This is what she wants to teleport



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Teleportation Scheme

- The unknown state has the generic form: $|\phi_1\rangle = a\,|\uparrow\rangle_1 + b\,|\downarrow\rangle_1$

$$where: |a|^2 + |b|^2 = 1$$

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- Alice performs an EPR measurement on her particle and the unknown particle
    - Basically an entanglement process
    - The following Bell State operator basis elements are needed
        - Singlet and Triplet states for particles i and j

$$|\Psi^{\pm}_{ij}\rangle = \frac{|\uparrow\rangle_i\,|\downarrow\rangle_j \pm |\downarrow\rangle_i\,|\uparrow\rangle_j}{\sqrt{2}}$$

$$|\Phi^{\pm}_{ij}\rangle = \frac{|\uparrow\rangle_i\,|\uparrow\rangle_j \pm |\downarrow\rangle_i\,|\downarrow\rangle_j}{\sqrt{2}}$$

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    - Singlet and Triplet states for particles i and j
    - Alice and Bob's state:

$$|\Psi_{23}^{-}\rangle = \frac{|\uparrow\rangle_2\,|\downarrow\rangle_3 - |\downarrow\rangle_2\,|\uparrow\rangle_3}{\sqrt{2}}$$

$$|\Psi_{ij}^{\pm}\rangle = \frac{|\uparrow\rangle_i\,|\downarrow\rangle_j \pm |\downarrow\rangle_i\,|\uparrow\rangle_j}{\sqrt{2}}$$

$$|\Phi_{ij}^{\pm}\rangle = \frac{|\uparrow\rangle_i\,|\uparrow\rangle_j \pm |\downarrow\rangle_i\,|\downarrow\rangle_j}{\sqrt{2}}$$

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- Rewrite the "representations" in terms of their basis elements:

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$$= \frac{a}{\sqrt{2}}\big(\,|\uparrow\rangle_1\,|\uparrow\rangle_2\,|\downarrow\rangle_3 - |\uparrow\rangle_1\,|\downarrow\rangle_2\,|\uparrow\rangle_3\,\big)$$

$$+\frac{b}{\sqrt{2}}\big(\,|\downarrow\rangle_1\,|\uparrow\rangle_2\,|\downarrow\rangle_3 - |\downarrow\rangle_1\,|\downarrow\rangle_2\,|\uparrow\rangle_3\,\big)$$

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$$+ \frac{b}{\sqrt{2}}\big(\,|\downarrow\rangle_1\,|\uparrow\rangle_2\,|\downarrow\rangle_3 - |\downarrow\rangle_1\,|\downarrow\rangle_2\,|\uparrow\rangle_3\,\big)$$

- Notice the products of spins of particles 1 and 2
  - We can rewrite these with the Bell State Basis elements
  - e.g.:

$$|\uparrow\rangle_1\,|\uparrow\rangle_2 = \frac{|\Phi_{12}^+\rangle + |\Phi_{12}^-\rangle}{\sqrt{2}}$$

Teleportation Scheme

- We arrive at: $$|\Psi_{123}\rangle = \frac{1}{2}\Big[\,|\Psi^-_{12}\rangle\,\big(-a\,|\uparrow_3\rangle - b\,|\downarrow_3\rangle\,\big) + |\Psi^+_{12}\rangle\,\big(-a\,|\uparrow_3\rangle + b\,|\downarrow_3\rangle\,\big)$$ $$+\,|\Phi^-_{12}\rangle\,\big(b\,|\uparrow_3\rangle + a\,|\downarrow_3\rangle\,\big) + |\Phi^+_{12}\rangle\,\big(-\,b\,|\uparrow_3\rangle + a\,|\downarrow_3\rangle\,\big)\Big]$$
- Alice makes a measurement, then communicates the result to Bob classically

Teleportation Scheme

- We arrive at:

$$|\Psi_{123}\rangle = \frac{1}{2}\Big[\,|\Psi_{12}^{-}\rangle\,(-a\,|\uparrow_3\rangle - b\,|\downarrow_3\rangle\,) + |\Psi_{12}^{+}\rangle\,(-a\,|\uparrow_3\rangle + b\,|\downarrow_3\rangle\,)$$
$$+\,|\Phi_{12}^{-}\rangle\,\big(b\,|\uparrow_3\rangle + a\,|\downarrow_3\rangle\,\big) + |\Phi_{12}^{+}\rangle\,(\,-\,b\,|\uparrow_3\rangle + a\,|\downarrow_3\rangle\,)\Big]$$

- Alice makes a measurement, then communicates the result to Bob classically
  - Bob can make the correction:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Context

Optimal Detection of Quantum Information

Asher Peres[(1),(2)] and William K. Wootters[(1),(3),(4)]
[(1)]*Santa Fe Institute, 1120 Canyon Road, Santa Fe, New Mexico 87501*
[(2)]*Department of Physics, Technion–Israel Institute of Technology, Haifa, Israel*[(a)]
[(3)]*Center for Nonlinear Studies and Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[(4)]*Department of Physics, Williams College, Williamstown, Massachusetts 01267*[(a)]
(Received 15 February 1990)

- Two identically prepared quantum systems the same way in different locations, want to determine their state
- Detailed sequence of observations alternating between two systems can be proven to be the most efficient way
- Can be done better in the same lab than other labs… Entanglement!

More context

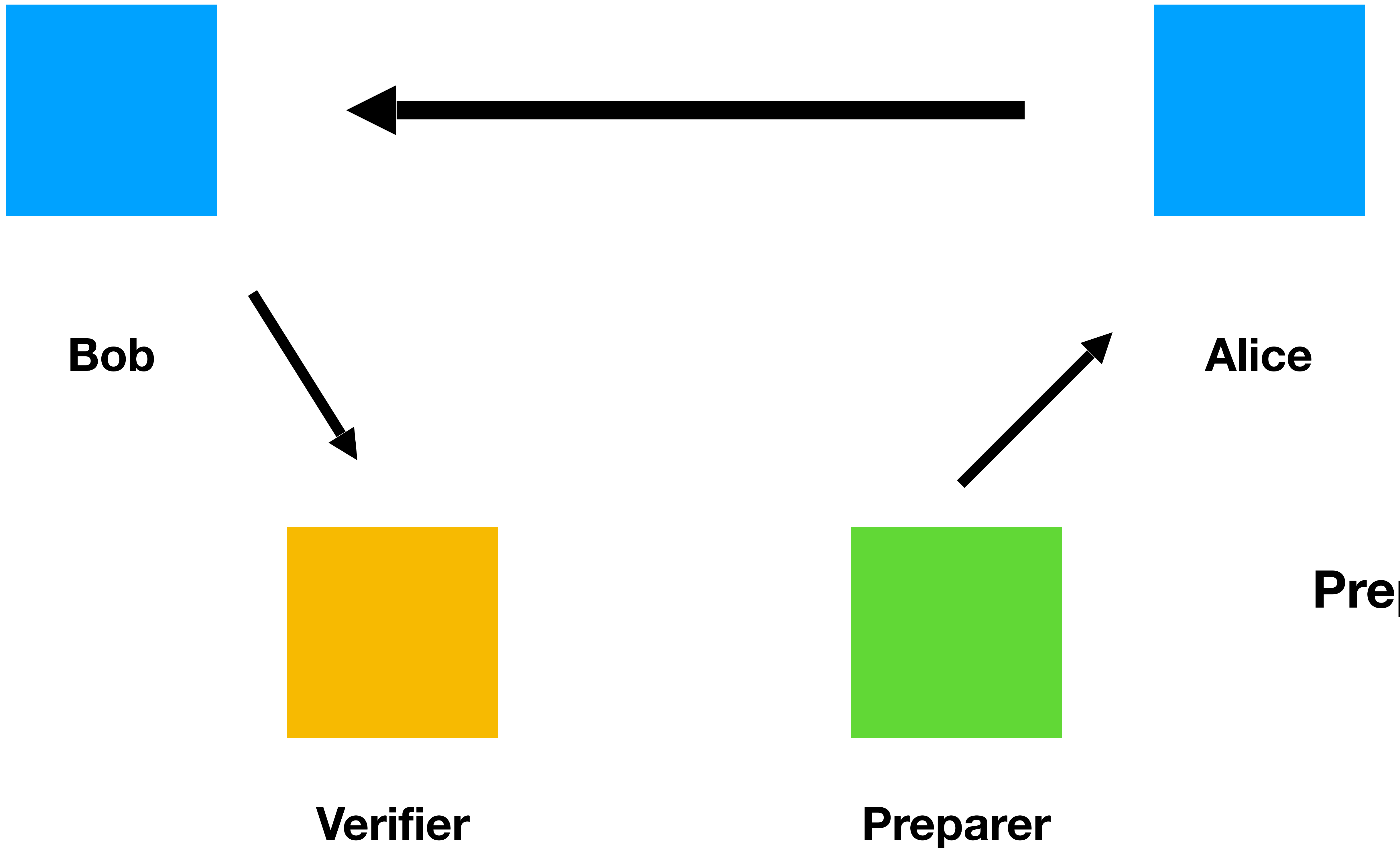
- Bennett, et. al. motivated by entanglement explanation of Peres paper
- Devised quantum teleportation in 1993 paper (feat. humans to the right)
- Experimentally verified:
    - This paper (photons, 1998)
    - Riebe, et. al. (trapped calcium ions, 2004)
    - Recently performed over 100km

(top, left) Richard Jozsa, William K. Wootters, Charles H. Bennett. (bottom, left) Gilles Brassard, Claude Crépeau, Asher Peres. Photo: André Berthiaume.

What can we measure?

Bob makes a guess of the state using info, sends to verifier

$$|\phi_l^c >$$



Alice measures the state and receives a set of outcomes, which she transmits to Bob classically

$$\sum_{l=1}^{L} |\epsilon_l >< \epsilon_l| = I$$

Preparer sends a polarized photon state

$$\phi_0, \phi_{2\pi/3}, \phi_{-2\pi/3}$$

What can we measure?

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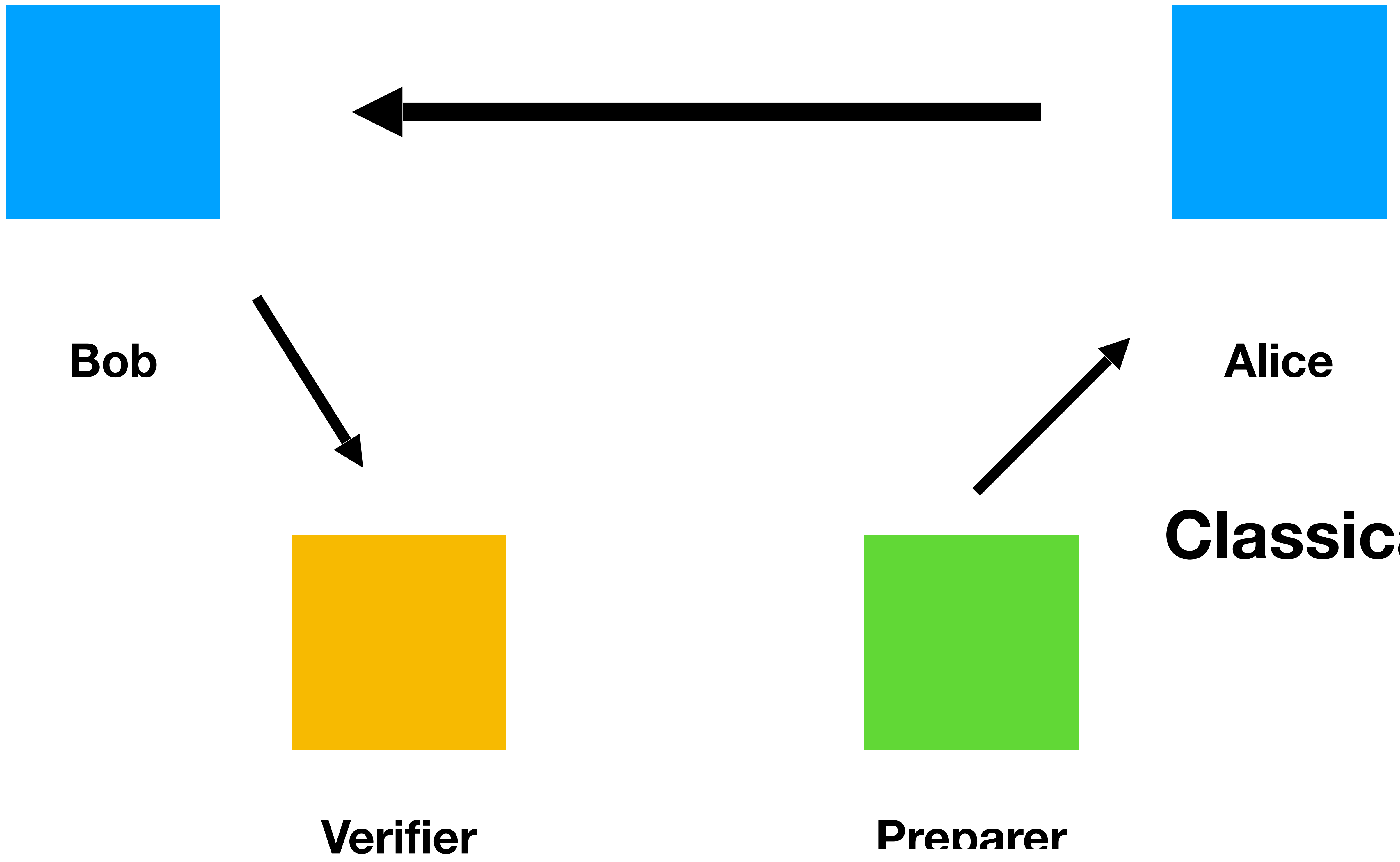
$$|\phi_l^c >$$



Alice measures the state and receives a set of outcomes, which she transmits to Bob classically

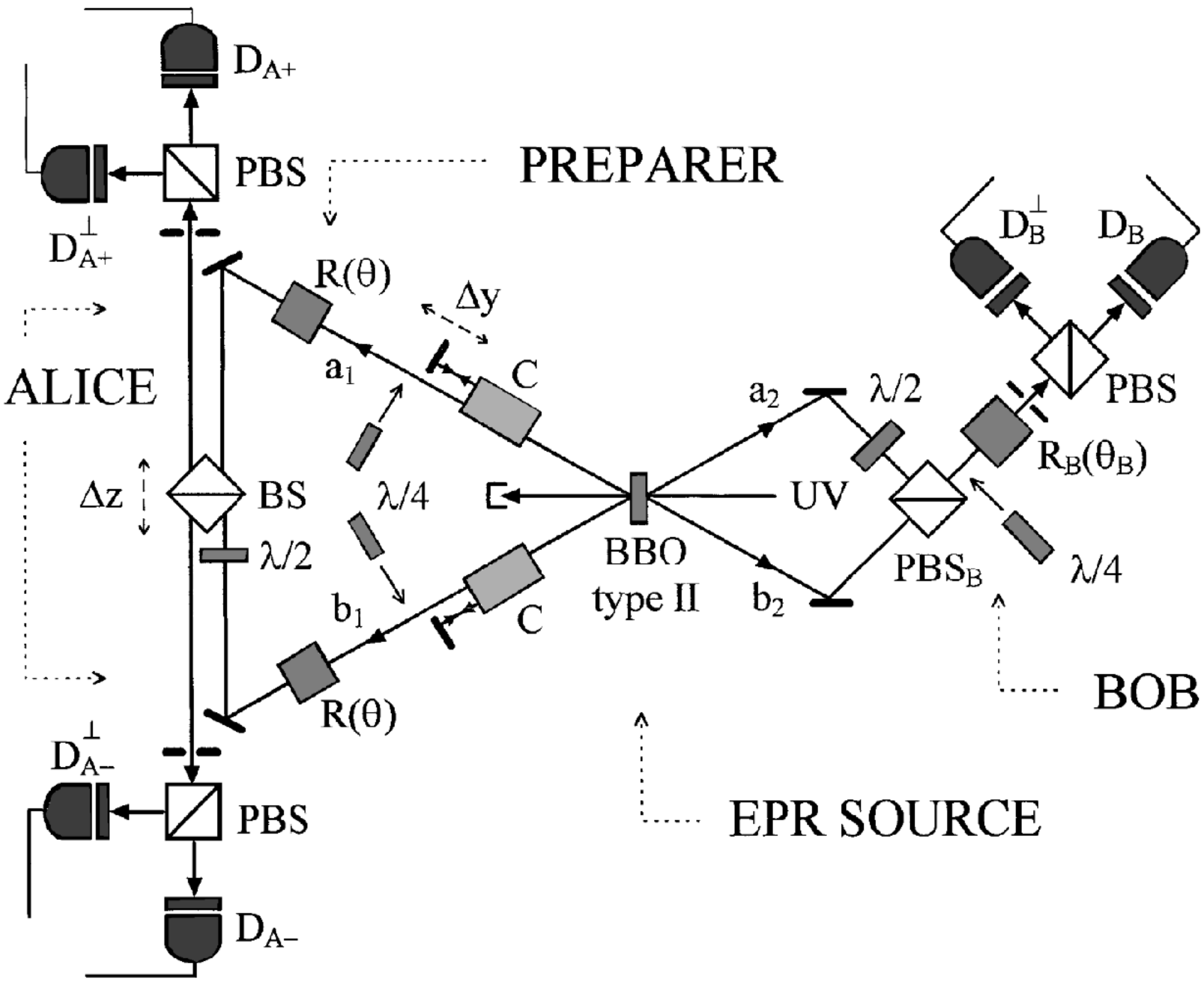
$$\sum_{l=1}^{L} |\epsilon_l >< \epsilon_l| = I$$

Classically (want to violate this bound):

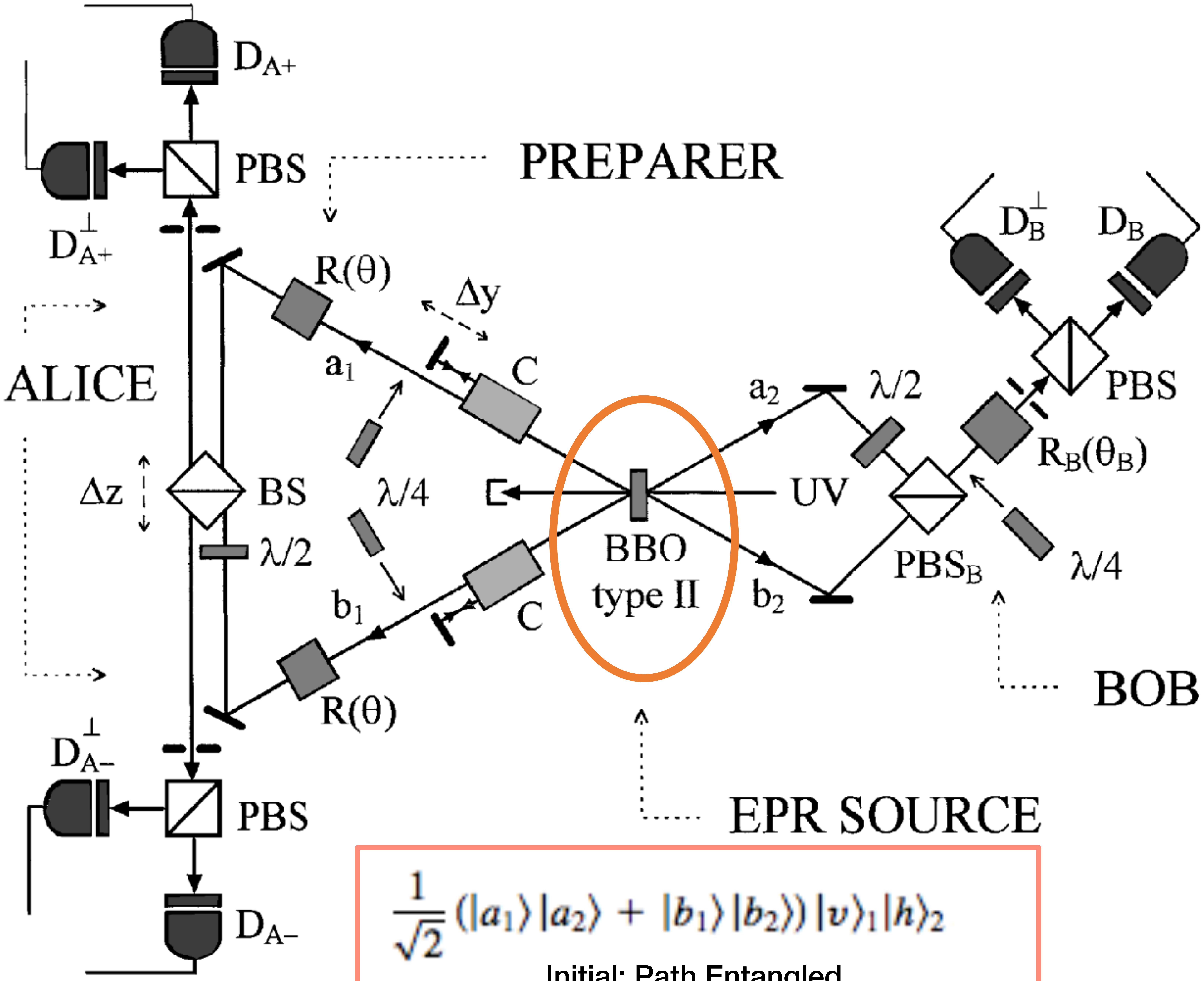
$$S \leq \frac{3}{4}\,.$$

Average probability of passing test: $$S = \sum_{a,l} \frac{1}{3} |\langle \phi_a | \phi_l^c \rangle|^2 \, |\langle \phi_a | \varepsilon_l \rangle|^2.$$

Experimental Setup

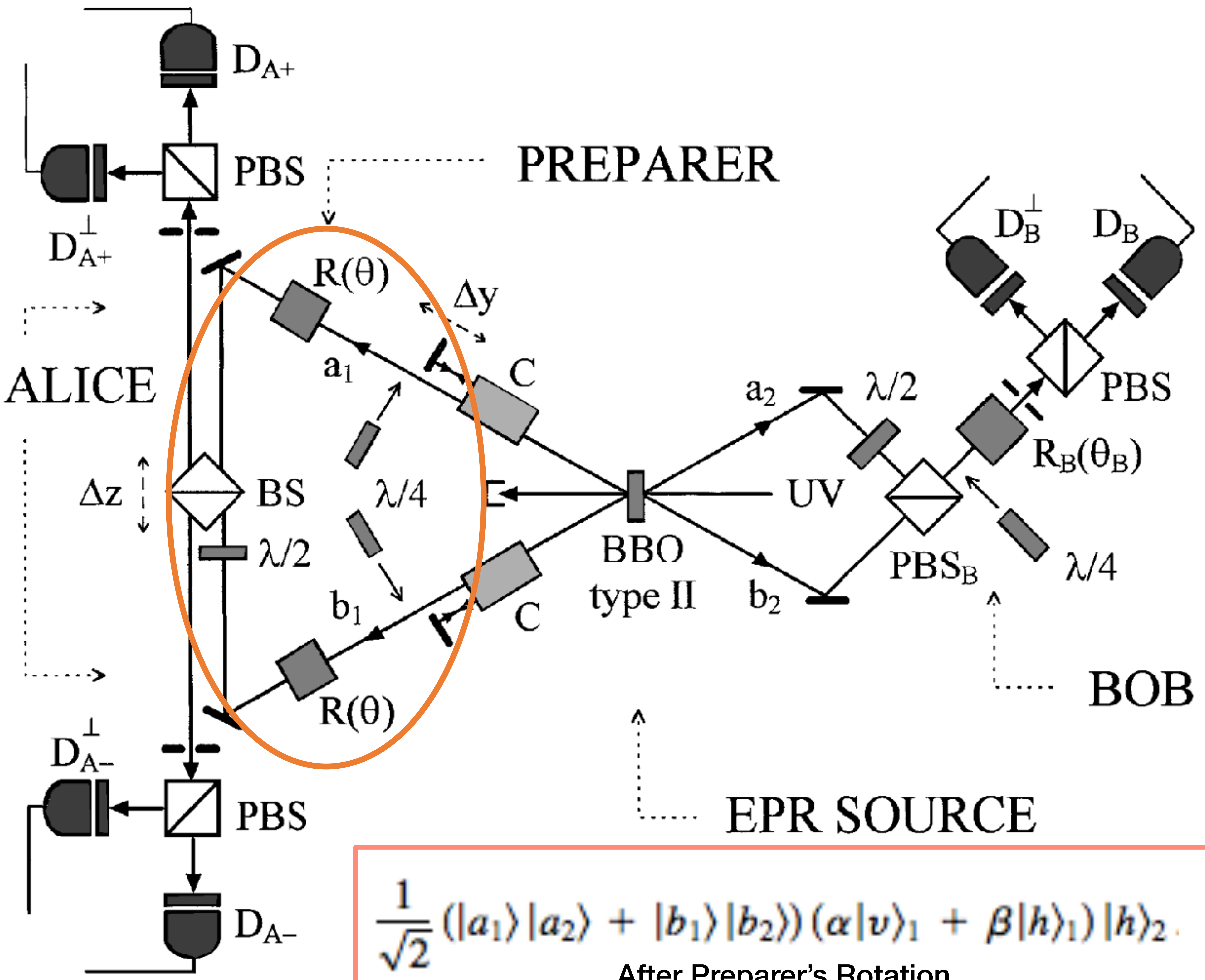


D_{A+}
PBS
PREPARER
$D_{A+}^{\perp}$
$D_B^{\perp}$
D_B
$R(\theta)$
Δy
a_1
C
ALICE
a_2
$\lambda/2$
PBS
Δz
BS
$\lambda/4$
UV
$R_B(\theta_B)$
$\lambda/2$
BBO
type II
PBS_B
$\lambda/4$
b_1
C
b_2
BOB
$R(\theta)$
$D_{A-}^{\perp}$
PBS
EPR SOURCE
D_{A-}



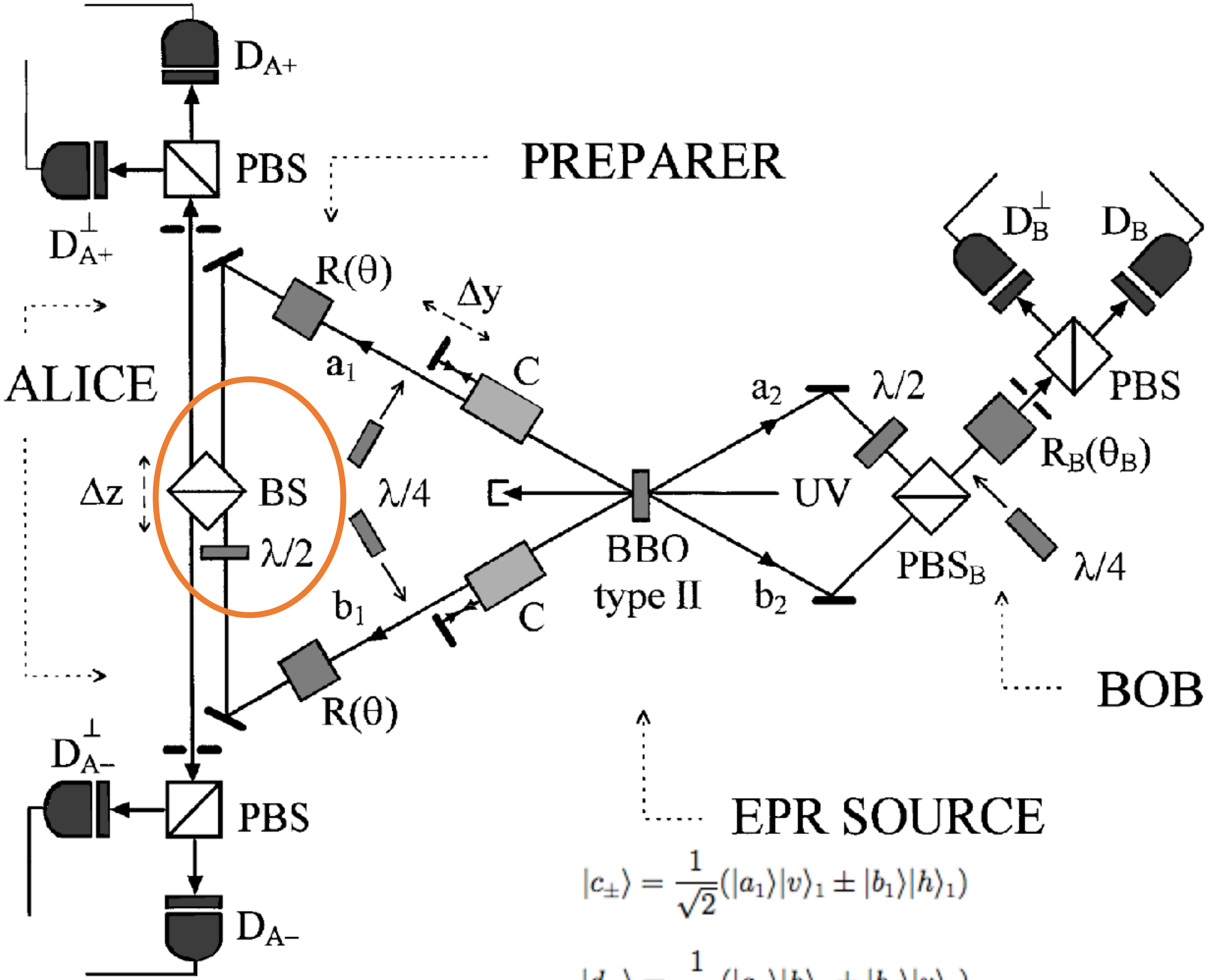
$$\frac{1}{\sqrt{2}}(|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle)|v\rangle_1|h\rangle_2$$

Initial: Path Entangled



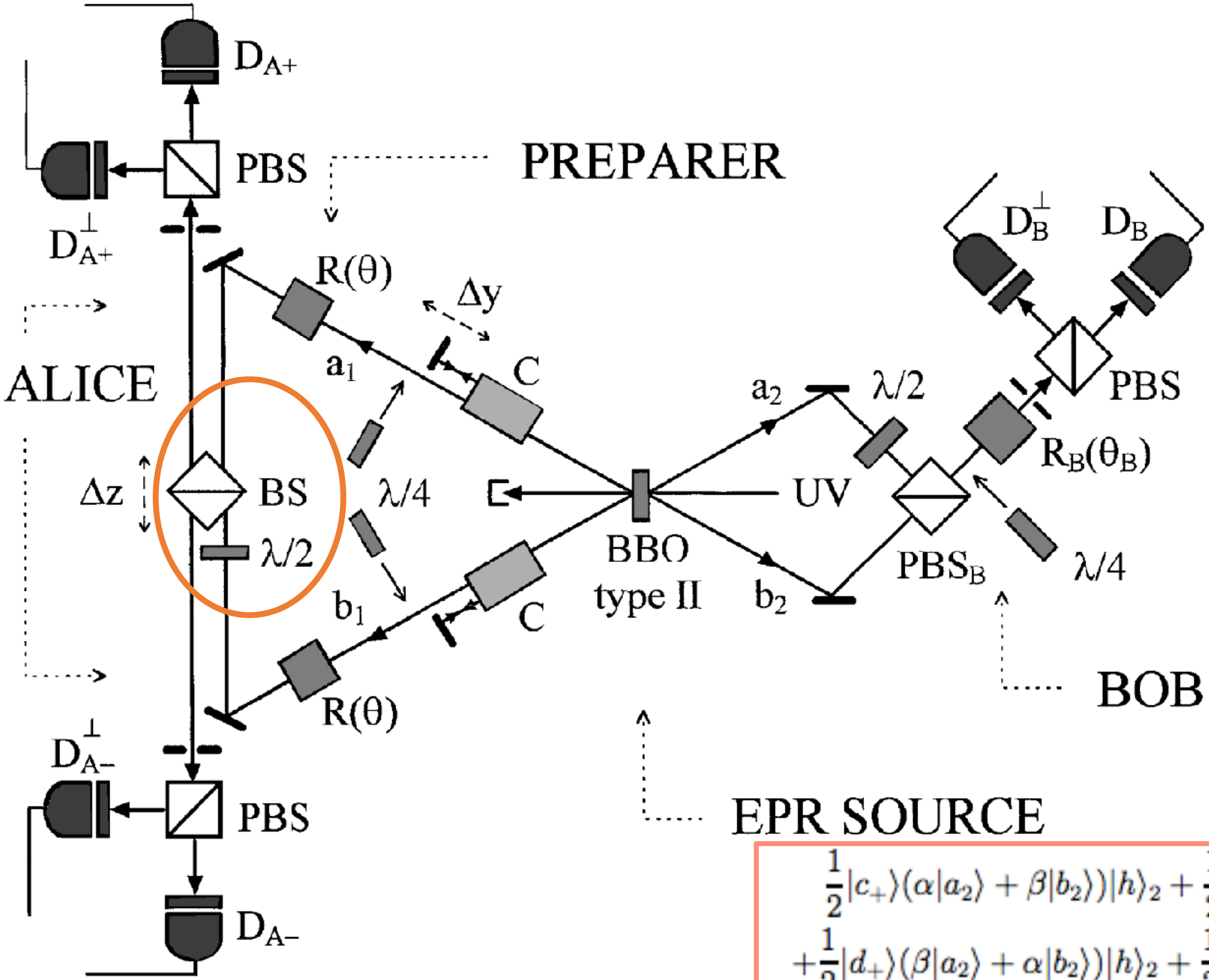
$$\frac{1}{\sqrt{2}}(|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle)(\alpha|v\rangle_1 + \beta|h\rangle_1)|h\rangle_2$$

After Preparer's Rotation



$$|c_{\pm}\rangle = \frac{1}{\sqrt{2}}(|a_1\rangle|v\rangle_1 \pm |b_1\rangle|h\rangle_1)$$

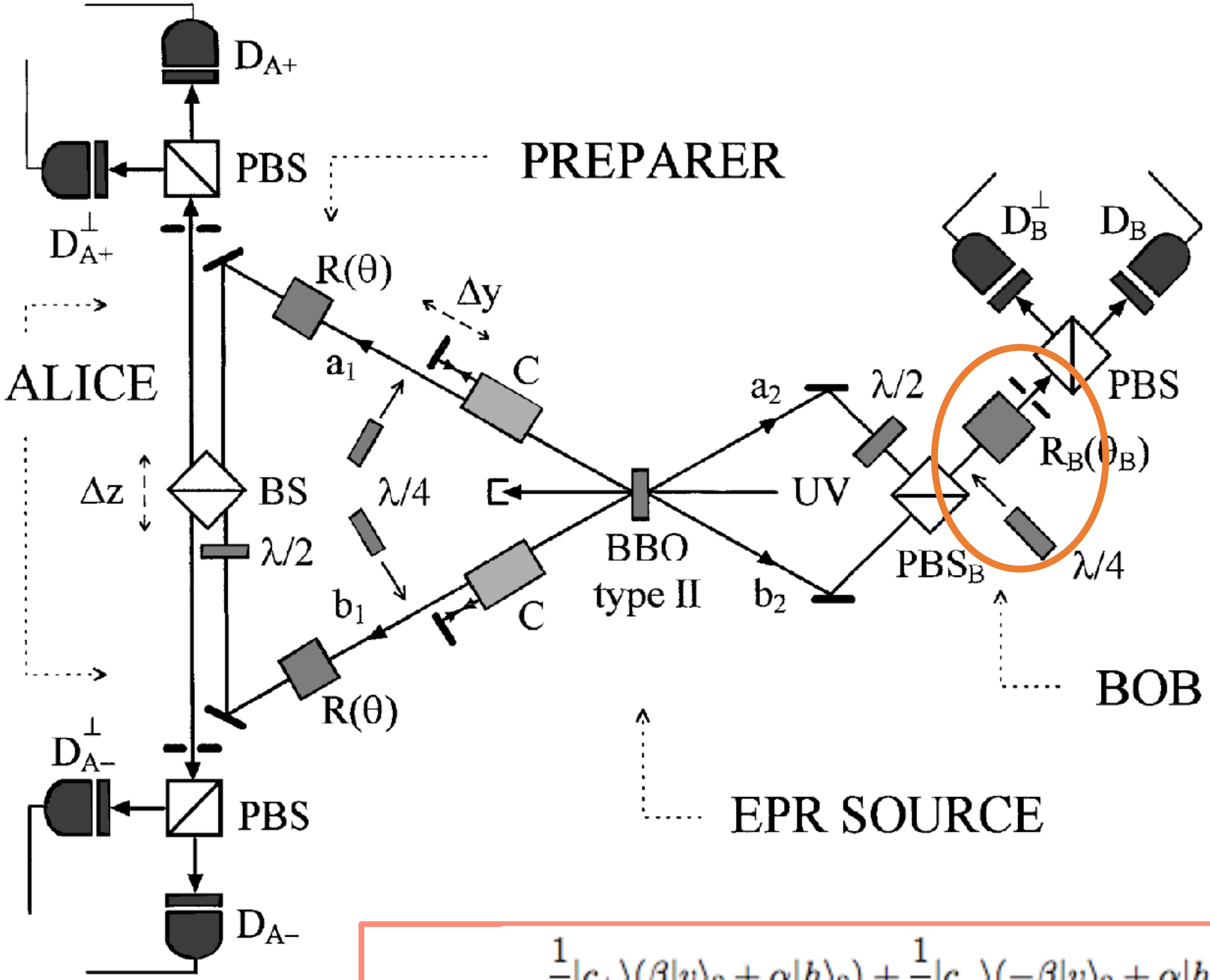
$$|d_{\pm}\rangle = \frac{1}{\sqrt{2}}(|a_1\rangle|h\rangle_1 \pm |b_1\rangle|v\rangle_1)$$



$$\frac{1}{2}|c_+\rangle(\alpha|a_2\rangle+\beta|b_2\rangle)|h\rangle_2+\frac{1}{2}|c_-\rangle(\alpha|a_2\rangle-\beta|b_2\rangle)|h\rangle_2$$

$$+\frac{1}{2}|d_+\rangle(\beta|a_2\rangle+\alpha|b_2\rangle)|h\rangle_2+\frac{1}{2}|d_-\rangle(\beta|a_2\rangle-\alpha|b_2\rangle)|h\rangle_2$$

In Alice's measurement basis



$$\frac{1}{2}|c_+\rangle(\beta|v\rangle_2+\alpha|h\rangle_2)+\frac{1}{2}|c_-\rangle(-\beta|v\rangle_2+\alpha|h\rangle_2)$$
$$+\frac{1}{2}|d_+\rangle(\alpha|v\rangle_2+\beta|h\rangle_2)+\frac{1}{2}|d_-\rangle(-\alpha|v\rangle_2+\beta|h\rangle_2)$$

After Bob Beam-splitter: back to polarization d.o.f

Results: an unknown quantum state can be teleported through EPR channel.

- Two sets of experiments are done, where the prepared unknown state is linearly polarized state and elliptically polarized state respectively.

- Linearly polarized state
  - The probability of the teleported state passing verification exceed the classical limit.
  - The unknown state can be reconstructed passively at Bob’s side.

- Elliptically polarized state
  - The unknown state can be reconstructed passively at Bob’s side.

Results: Linearly Polarized

1. The probability of the teleported state passing verification exceed the classical limit 3/4:

$$S = \frac{I_{\|}}{I_{\|} + I_{\perp}} = 0.853 \pm 0.012$$

This result exceeds the classical limit by eight standard deviation.

Results: Linearly Polarized

2. The unknown state can be reconstructed passively.

The count rates for four simultaneous coincidence experiments between Alice's Bell measurement and Bob's detection agrees with prediction.

The results are verified by reconstruction of the unknown state by a passive unitary transformation.

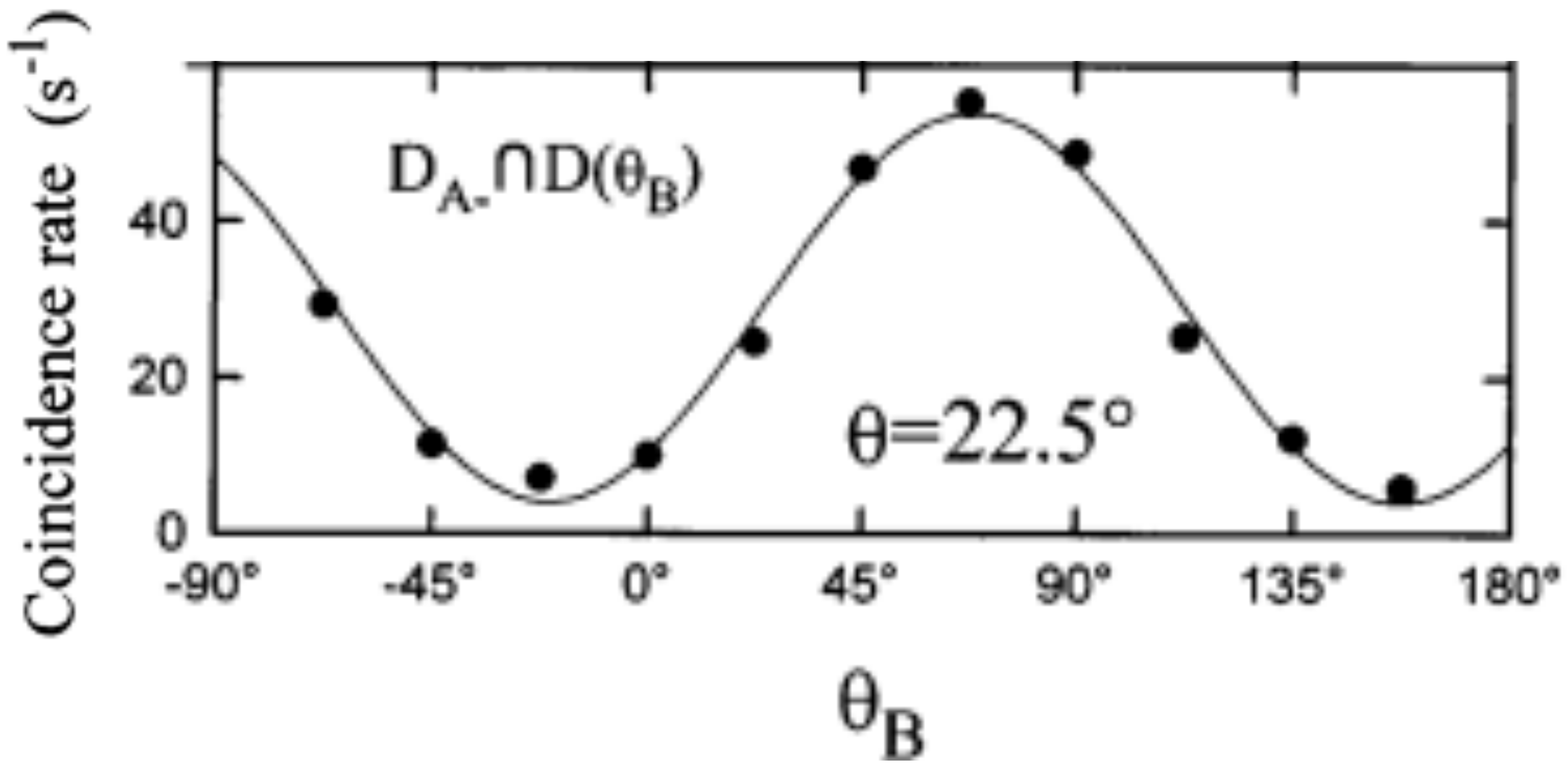


Fig. 2 Coincidence rate (solid curve-theory, solid dot-experiment)

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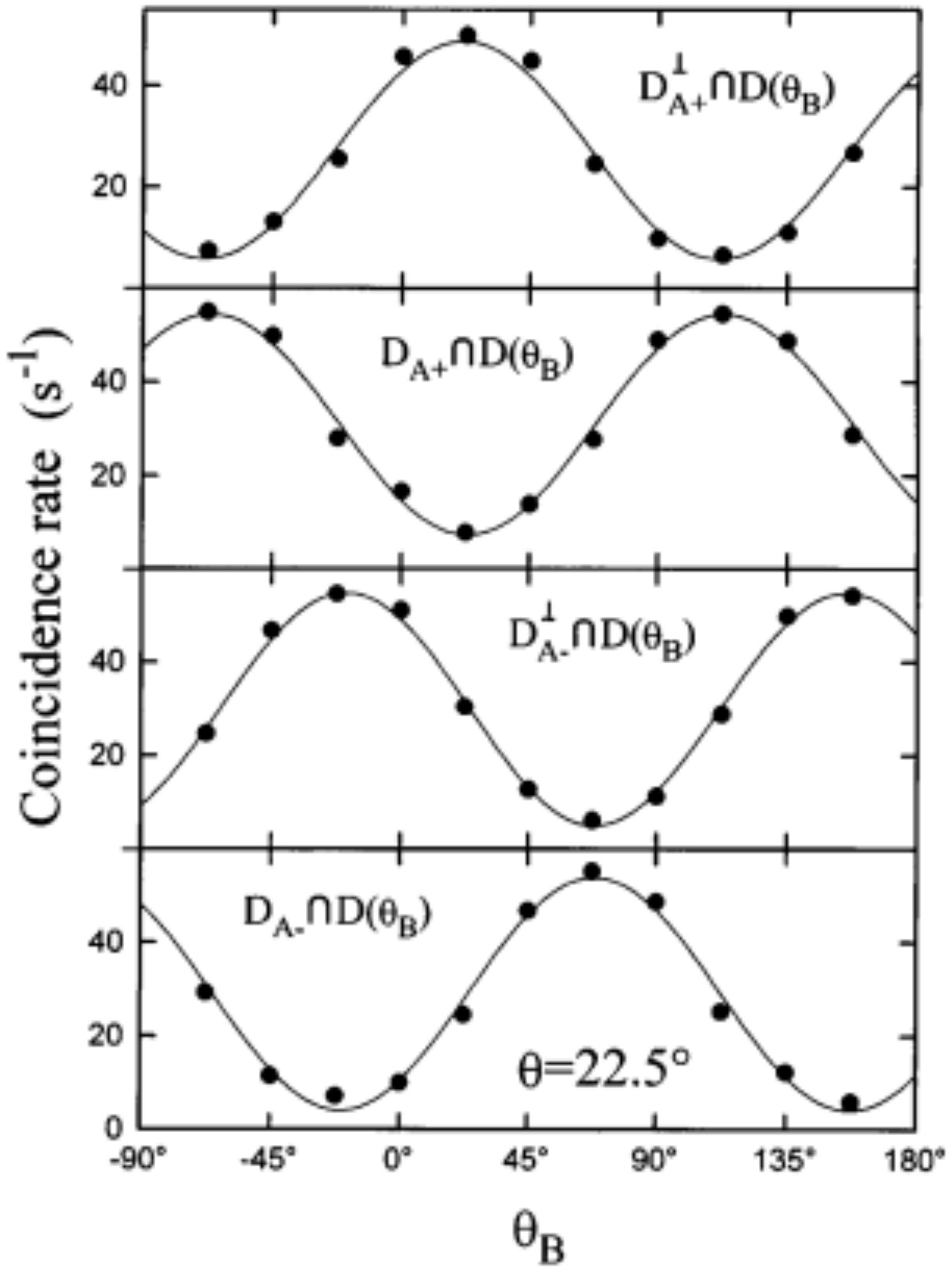


Fig. 2 Coincidence rates from four simultaneous Bell measurements (solid curve-theory, solid dot-experiment)

Results: Elliptically Polarized

1. The unknown state can be reconstructed passively.

The count rates for four simultaneous coincidence experiments between Alice's Bell measurement and Bob's detection (Fig.3) agrees with the prepared unknown state (ɣ = 20°):

$$\frac{1}{\sqrt{2}}[(1 + icos(2\gamma))|v\rangle + sin(2\gamma)|h\rangle]$$

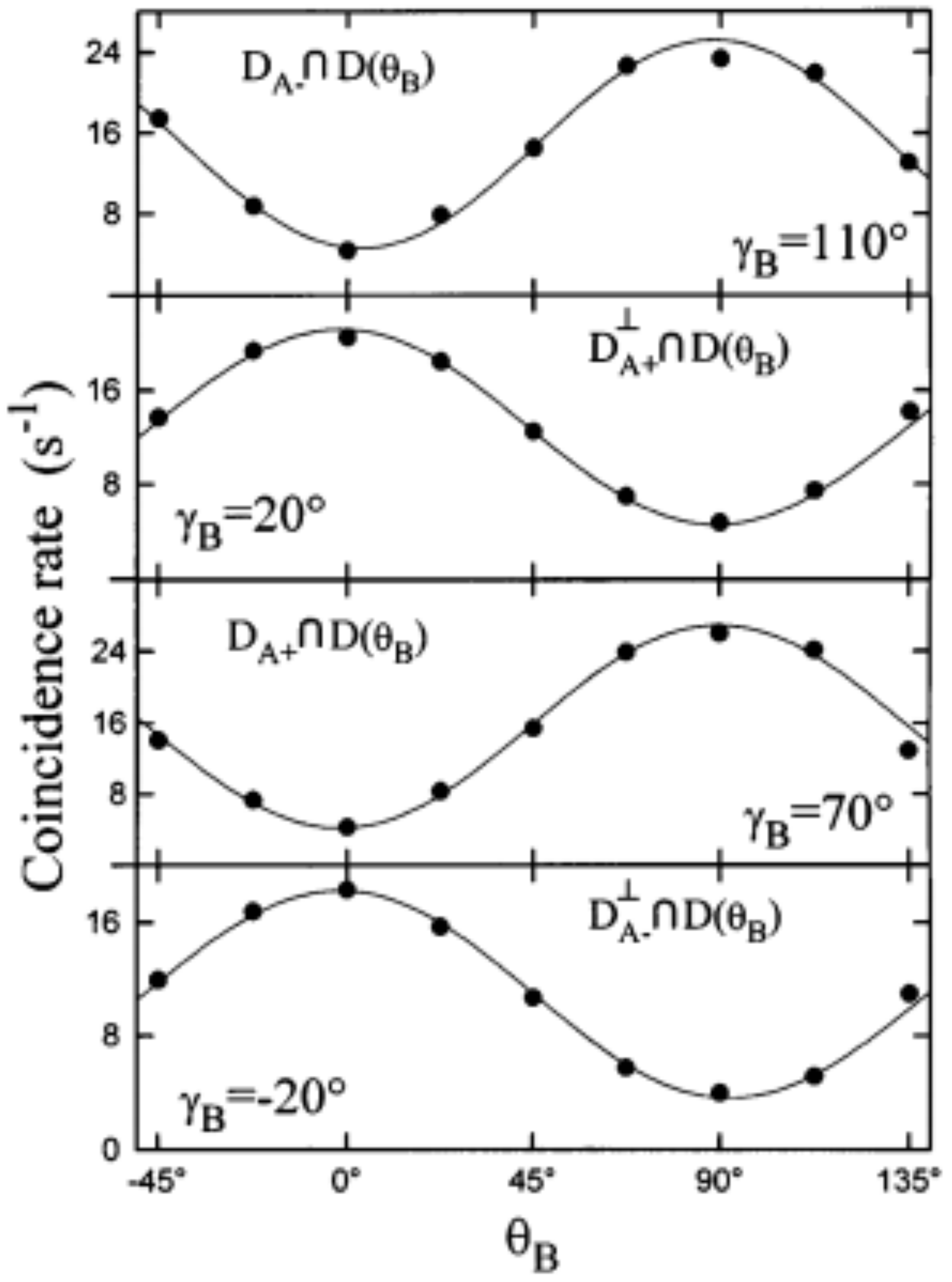


Fig. 3 Coincidence rate (solid curve-theory, solid dot-experiment)

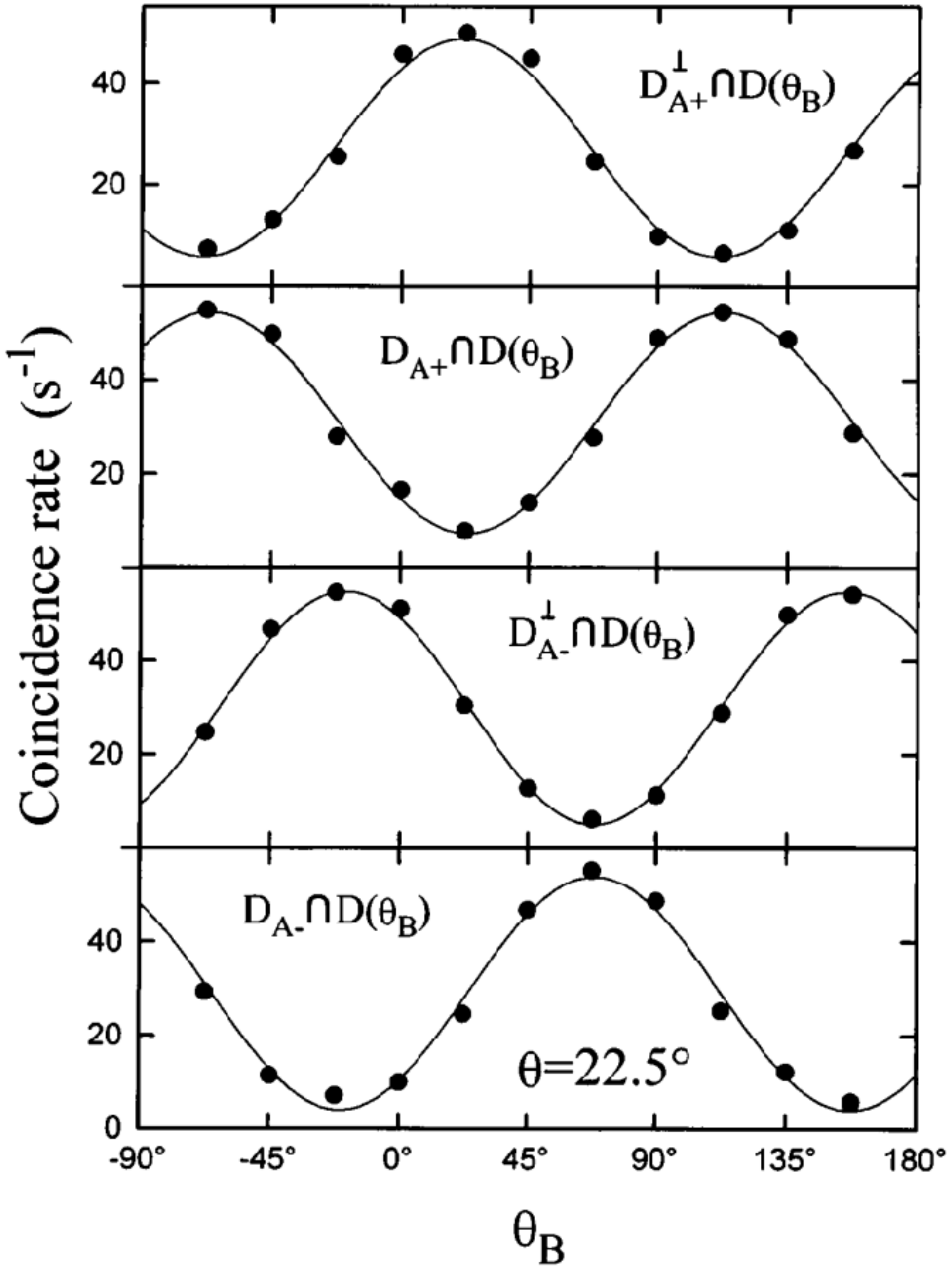
Citation Analysis

- Boschi, et. al. cited 1221 times* (since 1998)
- “A scheme for efficient quantum computation with linear optics”, Knill et. al (2001)
  - Cited 3300 times*
- “Quantum entanglement”, Hordecki, et. al.
  - Cited 2918 times*
- “Unconditional quantum teleportation”, Furusawa, et. al. (1998)
  - Cited 2067 times*

*Citation statistics retrieved from SCOPUS.

Critique

- The Good:
  - All quantities defined clearly in text => Non experts able to follow discussion of quantum teleportation
- The Bad:
  - Lack of labelled sections => difficult to differentiate between theory, experiment, and results sections
- The Ugly:
  - Data points larger than error bars => Error bars invisible to human eyes
- Overall:
  - Excellent paper with some minor gripes



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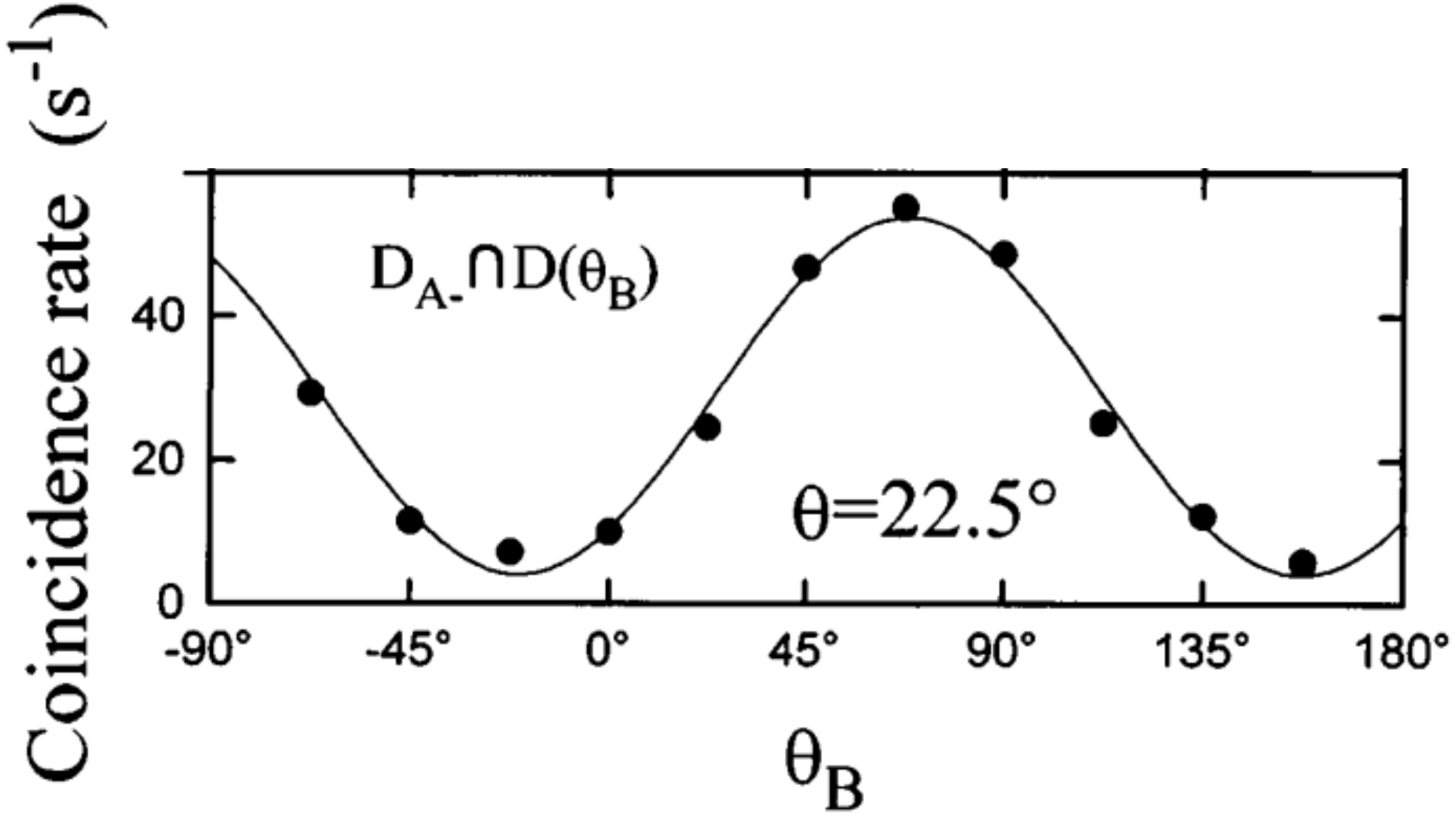

Conclusions

- Defined what constitutes non-classical EPR teleportation
- Designed experiment to observe EPR teleportation
- Experimentally confirmed non-classical teleportation
- Set the stage for future work in EPR teleportation