Problem 1 (20 points)

A neutral meson $X$ has $J = 1$ ($J$ is the total angular momentum) and a neutral meson $Y$ has $J = 2$. Both $X$ and $Y$ can decay to $\pi^+\pi^-$ by strong interaction. What are the $C$ and $P$ parities of $X$ and $Y$? Is $X \to \pi^0 + \gamma$ decay allowed by electromagnetic interaction? Is $Y \to \pi^0 + \gamma$ decay allowed by electromagnetic interaction?

Problem 2 (20 points)

The dominant decays of the $\eta$ meson are

$$\eta \to 2\gamma \,(39\%),\ \eta \to 3\pi \,(56\%),\ \eta \to \pi\pi\gamma \,(5\%)$$

and it’s classified as a “stable” particle, so evidently none of these is a purely strong interaction. Offhand, this seems odd, since at 549 MeV/c$^2$ the $\eta$ has plenty of mass to decay strongly into $2\pi$ or $3\pi$.

a) Explain why the $2\pi$ mode is forbidden, for both strong and electromagnetic interactions.

b) Explain why the $3\pi$ mode is forbidden as a strong interaction, but allowed as an electromagnetic decay.

Problem 3 (15 points)

You are planning for a new experiment to search for an exotic meson which can not be described as a quark-antiquark bound state. Which of the following decay channel(s) for $X$ would prove unambiguously that $X$ is indeed an exotic meson? Assume that these decays proceed via strong interaction.

a) $X \to \pi^+ + \pi^0$

b) $X \to K^- + \pi^+ + K^-$

c) $X \to \pi^+ + \pi^0 + \pi^+$
Problem 4 (25 points)

Assuming isospin conservation, are the following reactions allowed?

(a) $d + d \rightarrow ^4He + \omega$
(b) $d + d \rightarrow ^4He + \eta$
(c) $d + d \rightarrow ^4He + \rho^0$
(d) $p + d \rightarrow ^3He + \eta$
(e) $p + p \rightarrow d + \rho^+$

Problem 5 (20 points)

Find the effect of space-inversion (P) and time-reversal (T) operations on the following quantities:

a) position ($\mathbf{r}$)
b) momentum ($\mathbf{p}$)
c) spin ($\sigma$)
d) electric field ($\mathbf{E}$)
e) magnetic field ($\mathbf{B}$)
f) magnetic dipole moment ($\sigma \cdot \mathbf{B}$)
g) electric dipole moment ($\sigma \cdot \mathbf{E}$)
h) spin correlation ($\sigma_1 \cdot \sigma_2$)
i) longitudinal polarization ($\sigma \cdot \mathbf{p}$)
j) transverse polarization ($\sigma \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$)