Phys 487 Discussion 15 – Sudden, Adiabatic, Magnetic Moments

Given • $H(t) = H^{(0)} + H'(t)$, • $\left\{ E_n^{(0)}, |n^{(0)}\rangle \right\}$ = the eigen-* of $H^{(0)}$ • initial state $|\psi(t=0)\rangle = |i^{(0)}\rangle$

then $\boxed{|\psi(t)\rangle = \sum_n c_n(t)\, e^{-i\omega_n t} |n^{(0)}\rangle}$ with $\boxed{i\hbar \dot{c}_f(t) = \sum_n H'_{fn}\, e^{i\omega_{fn} t} c_n(t)}$

• $\omega_{fn} \equiv \left(E_f^{(0)} - E_n^{(0)}\right)/\hbar$

• $H'_{fn} \equiv \langle f^{(0)} | H' | n^{(0)} \rangle$

& to 1st order in $H' \ll H^{(0)}$, $\boxed{c_f(t) \approx \delta_{fi} + \frac{1}{i\hbar}\int_0^t H'_{fi}(t') e^{i\omega_{fi} t'} dt'}$ $\rightarrow$ $\boxed{P_{i\to f} = \left| c_f(t) \right|^2}$

• Fermi's Golden Rule : $\boxed{R_{i\to f} \equiv \frac{P_{i\to f}}{t} = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho\left(E_f\right)}$ • Larmor frequency $\omega = \gamma B$

Problem 1 : Atom in a rotated B field *Qual problem*

An atom with J = ½, m_J = ½ is in a uniform magnetic field.

(a) **Suddenly** the field is rotated by ϕ = 60°. Find the probability that the atom is in the sublevels m_J = +½ or m_J = –½, relative to the new field direction, immediately after the change in field direction.

(b) Suppose the field were instead rotated **very slowly** from its initial orientation to ϕ = 60°. In the limit of an infinitely slow rotation, what then would be the probability that the atom is in the sublevels m_J = ±½ relative to the new field direction once the field finally reaches its new ϕ=60° direction?

Problem 2 : Hyperfine interaction *Qual problem*

An isolated hydrogen atom has a hyperfine interaction between the proton and electron spins ($\boldsymbol{S}_1$ and $\boldsymbol{S}_2$, respective) of the form $J\,\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$. The two spins have magnetic moments $\alpha \boldsymbol{S}_1$ and $\beta \boldsymbol{S}_2$ respectively, and the system is in a uniform static magnetic field $\boldsymbol{B}$. Consider only the orbital ground state.

(a) Find the exact energy eigenvalues of this system and sketch the hyperfine splitting spectrum as a function of magnetic field.

(b) Calculate the eigenstates associated with each level.

NOTE: This is exactly how the qual problem is written. I don't particularly *like* how it is written (I think it is sloppy with its phrasing in places), and I don't think we've used the exact phrase "orbital ground state" before ... but that's always going to be the case: different books/groups/people, different phrasing. I'm sure you can figure out what is meant. Also, if you're not sure what technique to use in a qual problem, or in real life(!!!), just **do! something! reasonable!** Explain what you are doing, e.g. "I will assume that …, in which case … This assumption is fairly likely because … (example) …" That's what physicists do: turn chaos into something reasonable. ☺ For example :

Problem 1′ : Atom in a rotated B field again

Return to the situation described in Problem 1. What **time scales** would you compare with each other to decide if it is appropriate to apply the adiabatic approximation (part b) or sudden approximation (part a) to answer this question?