Making Estimates in Research and Elsewhere

Lance Cooper and Taylor Hughes

Q: How many cows can you fit in a barn?

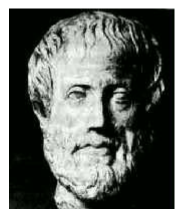
A: *assume* spherical cows of radius ~R
&
assume rectangular barn w/ volume ~ LxW

$N \sim (L \times W)/\pi R^2$

Making Estimates in Research: Why?

It is the mark of an instructed mind to rest assured with that degree of precision that the nature of the subject admits, and not to seek exactness when only an approximation of the truth is possible.

- Aristotle



Why make estimates in science?

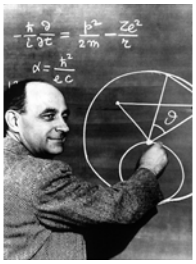
The ability to estimate – to within an order of magnitude or so – the size or probability of various quantities is useful in science as well as in many other endeavors:

- To provide a rough check of more exact calculations
- To provide a rough check of research results or hypotheses
- To obtain estimates of quantities when other resources aren't available
- To obtain estimates of quantities that are difficult to measure precisely
- To obtain estimates of quantities for which no firm theoretical prediction exists
  ⇒ particularly important in interdisciplinary sciences, soft matter, astrophysics
- To provide bounds for possible design alternatives

Making Estimates in Research: How?

How do you estimate the answer to a question that appears impossible to determine at all, or at least without access to an encyclopedia, internet connection, or omniscient being?

e.g., how many grains of sand are there on earth's beaches?
how many piano tuners are there in Chicago?
how many atoms are in your body?



Enrico Fermi

These problems are sometimes referred to as Fermi problems, after the physicist Enrico Fermi, who was famous for (among other things) posing and solving such problems.

Getting started:

(1). Don't panic when you see the problem

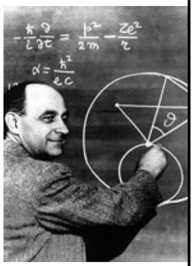
(2). Write down any fact you *do* know related to the question

(3). Outline one or more possible procedures for determining the answer

(4). List the things you'll need to know to answer the question

(5). Keep track of your assumptions

Making Estimates in Research: How?



Other general guidelines for making order-of-magnitude estimates:

Make everything as simple as possible!

(1). Don't worry about specific values: round numbers to "convenient values"
e.g., $\pi \approx 3$; 8.4 $\approx$10; etc.

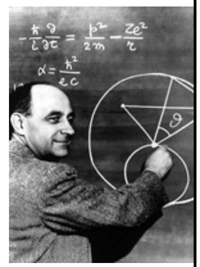
(2). Choose convenient geometries when modeling
e.g., a spherical cow, a cubic grain of sand, etc.

(3). Make "educated" guesses or even upper and lower bounds of quantities you don't know.
try to make good guesses, and keep track of these guesses, as they will set bounds on the fidelity of your estimate

(4). Use ratios when possible – by comparing the value of one quantity (e.g., force, energy, etc.) in comparison to a related quantity – in order to eliminate unknown parameters and get a dimensionless parameter

(5). If possible, exploit plausible scaling behavior of some quantity, i.e., estimate an unknown quantity by assuming it scales linearly - from known values – with some parameter

Making Estimates in Research: How?



More guidelines for making order-of-magnitude estimations:

Checking your estimates:

(1). Make sure that your estimates and calculations are dimensionally correct! ⇒ This is a very powerful tool!

(2). Check the plausibility of your estimate, if possible
e.g., if your answer exceeds the speed of light or the size of the universe, you've got a problem!

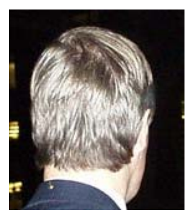
(3). Check the plausibility of your estimate using an alternate calculation method
do the two methods agree to within an order of magnitude?

(4). Perform a "reality check" on your estimate based on the number and size of the approximations you made

(5). More quantitatively - place "bounds" on your estimate:
To obtain an "upper bound" – in equations, put largest estimated values of quantities in the numerator and the smallest estimated values in the denominator

To obtain a "lower bound" – in equations, put smallest estimated values of quantities in the numerator and the largest estimated values in the denominator

Making Estimates: Hairs on a Human Head



How many hairs on a human head?

(1). What do we need to know? Size of a typical scalp, and approximate number of hairs per square inch

Hairs per inch: 20 – 40 (guess or measure) ⇒ 400 – 1600 per $inch^2$

(2). Do we need a model? We don't want a precise, specific answer, so assume "average" head that is a hemisphere (keep it simple!)

Radius of "typical" head: ~ 5 inch (guess or measure)
⇒ area of scalp ~ $\frac{1}{2}(4\pi r^2)$ ~ 150 $inch^2$ (you can take π ~ 3! Excellent!)

(3). Make your estimate!

Note dimensional consistency!

\# of hairs ~ area of scalp * hairs/unit area ~ 60,000 – 240,000 hairs

OR $N \sim 10^5$ hairs

Rounding up is OK, even encouraged! ⇒

(4). Now, check: How good were our approximations?

Chemistry Estimates: Atoms in a Grain

How many atoms in a grain of sand?

(1). What do we need to know? Size of a grain of sand, size of an atom

Dimension of a grain of sand: assume L_{sand} ~ 1 mm = 10^{-3} m × 10^{10} Å/m = 10^{7} Å

Dimension of an atom: assume L_{atom} ~ 1 Å

(2). Do we need a model? Don't worry about geometry…assume a cubic grain of sand!

Volume of grain of sand: V_{sand} ~ $(10^7)^3$ Å^3 ~ 10^{21} Å^3

Volume of an atom: V_{atom} ~ $(1)^3$ Å^3 ~ 1 Å^3

(3). Make your estimate!

# of atoms in 1 grain ~ (volume of 1 grain)/(volume of 1 atom) ~ 10^{21} atoms per grain

(4). Now, check: How good were our approximations?

Close to N_A… so this is reasonable

"Musical" Estimates: Piano Tuners in CU

How many piano tuners are there in Champaign?

(Similar to an original Fermi problem!)

"Let me through - I'm a piano tuner!"

(1). What do we need to know? How many families are there in Champaign? How many families own a piano? How often are pianos tuned? How many tuners are needed?

# of people in CU: Estimate 150,000

# of families in CU: Estimate 150,000/4 ~ 38,000

# of families owning a piano: Estimate 1 in 10 ⇒ ~ 4,000 pianos in Champaign

rate at which pianos are tuned: Estimate 1 time each year ⇒ ~ 4,000 tunings/year

rate at which piano tuners can perform a tuning: Estimate ~ 4 tunings/day, working ~ 200 days/year (exclude weekends and holidays) ⇒ ~ 800 tunings/year per tuner

(2). Make your estimate!

# of tuners ~ (rate at which piano tunings are needed)/(rate at which each tuner performs tunings)

⇒ (4000 tunings/year)/(800 tunings/year*tuner) ~ 5 tuners (± 3 tuners)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PICTURE FRAMES–DEALERS

From C/U Yellow Pages…

Estimate: 2-8
Actual: 7

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Culinary Estimates: Supersize Me

How many McDonalds franchises are there in the US?

(1). What do we know? # of McDonalds in Champaign, population of Champaign, population of US

# of McDonalds in Champaign: estimate $N_{champaign} \sim 6$

population of Champaign: ~ 150,000

population of US: ~ 250,000,000

⇒ Reasonably assume that # of McDonalds franchises scales with population!

(2). Make your estimate!

Assume simple scaling relationship

$N_{USA} \sim$ (population of US/population of Champaign)*(# of McDonalds in Champaign)

$N_{USA} \sim 10{,}000$

Actual: 13,000

Technological Estimates: Storage Capacity of CDs

Introduction: CDs store information as a high density of “pits” or bumps in an aluminum layer

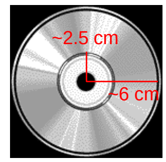
(a) How many “pits” in a typical CD, and what is their spacing?

(1). What do we need to know for (a)? Active area of CD, storage capacity of CD

Consider 700 Mbyte CD $\Rightarrow$ (8 bits = 1 byte) 700 Mbytes = 5.6×10^9 bits! $\Leftarrow$ # of pits!

To get the pit spacing, we need to estimate the active area on the CD:

$A_{CD} = \pi(0.06)^2 - \pi(0.025)^2$ m^2 (guess or measure)



$A_{CD} \sim 9 \times 10^{-3}$ m^2

(2). Make your estimate!

Note dimensional consistency!

Area associated w/ each bit, $A_{bit} = A_{CD}/(\text{\# of bits}) \sim 1.6 \times 10^{-12}$ m^2/bit

$\Rightarrow$ Separation between each bit, $d \sim (A_{bit})^{1/2} \sim 1.3 \times 10^{-6}$ m = 1.3 microns!

λ=0.65 microns

Blu-ray Disc™

The benefit of using a blue-violet laser (405nm) is that it has a shorter wavelength than a red laser (650nm), which makes it possible to focus the laser spot with even greater precision. This allows data to be packed more tightly and stored in less space, so it's possible to fit more data on the disc even though it's the same size as a CD/DVD. This together with the change of numerical aperture to 0.85 is what enables Blu-ray Discs to hold 25GB/50GB.

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Other resources on making estimates

A View From the Back of the Envelope
http://www.vendian.org/envelope/

University of Maryland Fermi Problems Site
http://www.physics.umd.edu/perg/fermi/fermi.htm

Old Dominion University Fermi Problems Site
http://www.physics.odu.edu/~weinstei/wag.html

Order of Magnitude Astrophysics
http://www.astronomy.ohio-state.edu/~dhw/Oom/questions.html

Back-of-the-Envelope Physics, Clifford Swartz (Baltimore, Johns Hopkins University Press, 2003).

The Back of the Envelope, E.M. Purcell, monthly column in the *American Journal of Physics,* July 1984 – Jan. 1993.

Consider a Spherical Cow : A Course in Environmental Problem Solving, John Harte (Berkeley, University Science Books, 1988).

Powers of Ten : About the Relative Size of Things in the Universe, and the Effect of Adding Another Zero, Philip Morrison and Phylis Morrison (Scientific American Library, 1982, 1994).

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In-Class Activity Questions*

(1). How much money did American consumers pay to fill up their cars with gas last year?

(2). How many nails are needed to make sleeping on a bed of nails safe (but still crazy!)?

(3). How much kinetic energy does the earth have due to its rotation about the sun? How much rotational kinetic energy does the earth have?

(4). Legend has it that water in Southern Hemisphere washbasins drains in the opposite sense to that of water draining in Northern Hemisphere washbasins, i.e., that the Coriolis force, $|F_c| \sim 2m(\omega \times v)$ (where ω is the rotational frequency of the earth, v is the radial speed of the water, and m is the mass of the water), governs the behavior of draining water. Is this legend reasonable? (Hint: Compare your estimate of $|F_c|$ with another force in the problem, i.e., the gravitational force $F_{grav} = mg$.)

(5). How fast does human hair grows (on average) in mph?

(6). How much sand is there on all the beaches of the earth?

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In-Class Activity Questions*

(7). Estimate the surface area and volume of a typical human.

(8). Estimate the number of cells in the human body.

(9). Estimate the weight of earth's atmosphere. [Hint: Atmospheric pressure at sea level is ~ 14.7 psi]

(10). In a letter to the Royal Society in 1774, Ben Franklin reported that the equivalent of 0.1 cm^3 of oil dropped on a lake spread to a maximum area of 40 m^2. What is the thickness of the oil and does this make sense?

(11). Estimate the number of leaves on a tree.

(12). Estimate the weight of Mt. Everest [height ~ 29,000 ft].

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