

# Writing Numbers in Technical Documents 

Perhaps you think these comments are pedantic. Some may be. But scientific reports, written or spoken, should reflect the precision of the experiments. Numbers and values submitted to editors or presented at meetings are sometimes hardly better than "umpteen zillion" or "every so often."
-Vernon Booth
Communicating in Science

Celia M. Elliott Department of Physics University of Illinois

Physics is a quantitative science. Thus, numbers must be represented in a way that ensures a reader interprets them correctly. To make sure that we're all speaking the same quantitative language, rules have evolved about how numerical values are presented in writing.

Learn the rules, and hew to them witlessly.

Caveat lector! While these rules generally apply to papers published in the physical sciences, different disciplines may have different rules. The style guides for individual journals may also differ in their rules for expression of numbers.

Use this document as a general style guide, be consistent, and always strive to express numbers in way that avoids ambiguity.

# First, let's define our terms 

A number (872) is a scholarly convention to define the size or amount of something that has been measured, calculated, or counted

The special characters that represent
numbers are numerals ( 8,7 , and 2 )
87.2 is a quantity*
*unless it appears in a table, where it is a "value"

## Follow two fundamental rules for expressing numerical quantities

## 1. Use a number style that conveys information unambiguously

A realistic quantum simulator involving a hundred-thousand interacting spins that could be constructed in a few years has the potential to explore outstanding theoretical issues in condensed matter physics.

## 2. Maintain the style consistently throughout the text

In this example, a reader might interpret "hundred-thousand" as a range (100 to 1000 ), a collection of one hundred 1000 -spin sets, or a single number (100 000).

Fortunately, we have standard rules about the expression of numbers in scientific writing to avoid ambiguity and to ensure that a reader assigns the same meaning to a number that the writer intended.

# Numbers <10 are usually written as words 

Spell out cardinal numbers one to nine for things that are counted two-state quantum system nine separate experimental runs three-body problem
Spell out ordinal numbers first to ninth first occurrence second question seventh data run
"Zero" may be written as a numeral or as a word the $x$ and $y$ axes intersect at 0 Zero evidence was seen for the predicted effect.

Numbers are written as words only for quantities that have been counted. If the number is the result of a measurement or a calculation, it must be expressed in numerals.

# Numbers $>9$ are written in numerals 

Use numerals for quantities >9
for things that have been counted
18 months
128-processor Paragon ${ }^{\circledR}$ supercomputer
$\$ 2.1$ million write out million, billion, trillion in words

Use numerals for ordinal numbers greater than ninth
50th anniversary
21 st century

REMINDER: Numbers are written as words only for quantities that have been counted. If the number is the result of a measurement or a calculation, it must be expressed in numerals.

# Always express numbers that have been measured or calculated in numerals, even if they're <10 

## 830 MHz If an exact number is

$6 \mu \mathrm{~g} \quad$ followed by a unit of 77 K measure, abbreviate the $200 \AA$ unit and write the number in
$4 \mathrm{~cm} \times 4 \mathrm{~cm}$ numerals

Note the use of the "times" sign ( $\times$ ), not a letter " x "
Units of measure are set in Roman type; they are never italicized

Note that the unit of measure is given after each number, and the "times" symbol is used instead of the letter $x$.

To obtain the times symbol, use the "insert symbol" function in MS Word or PowerPoint, or type \times in LaTeX.

To avoid awkward breaks that maroon the number at the end of a line of text (e.g., 7 mm ) and the unit at the beginning of the next line, as in this example, train yourself to type a nonbreaking space between all numbers and units. Do it every time until it's automatic.

# A physical quantity is the product of a numerical value (a pure number) and a unit 

1852 m (nautical mile)
133.322368 Pa (1 torr)

Symbols (usually Latin or Greek letters) for physical quantities are italicized in text

The force, $f$, varied between 15 pN and 17 pN . The electric potential, $\varphi$, is 12 V .

## Abbreviations for units are never italicized

7

It is important to set letters used to symbolize physical quantities in italics to distinguish them from the rest of the text.
In typography, "Roman" text is straight up and down, like this text, and italics text is slanted, like this. "Roman" refers to the text style, not the font or typeface name (e.g. Helvetica, Times, Century Schoolbook).

Note that abbreviations for units are never italicized.

## A space is usually inserted between a number and the unit of measure

## Examples:

$77 \mathrm{~K}, 250 \mathrm{~kJ}, 10 \mu \mathrm{~m}, 4 \mathrm{~T}$
Use a non-breaking space to keep the number and the unit on the same line Word-123Crtl+Shift+Spaceunit or $\boldsymbol{\Omega}$ TeX-123~unit

## Do not use an intermediate space in a few exceptions:

$70 \%, \$ 100 \mathrm{k}, 15^{\circ}, 45^{\circ} \mathrm{C}$

Note that the degree symbol $\left({ }^{0}\right)$ is used only for temperatures in the Celsius or Fahrenheit ranges. The SI unit, kelvin (K), is an absolute unit, not a "degree" based on some arbitrary scale. Thus, writing " K " or "degrees Kelvin" is incorrect.
Some older papers may show temperature as ${ }^{\circ} \mathrm{K}$, but that use was changed by The General Conference on Weights and Measures (French: Conférence générale des poids et mesures - CGPM) in 1967.

Note also that units are abbreviated when they describe a quantity that has been calculated or measured.

## The US is gradually adopting European style in breaking 000s <br> According to IUPAP, a comma (,) should no longer be used to separate numbers having more than four digits into groups of three digits

12578896 NOT 12,578,896
Ideally, narrow or half spaces should be used
Be sure to use non-breaking spaces to avoid having part of the number marooned on a separate line

Some subdisciplines of physics are adopting the change faster than others; you'll see numbers presented both ways.

# Approximate numbers follow the same rules as exact numbers 

## Same guidelines as exact numbers

Approximately 50000 discrete events were recorded.
Approximately one sample in seven had to be discarded because of poor adherence of the thin film to the silicon substrate.
Do not abbreviate a unit that follows an approximate number tens of kilohertz thousands of volts several millimeters

# Very large approximate numbers are written as numerals followed by the word million, billion, or trillion 

The renovation of the microanalysis laboratory will cost $\$ 3.7$ million and take nearly four years.
Overall, NSF funding increased by $\$ 372.5^{*}$ million to $\$ 4.789$ billion, an 8.4-percent increase over the previous year.
*Avoid awkward line breaks
The indefinite article preceding a number is chosen based on what the number sounds like when it is spoken

Again, train yourself to insert a non-breaking space between the number and the word to avoid awkward line breaks.

## Mathematical operations are expressed in numerals

a factor of 4
a probability approaching 0
$3 \times 3$ matrix
6 orders of magnitude
Fractional numbers written as decimals must have a zero preceding the decimal point
(3) $\mathrm{cm} \quad 0.3 \mathrm{~cm}$

## A sentence may not begin with a number expressed in numerals

(35)experimental runs were made. Thirty-five experimental runs were made.
75.) mm holes were drilled in the sample. Sovonty five-mm holes were drilled... Holes 75 mm in diameter were drilled...

Double penalties attach for beginning a paragraph, a figure caption, or a title with a number expressed in numerals

Notice that the example "Seventy-five mm holes were drilled...," which attempts to observe the "don't begin a sentence with a number expressed in numerals" rule, is tantalizingly ripe for misinterpretation.

Is it 75 holes, 1 mm in diameter each?
Or is it some indeterminate number of $75-\mathrm{mm}$ holes that were drilled in the sample?

# Qualifiers should not be used with exact numbers* 

# Approximataly 17 samples were contaminated with aluminum oxide. 

 Seventeen samples were contaminated with aluminum oxide.*They sound ridiculous

In general, all wimpy qualifiers should be avoided in scientific writing-be specific and quantitative.

The American author Mark Twain had some strong opinions about qualifiers; he was particularly opposed the the profligate use of "very." His advice-every time you are tempted to write "very," substitute "damn." Then your editor will remove all the damns for the sake of propriety, and your writing will be much improved.

Heed Mark Twain.

## Numerals for quantities <10 are used in special cases

## Names of parts of anything printed

Chapter 2, Vol. 3, No. 7, Fig. 4, Eq. 8
Table IV, Section 6.3.7, 2nd ed.
Locations
Row 3, Area 51
Time
$17 \mathrm{~ms}, 5 \mathrm{~s}, 3 \mathrm{~min}$-except days, months, years, and centuries (they're counted)
Money
$\$ 0.26$ per unit, $\$ 1.3$ billion
(\$100k and \$13M okay for informal writing)

Note that " k " (kilo) is always written lower case when it means "thousands." A capital $K$ is the abbreviation for the $S I$ unit "kelvin."

## Capitalization depends on position

> When the noun comes before the number, capitalize it

Figure 7, Equation 21
Section 5
Model No. 34001x
When the noun lags behind the number, leave it in lower case the seventh figure the 5th edition the 34001x model

If the number comes after the noun, the phrase is treated as a title and is capitalized accordingly.

# Numbers expressed in percentages and decimals require numerals 

Write out the word "percent" in tovt" 98.5 percent 3 percent
Use the pereent symbol (\%) only in headlines, tables, or graphics to save space Decimals also require numerals (the quantities had to be calculated or measured, not counted)
$4.39 \mathrm{eV}, 1.5 \mathrm{~cm}, 0.22 \mathrm{ml}, 1.5 \mathrm{~s}$
*"The times, they are a-changin"-Bob Dylan 17
*Although the "rule" is that "percent" should be written out in running text, usage is evolving, and the \% sign appears more and more frequently in physics papers. Although Ms. P is loath to abandon a principle, she thinks this fight is probably lost and has decided this hill in not the one to die on and thus reluctantly abandons the rule.

Reminder: Any physical quantity that has been measured or calculated must be expressed in numerals.

# Use numerals to express ratios <br> The ratio of epoxy to pigment should be 

 15 to 2." $1: 4$ " is read "one to four" and means "one out of five"

Read Vernon Booth's chapter on

Highly recommended reading: Vernon Booth, Communicating in Science: Writing a scientific paper and speaking at scientific meetings, $2^{\text {nd }}$ edition (Cambridge, UK, Cambridge University Press, 1993).

## Avoid ambiguous ratios

"three times more than" =
"four times as much as"
A hasty reader may interpret "times more than" to mean "times as many as"avoid this ambiguity!
The footprint of our device is one-fourth smaller because of our integrated heating and cooling system.
The results are $\mathbf{1 0}$ times smaller than expected.

# Forming the plural of a number written in numerals 

## Plurals of single-digit numbers are

 formed by adding an apostrophe plus an $s$"Binary code comprises 1's and 0's."
Plurals of numbers $>9$ are formed by adding only the $s-$ no apostrophe
Boeing 767s
expressed in 100s

## Hyphenating numbers

Hyphenate numbers and units of measure only when they form a modifier that describes something else
The beam diameter is $25 \mu \mathrm{~m}$.
The $25-\mu \mathrm{m}$ beam provides excellent resolution.
Hyphenate numbers 21 through 99 when they are written as words
Forty-five days is the maximum the unit should be used without replacing the $\mathbf{J} 17$ filter.

More than you ever wanted to know about hyphenation: http://people.physics.illinois.edu/Celia/Lectures/Dashes.pdf.

More than you ever wanted to know about hyphenation:
http://people.physics.illinois.edu/Celia/Lectures/Dashes.pdf.

## Fractions and -fold numbers

Mixed integers and fractions are always written in numerals $21 / 2$ years, $31 / 4$ percent
Hyphenate fractions written as words two-thirds, three-quarters, one-fourth as many
Hyphenate "fold" numbers when written in numerals (numbers $>9$ )
20-fold
100-fold
Do not use hyphens when written as words threefold sevenfold

## Writing ranges of numbers requires special rules

Use an en dash ( - ), not a hyphen (-) 1985-1993; pp. 11-18; $4.38 \mathrm{eV}-4.54 \mathrm{eV}$
Include all numerals to ensure accuracy 1348-1458, not 1348-458
Include the units of measure for both quantities in the range to avoid ambiguity $\$ 400-\$ 600 ; 10 \mu \mathrm{~m} \times 20 \mu \mathrm{~m}$
The electron-beam sculpting technique was used to fabricate superconducting nanowires having widths of $<8 \mathrm{~nm}$ and lengths of $30 \mathrm{~nm}-50 \mathrm{~nm}$.

For more information on the difference between hyphens and dashes in scientific writing, see http://people.physics.illinois.edu/Celia/Lectures/Dashes.pdf.

A dash means "to" or "through," not "between"

Use to or through instead of a dash with negative numbers to avoid confusion with temperatures of $-5-25^{\circ} \mathrm{C}$
(is the second number " $+25^{\circ} \mathrm{C}$ " or " $-25^{\circ} \mathrm{C}$ "?)
Do not use from or between before a range; it's meaningless
from 2008 to 2013 not from 2008-2013 between 11 and 17 not between 11-17

# Express adjacent numbers in a combination of words and numerals to avoid confusion 

154-mg doses

| fifteen 4-mg doses |
| :--- |
| $180-\mathrm{mour}$ rums samples |
| thirty $20-\mathrm{mm}$ samples |
| eighteen 6-hour runs |

In these examples, the numbers that
express quantities that have been
counted (doses, samples, runs) are
expressed in words

Never write two adjacent numbers in numerals; use a combination of numerals and words to avoid ambiguity and mistakes in interpretation.

## What if I have multiple numbers in a sentence, some $<10$ and some $>9$ ?

All the samples were annealed at $400^{\circ} \mathrm{C}$; six samples were annealed for $8 \mathrm{~min}, 12$ samples were annealed for 15 min , and nine samples were annealed for 25 min .
It depends. Time is always written in numerals; the count for the samples could be written in either numerals or words but should be consistent. I'd probably write the 12 as twelve, to distinguish the times from the counts. However, if I thought the reader would want to sum the number of samples, I'd probably write all three sample counts in numerals to facilitate addition.

If you have multiple numbers in the same sentence, for items that are counted, write all the numbers in numerals or all in words. In general, I would usually write the numbers in numerals, since doing so makes it easier for the reader to sum the total number of things.

## Use correct descriptors

For quantities that are measured
less than, more than, amount of
"Less than 10 percent of the solution ..."
For quantities that are counted
fewer than, greater than, number of "Fewer than half of the samples ..."
For dimensions
smaller than, larger than
Use "more than"-not "over"-to indicate the larger of two quantities

# Keep the number and the unit or the thing it is describing on the same line of text 

"Joseph Lykken attempted to lower the string scale to the vicinity of $10^{-17}$ cm , the TeV scale."
"In a 1995 paper, James Hurrell noted the marked similarity between the spatial patterns in surface air temperature trends during the previous 30 years and the winter-to-winter . . ."

# To recap... <br> Use numerals for physical quantities that have been measured or calculated <br> Write out in words quantities <10 that have been counted <br> Observe standard conventions to ensure that your reader interprets numbers as you intended 

cmelliot@illinois.edu
http://physics.illinois.edu/people/Celia/

Recommended References:
Scientific Style and Format: The CBE Manual for Authors, Editors, and Publishers, 6th ed. (Cambridge University Press, New York, 1994). (Later editions are available, but the 6th ed. contains everything you'll need to know about physics conventions.)

