

R. P. Feynman:

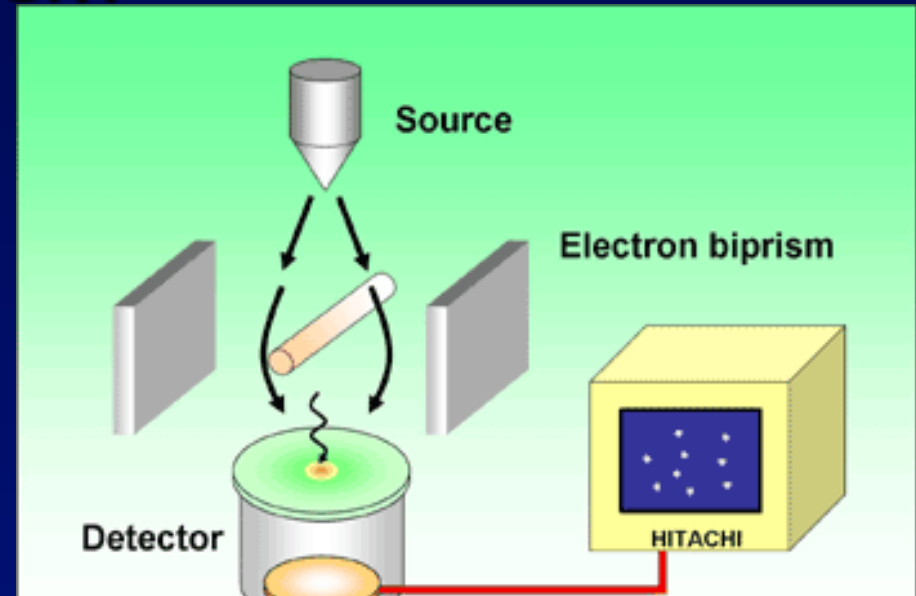
Interference is ... “the heart of QM.

In reality it contains the *only* mystery.

We cannot make the mystery go away  
by ‘explaining’ how it works.”

# “Double-slit” Experiment for Electrons

- Electrons are accelerated to 50 keV  $\rightarrow \lambda = 0.0055$  nm
- Central wire is positively charged  $\rightarrow$  bends electron paths so they overlap
- A position-sensitive detector records where they appear.
- $\ll 1$  electron in system at any time



[A. TONOMURA (Hitachi) -- pioneered electron holography]

# Application: Electron Holography

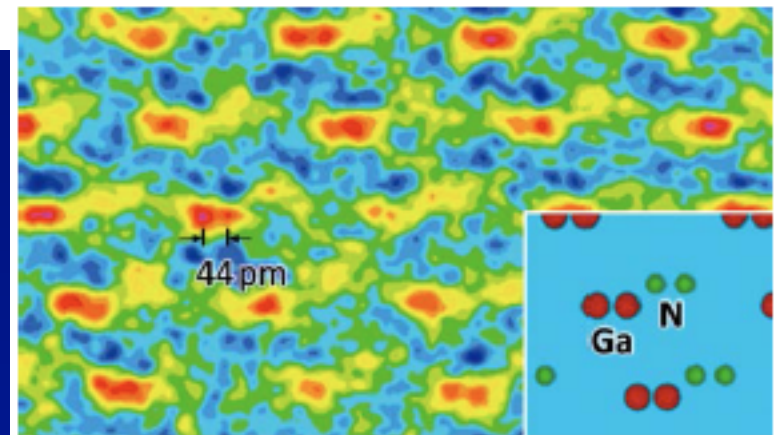
## Development of an Atomic-Resolution Holography Electron Microscope<sup>\*1</sup> with the World's Highest Point Resolution (43 picometers)



Tokyo, February 18, 2015 --- Hitachi, Ltd. today announced that it has developed an atomic-resolution holography electron microscope accelerated at a 1.2-megavolt ("MV") under the government-sponsored FIRST Program<sup>\*2</sup> project named "Development and Application of an Atomic-resolution Holography Electron Microscope" (Principal Investigator: the late Dr. Akira Tonomura, Acting Principal Investigator: Dr. Nobuyuki Osakabe), and has achieved the world's highest point resolution of 43 picometers ("pm"), i.e., 43 trillionths of a meter. With its ability to measure electromagnetic fields at the atomic resolution, the developed microscope will contribute to the advancement of fundamental sciences by supporting the development of cutting-edge functional materials, through elucidating quantum phenomena that cause the functions and properties of high-performance materials, such as magnets, batteries, and superconductors.

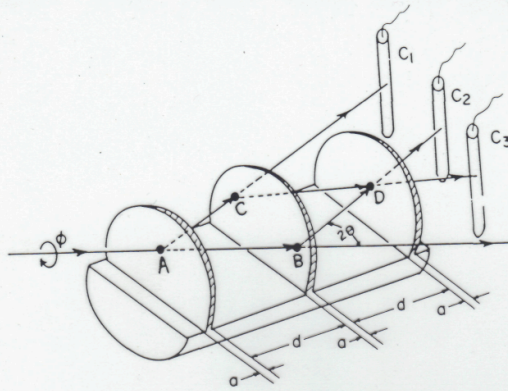
"Aberration corrected 1.2-MV cold field-emission transmission electron microscope with a sub-50-pm resolution."

Akashi et al., Appl. Phys. Lett. 106, 074101 (2015)

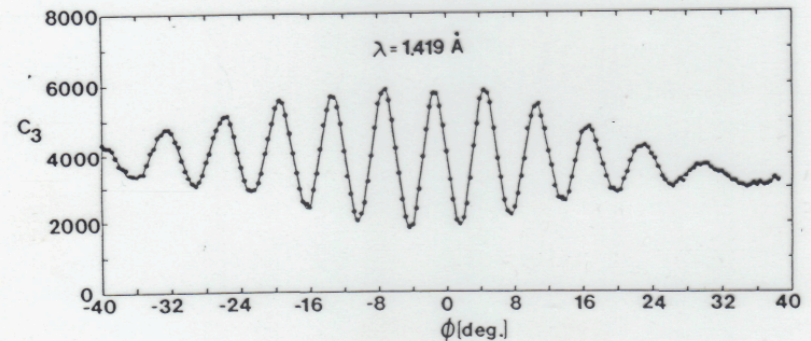
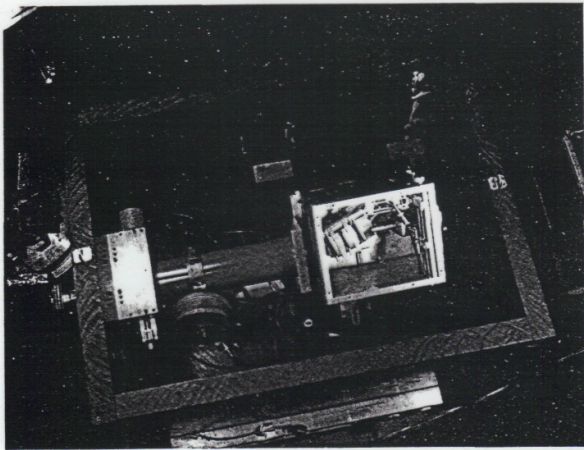


Low-pass filtered image Atomic positions  
Fig.2 Observation case of GaN crystal

# Neutron interference



Path lengths are fixed. As entire interferometer is rotated, the paths are at different heights  $\rightarrow$  different gravitational potential energies  $\rightarrow$  different speeds  $\rightarrow$  different phases  $\rightarrow$  fringes



COW experiment:  
Colella, Overhauser and Werner  
PRL **34**, 1472 (1975)

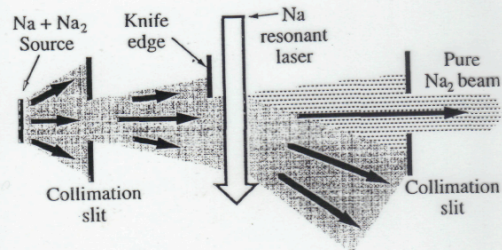
## Optics and Interferometry with Na<sub>2</sub> Molecules

Michael S. Chapman,<sup>1</sup> Christopher R. Ekstrom,<sup>1</sup> Troy D. Hammond,<sup>1</sup> Richard A. Rubenstein,<sup>1</sup> Jörg Schmiedmayer,<sup>1,2</sup> Stefan Wehinger,<sup>1,2</sup> and David E. Pritchard<sup>1</sup>

<sup>1</sup>*Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

<sup>2</sup>*Institut für Experimentalphysik, Universität Innsbruck, A-6020 Innsbruck, Austria*  
(Received 20 January 1995)

We have produced an intense, pure beam of sodium molecules (Na<sub>2</sub>) by using light forces to separate the atomic and molecular species in a seeded supersonic beam. We used diffraction from a microfabricated grating to study the atomic and molecular sodium in the beam. Using three of these gratings, we constructed a molecule interferometer with fully separated beams and high contrast fringes. We measured both the real and imaginary parts of the index of refraction of neon gas for Na<sub>2</sub> molecule de Broglie waves by inserting a gas cell in one arm of the interferometer.



$$\lambda_{dB}(\text{Na}_2) = 0.11 \text{ \AA}$$

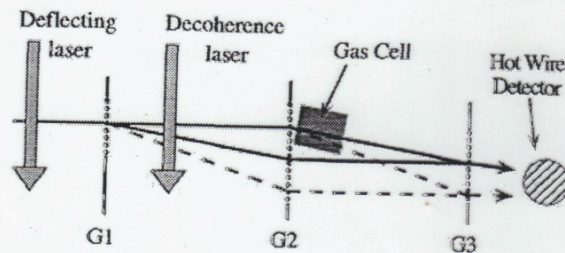
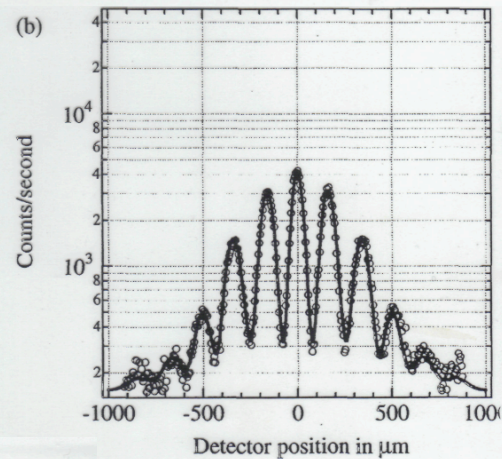


FIG. 3. Schematic of our interferometer showing the paths of Na (dashed line) and Na<sub>2</sub> (solid line). G1, G2, and G3 indicate the three diffraction gratings.

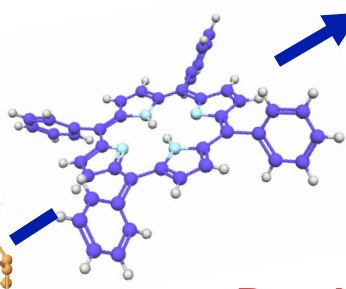
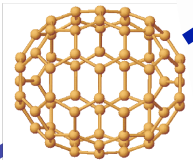
**Which-Path Information,  
Decoherence & Dephasing  
in Macromolecule Interferometry**

**Markus Arndt**

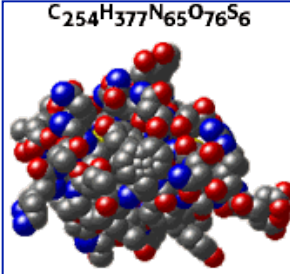
**Institut für Experimentalphysik, Universität Wien**

**BEC, Atom Lasers ...  
(1995 ...)**

**C<sub>60</sub> (1999)**



**Porphyrins (2003)**

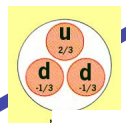
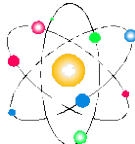


**Proteins ?**

**Na<sub>2</sub>, I<sub>2</sub>, He<sub>2</sub>-He<sub>26</sub> (1994-1996)**

**Cold atoms (1988 ...)**

**He & H<sub>2</sub> (1930)**



**Neutron (1936)**

**Electron (1927)**

# Interference of larger particles

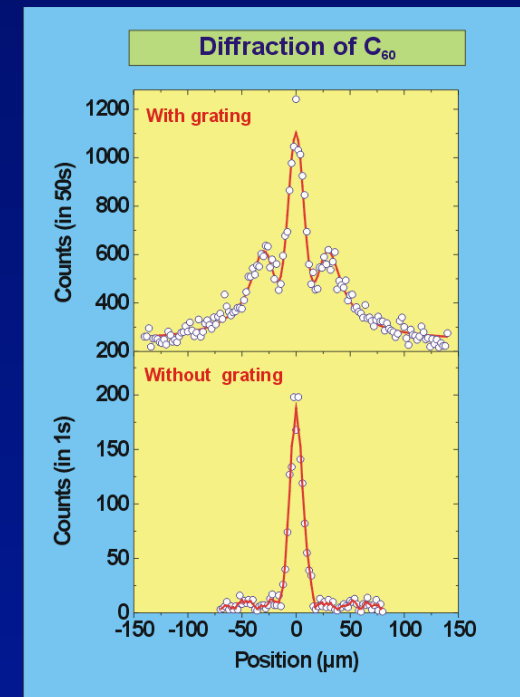
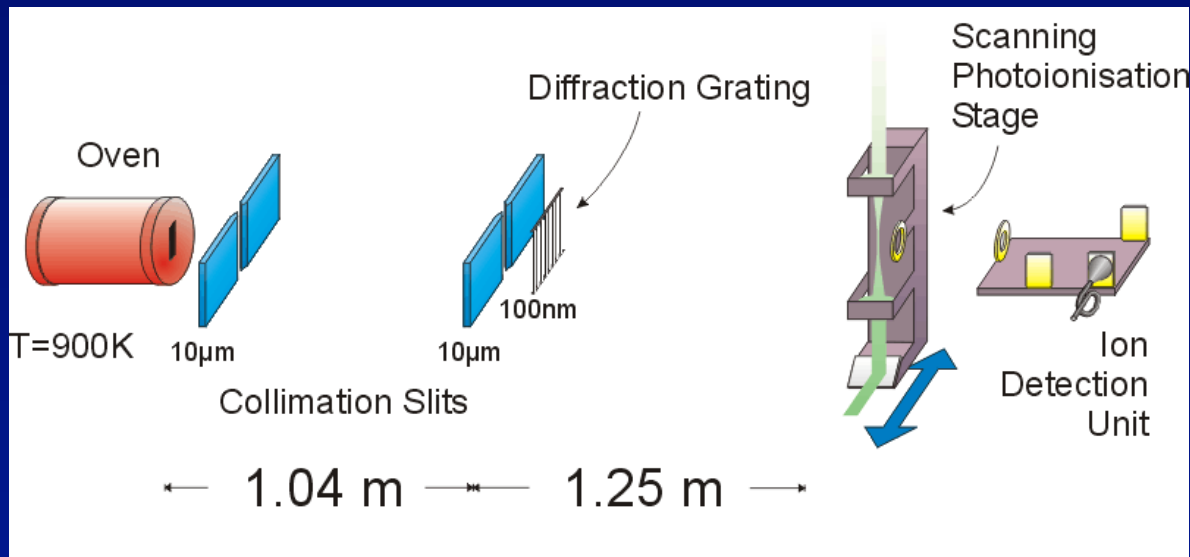
- Matter-wave interference has now been demonstrated with electrons, neutrons, atoms, small molecules, BIG molecules, & biological molecules
- Recent Example: Interference of  $C_{60}$ , a.k.a. “fullerenes”,



$$\text{Mass} = (60 \text{ C})(12 \text{ g/mole}) = 1.2 \times 10^{-24} \text{ kg}$$

$$\frac{\langle p^2 \rangle}{2m} = \text{K.E.} \approx \frac{3}{2} kT \Rightarrow \langle p \rangle = \sqrt{3kTm} = 2.1 \times 10^{-22} \text{ kg m/s}$$

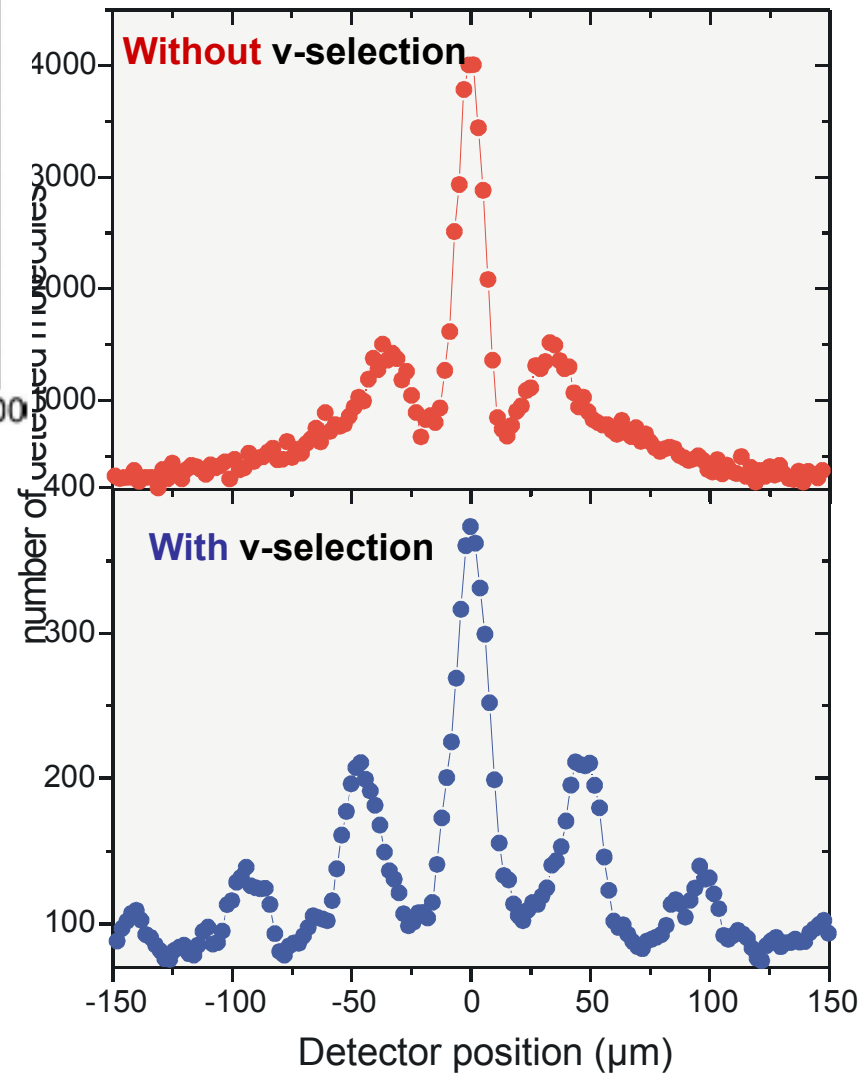
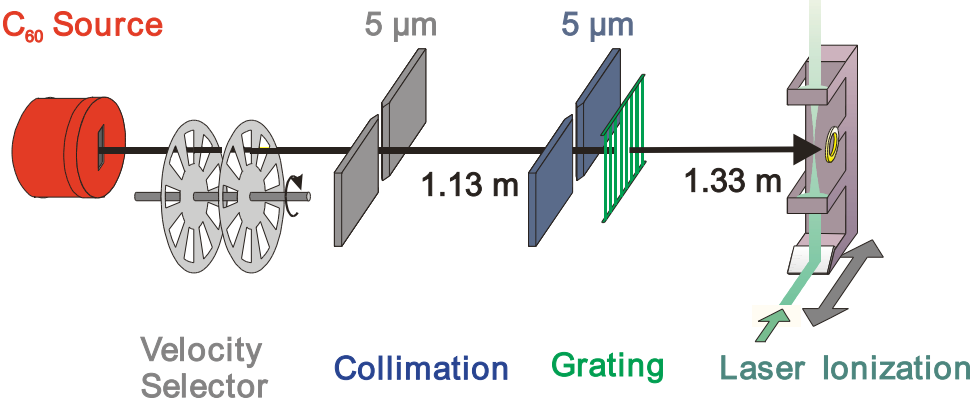
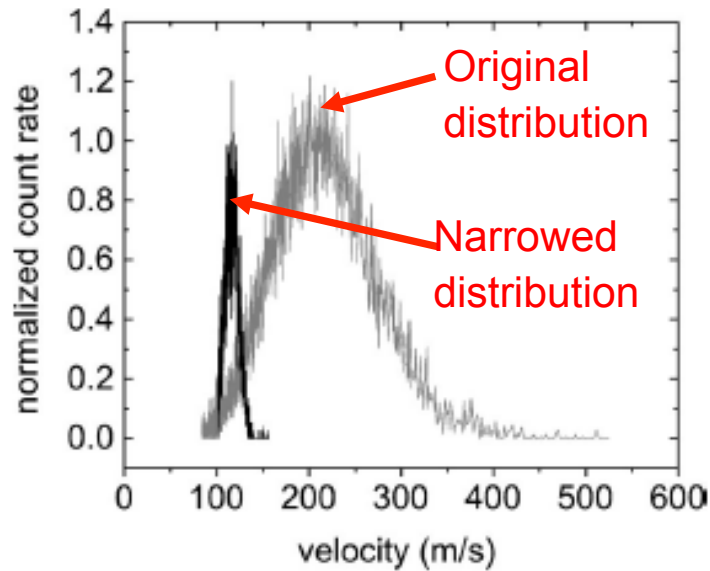
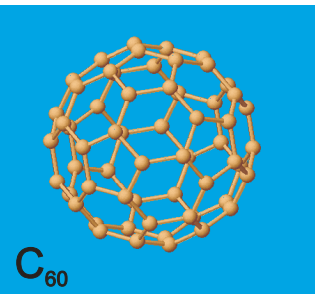
$$\lambda = h/p = 2.5 \text{ pm} \quad (\text{c.f. } C_{60} \text{ is } \sim 1 \text{ nm across!})$$



[A. Zeilinger (U. Vienna), 1999]



# Short reminder: Far-Field Diffraction at a nanograting



M. Arndt et al., *Nature* 401, 680 (1999).

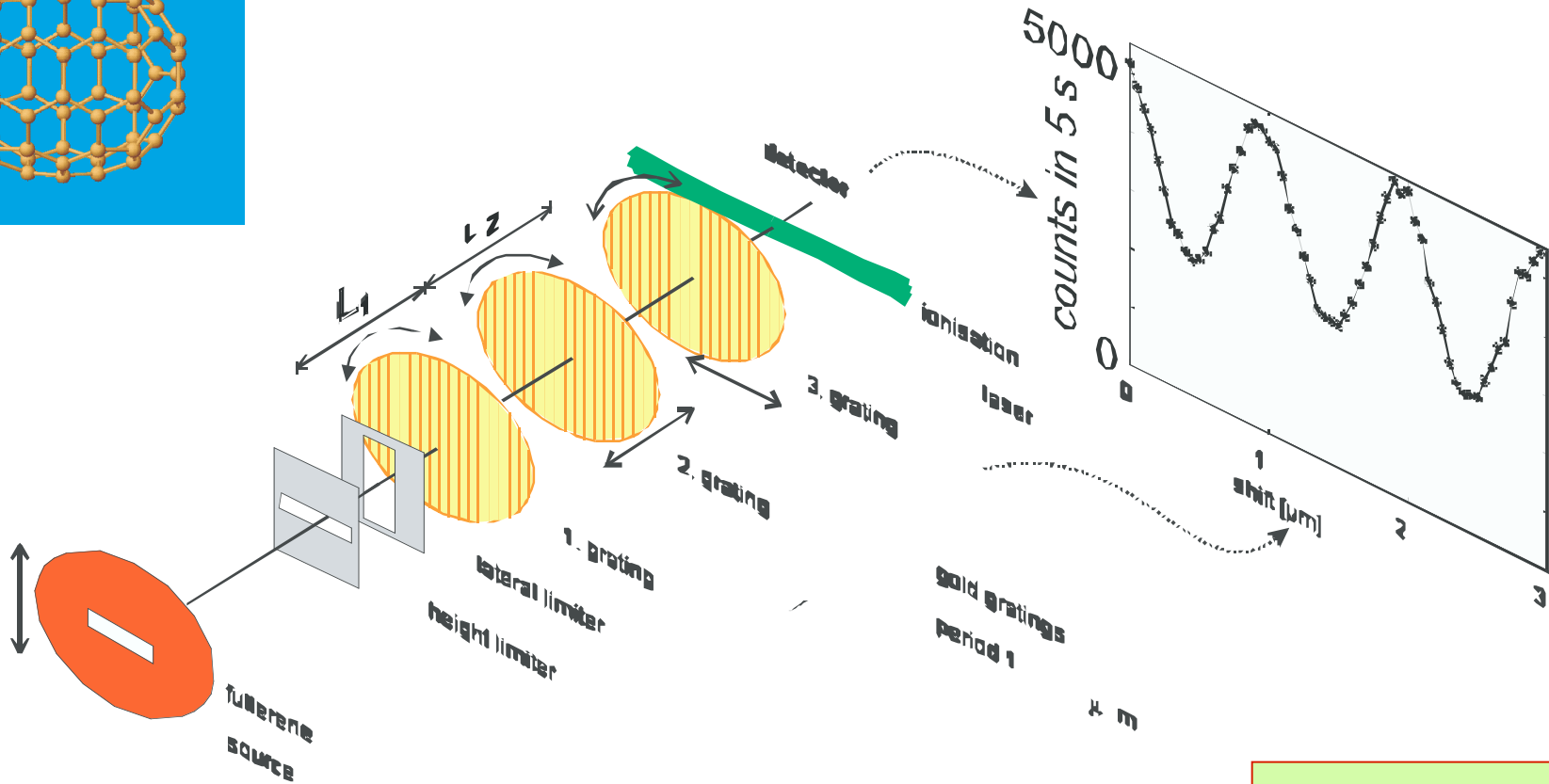
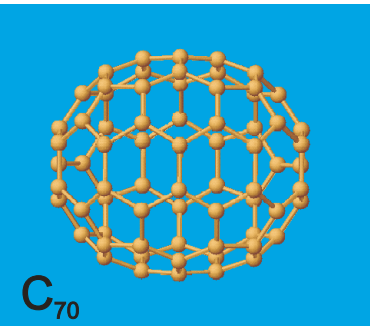
O. Nairz, M. Arndt, A. Zeilinger, *AJP* 71, 319 (2003).

## Challenges for very massive objects:

- smaller de Broglie wavelength
- tighter collimation
- smaller count rate
- high-resolution detector required, ...

## Solutions:

1. Slowing and Cooling: **still open field of research**
2. Near-field interferometry

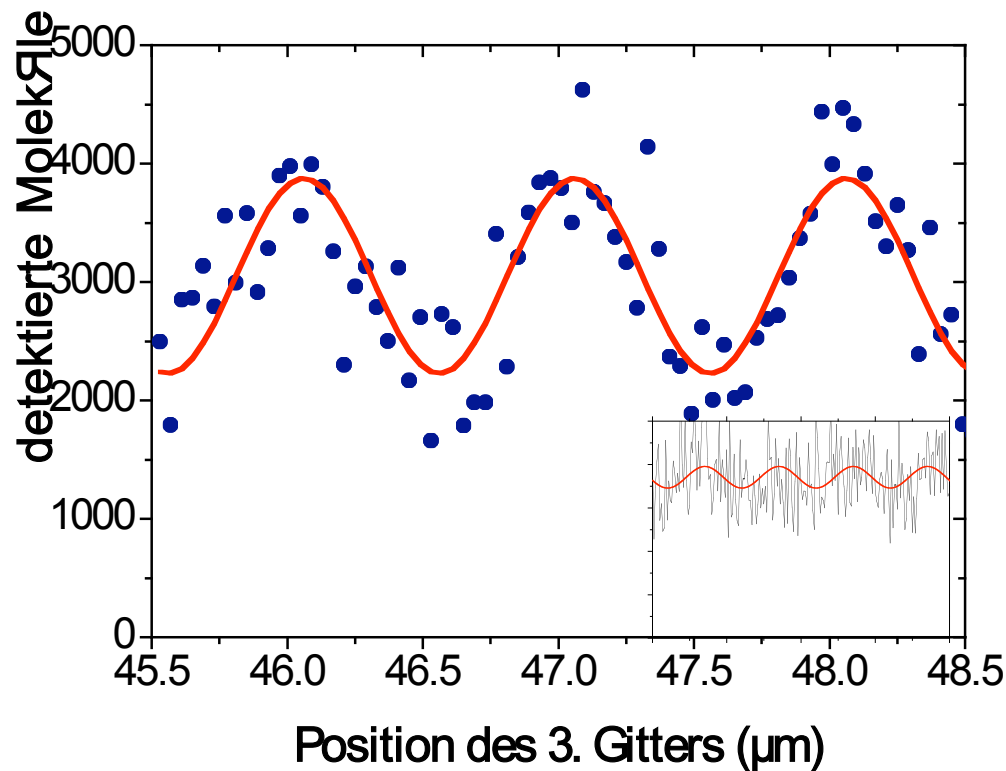
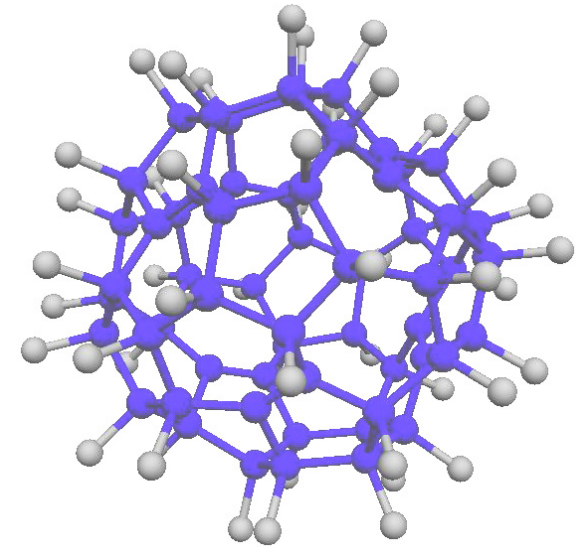


**Gratings:** Heidenhain, Traunreut, AXAF/CHANDRA

Visibility:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

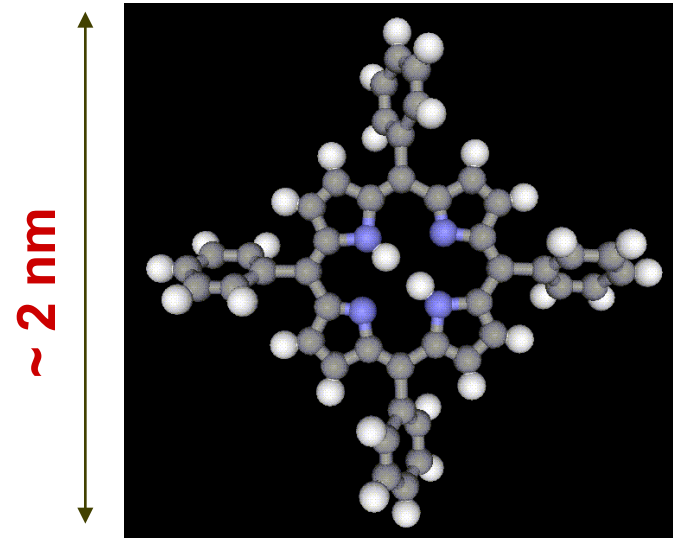
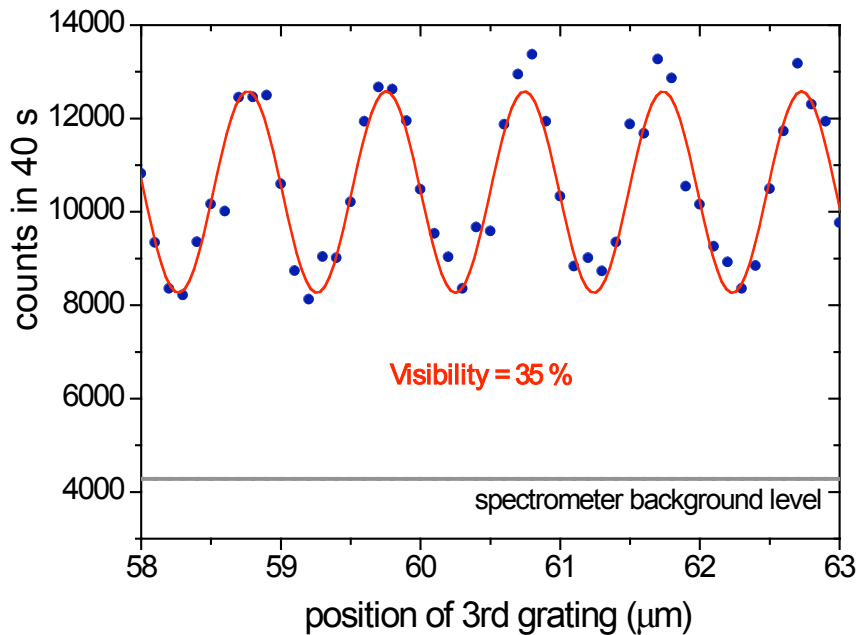
- ✦  $C_{60}F_{48}$
- ✦ **1632 amu !**
- ✦ **108 atoms in a *single object* !**
- ✦ **Several isomers with different symmetries**



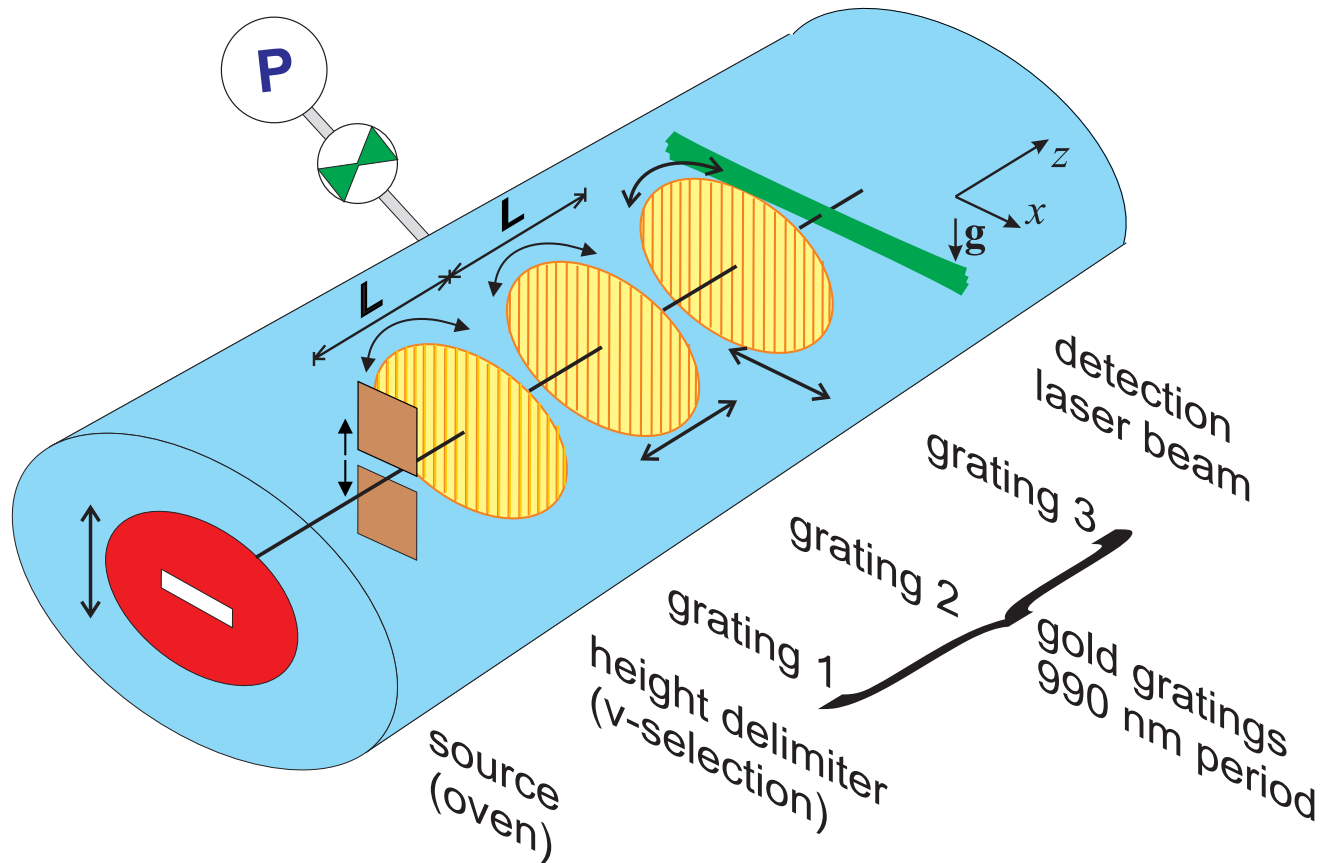
Low velocity →  
Vibrational Dephasing !!

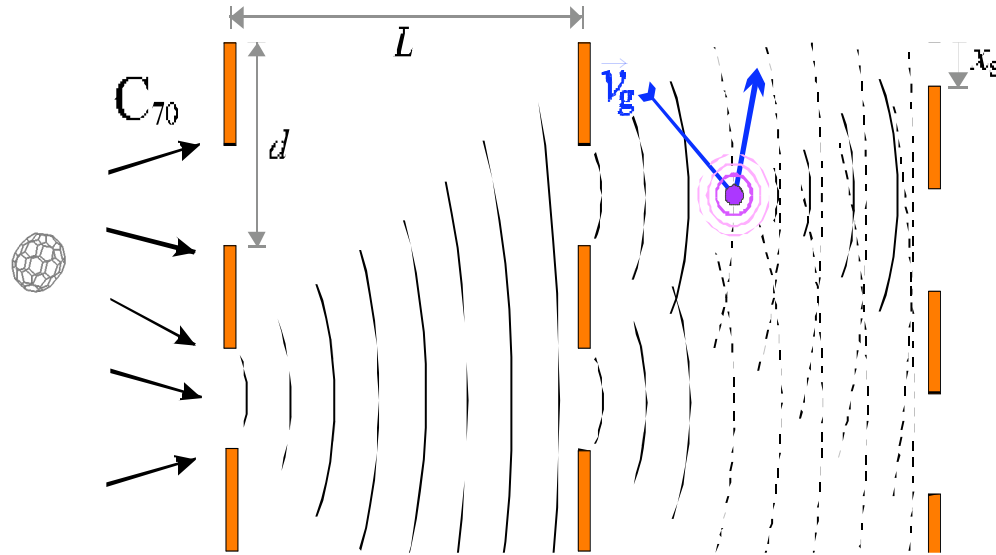
## Question:

Will high interference visibility  
vanish with reduced symmetry  
of the quantum object ?



Various gases can be added with a well controlled pressure

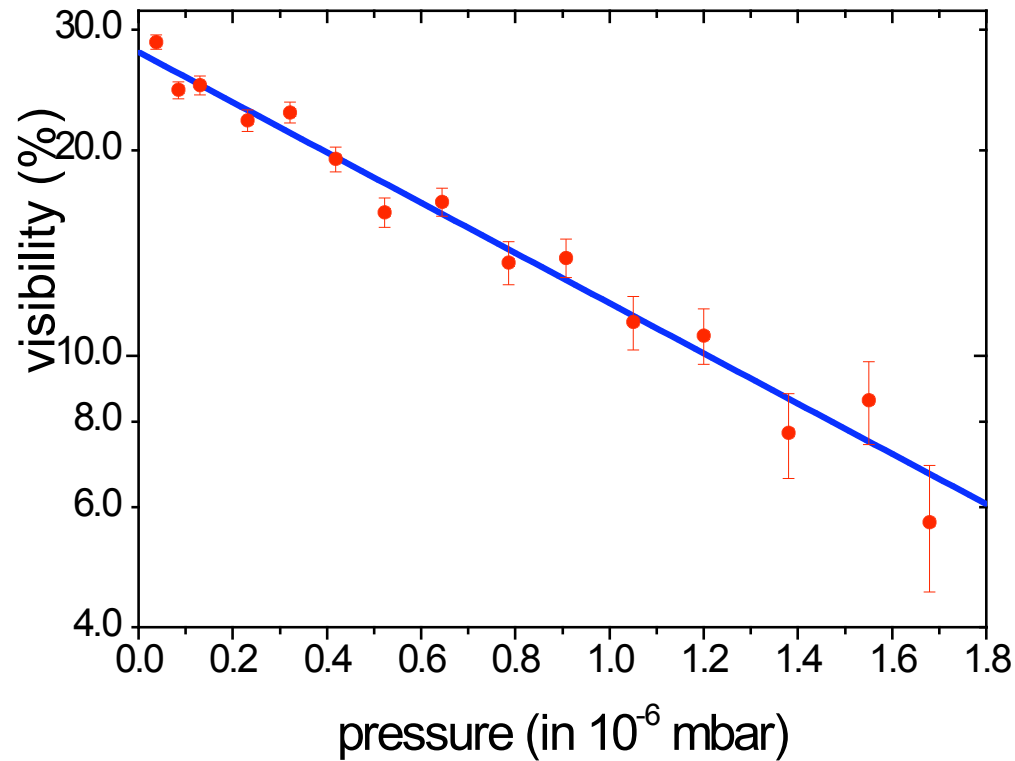




- Collision generates **entanglement**

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[ |C_{70}^{\text{left}}\rangle + |C_{70}^{\text{right}}\rangle \right] \otimes |g\rangle \xrightarrow{\text{coll.}} \frac{1}{\sqrt{2}} \left[ |C_{70}^{\text{left}}\rangle |g_{\text{scat}}^{\text{left}}\rangle + |C_{70}^{\text{right}}\rangle |g_{\text{scat}}^{\text{right}}\rangle \right]$$

# Fringe visibility: Exponential pressure dependence



$$V(p) = 2 \frac{|T_1|}{T_0} \exp\left(-\frac{2L\sigma_{\text{eff}}}{k_B T} p\right) =: V_0 e^{-p/p_0}$$



**Bohr: Complementarity**

Which-path information and interference contrast are complementary

**Heisenberg: „microscope“ argument**

Random recoils wash out the interference pattern

**Modern language: „Entanglement“ with the environment**

Leads to an uncontrolled extension of the quantum system.

May be regarded as a mathematical formulation of Bohr's view.

## Is a virus interferometer conceivable?

$$M = 5 \times 10^7 \text{ amu}$$

$$v_m = 10 \text{ m/s}$$

$$L = 1 \text{ m}$$



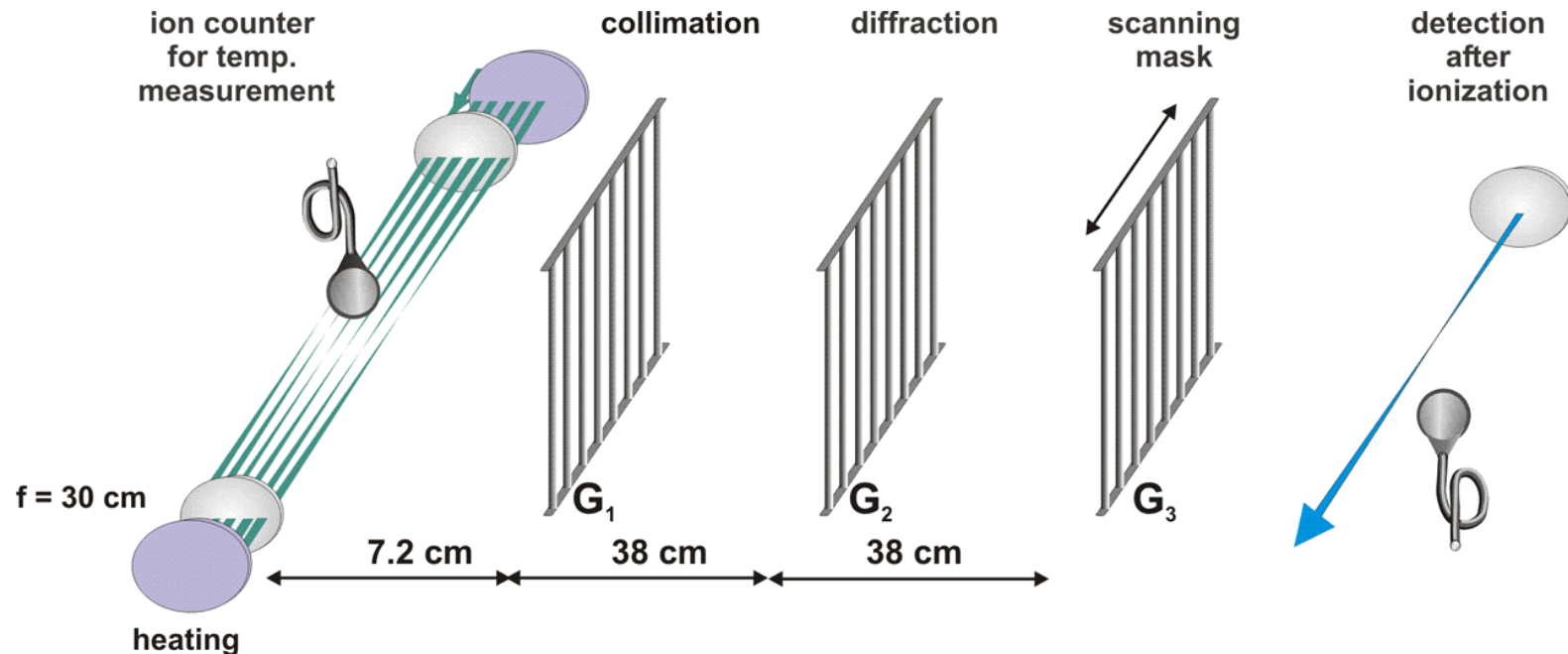
$$p \simeq 3 \times 10^{-10} \text{ mbar}$$

A „Virus Interferometer“ would not yet be limited by collisions

# Loss of interference due to Thermal Radiation

## Idea:

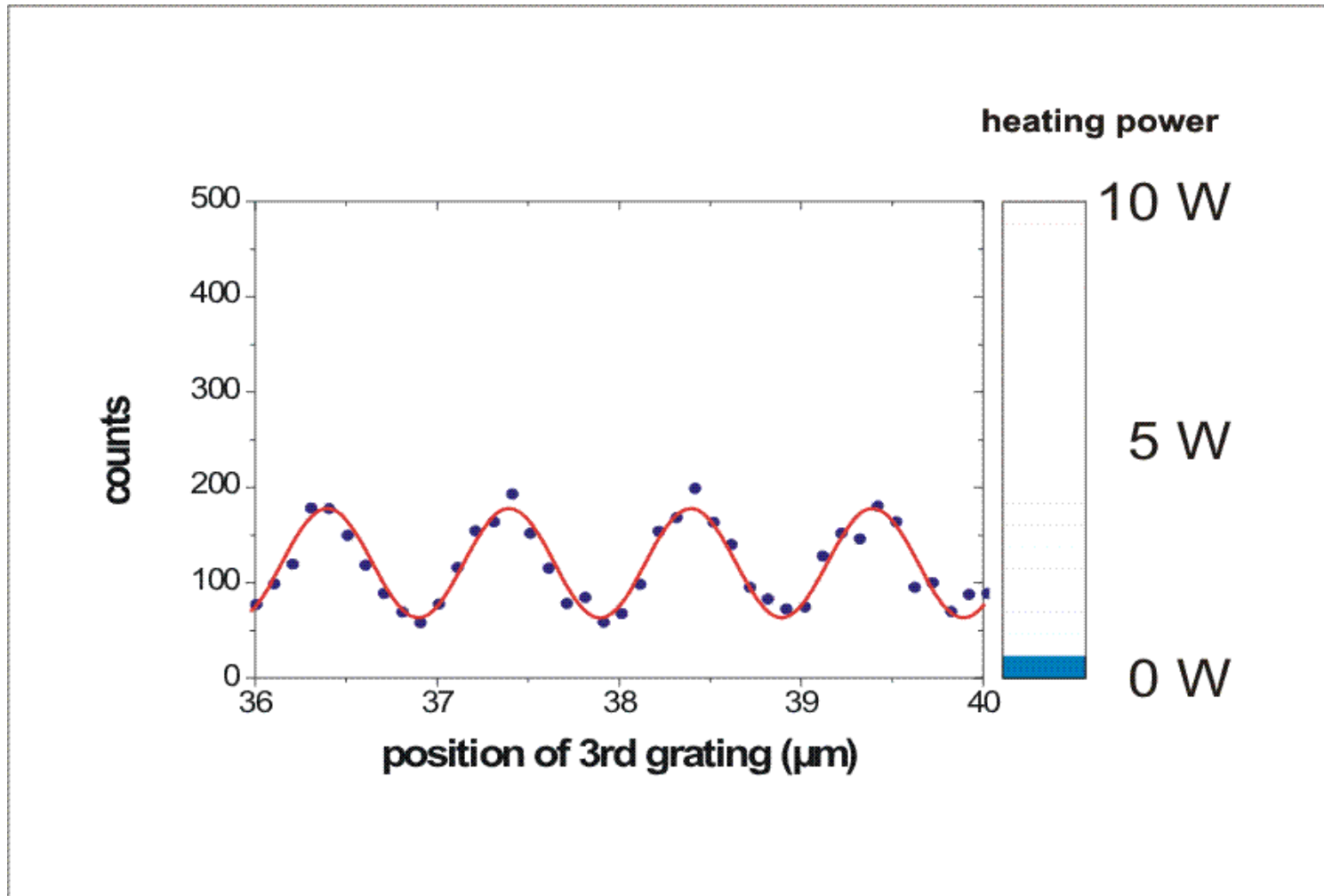
- Warm bodies emit light
- A single photon suffices to localize the emitter to within the photon's wavelength



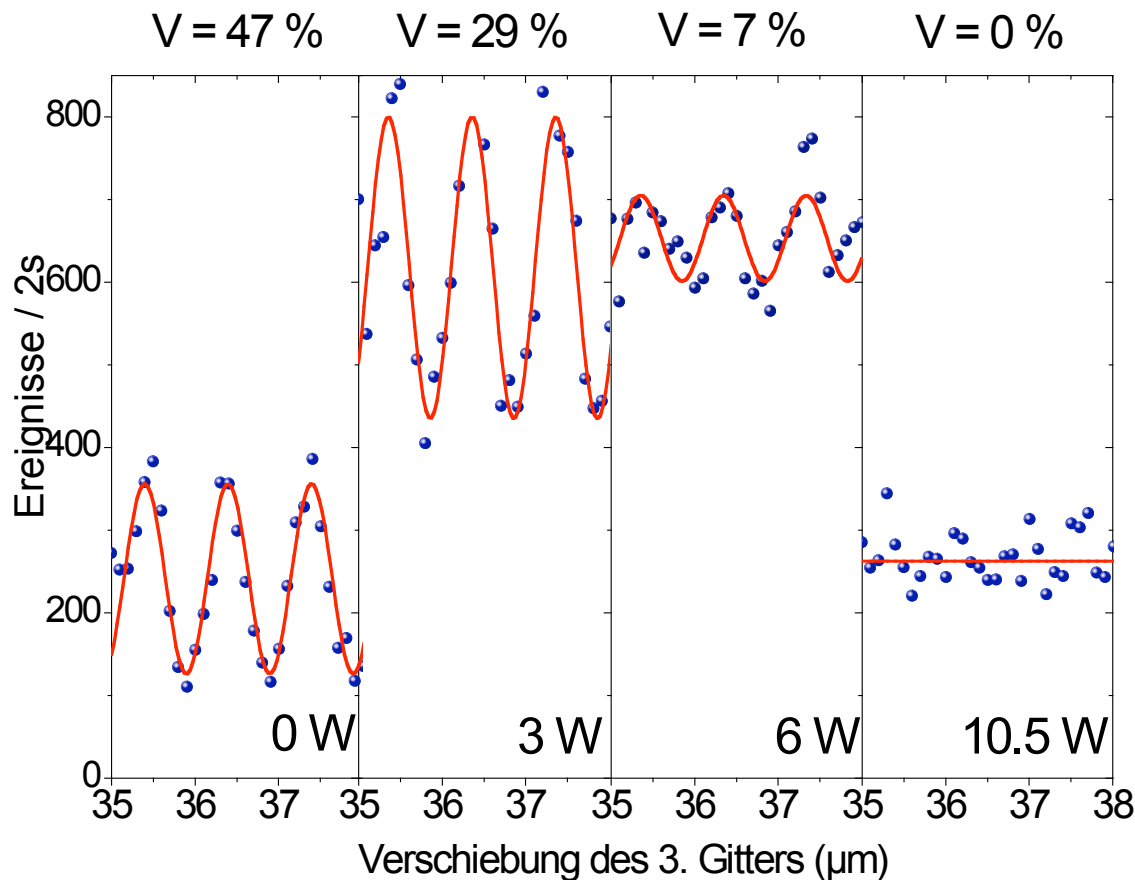
Now 1- $\mu\text{m}$  slit spacing

- ⊕ Heating of  $C_{70}$  before it enters the interferometer (up to  $\sim 3000 \text{ K}$ ).
- ⊕ Hot fullerenes emit **visible light** (see Mitzner & Campbell)
- ⊕ The **interference contrast decreases**, with increasing temperature

# Interference patterns for Increasing heating laser power



increasing temperature  
→  
vanishing quantum interference



Molecule fragments  
or ionizes (misses  
detector altogether)

# Roy Glauber Nobel 2005

## Quantum Theory of Optical Coherence & Photodetection



September 1, 1925 –  
December 26, 2018

(Semi-)classically

$$p \propto I \propto \langle E^2 \rangle$$

$$p(\Delta T) = \int_{-\Delta T/2}^{+\Delta T/2} I(t) dt$$

Classically, E is real, but we can write

$$\begin{aligned} \vec{E}(\vec{r}, t) &= \int_{-\infty}^{+\infty} d\omega \vec{E}(\vec{r}, \omega) e^{-i\omega t} \\ &= \int_{-\infty}^0 d\omega \vec{E}(\vec{r}, \omega) e^{-i\omega t} + \int_0^{+\infty} d\omega \vec{E}(\vec{r}, \omega) e^{-i\omega t} \\ &\equiv \int_{-\infty}^0 d\omega \vec{E}^{(-)}(\vec{r}, \omega) + \int_0^{+\infty} d\omega \vec{E}^{(+)}(\vec{r}, \omega) \end{aligned}$$

QM

$$\begin{aligned} \hat{E} &= \hat{E}^{(+)} + \hat{E}^{(-)} & \hat{E}^{(+)} &= \hat{E}^{(-)\dagger} \\ \hat{E}^{(+)}(\vec{r}, t) &= i \sum_{k, \sigma} \sqrt{\frac{\hbar \omega_k}{2 \epsilon_0 V}} \vec{u}_{k, \sigma}(\vec{r}) e^{-i\omega t} \hat{a}_{k, \sigma} & \text{photon absorption} \\ \hat{E}^{(-)}(\vec{r}, t) &= -i \sum_{k, \sigma} \sqrt{\frac{\hbar \omega_k}{2 \epsilon_0 V}} \vec{u}_{k, \sigma}^*(\vec{r}) e^{+i\omega t} \hat{a}_{k, \sigma}^\dagger & \text{photon creation} \\ \hat{E} &\propto (\hat{a} - \hat{a}^\dagger) \end{aligned}$$

Typical detector has atom size  $\ll \lambda$

→ electric dipole approximation:  $H = -e\vec{d} \cdot \vec{E}$

Photodetection ~ photoabsorption

Matrix element

$$\langle f | \hat{\vec{E}}^{(+)}(\vec{r}, t) | i \rangle$$

light + atom (assume initially in ground state, ready to count)

$w_{i \rightarrow f} = \frac{\text{prob}}{\text{unit time}}$  for a photon to be absorbed at  $\vec{r}, t$

$$= \sum_f \left| \langle f | \hat{\vec{E}}^{(+)}(\vec{r}, t) | i \rangle \right|^2 = \sum_f \langle i | \hat{\vec{E}}^{(-)}(\vec{r}, t) | f \rangle \langle f | \hat{\vec{E}}^{(+)}(\vec{r}, t) | i \rangle$$

sum over final states of atom

$$= \langle i | \hat{\vec{E}}^{(-)}(\vec{r}, t) \hat{\vec{E}}^{(+)}(\vec{r}, t) | i \rangle$$

Define “1<sup>st</sup>-order” correlation function

$$G^{(1)}(\vec{r}_1, t_1; \vec{r}_2, t_2) \equiv \left\langle \hat{\vec{E}}^{(-)}(\vec{r}_1, t_1) \hat{\vec{E}}^{(+)}(\vec{r}_2, t_2) \right\rangle$$

‘Ergodic’ fields: ensemble average = time average



$G^{(1)}$  measures field-field correlations

For stationary fields:  $G(t_1, t_2) = G(\tau = t_2 - t_1)$

$$p(t) = G^{(1)}(\vec{r}, t; \vec{r}, t) \propto \langle \hat{a}^\dagger \hat{a} \rangle = \langle \hat{n} \rangle$$

Prob detection  
per unit time

assume detector is  
localized at one place,  
and very fast

evaluated over  
the initial state  
of the light field

E.g.,  $|\psi\rangle = |1\rangle_\omega$  monochromatic field

or  $|\psi\rangle = \int d\omega \phi(\omega) e^{i\omega t} |1\rangle_\omega$  single-photon wavepacket

or  $\rho = \sum_i p_i |i\rangle\langle i|$  mixed state

$$\langle i | \hat{E}^{(-)} \hat{E}^{(+)} | i \rangle \Rightarrow \sum_i p_i \langle i | \hat{E}^{(-)} \hat{E}^{(+)} | i \rangle \equiv \text{Tr}(\rho \hat{E}^{(-)} \hat{E}^{(+)})$$