As we derived in class, if our Hamiltonian \( H \) consists of a time-independent part \( H_0 \) with known eigenvalues \( E_n^{(0)} \) and eigenstates \( |n^{(0)}\rangle \) plus and a much smaller time-dependent part \( H' \), then we get the following results to first order in the small perturbation:

Given

- \( H(t) = H^{(0)} + H'(t) \),
- \( \{ E_n^{(0)}, |n^{(0)}\rangle \} \) = the eigen-* of \( H^{(0)} \)
- initial state \( |\psi(0)\rangle = |i^{(0)}\rangle \)

then

\[
|\psi(t)\rangle = \sum_n c_n(t) e^{-i\omega_n t} |n^{(0)}\rangle
\]

with \( i\hbar \dot{c}_n(t) = \sum_n H'_{fn} e^{i\omega_{jn}} c_n(t) \)

- \( \omega_{fn} \equiv (E_f^{(0)} - E_n^{(0)})/\hbar \)
- \( H'_{fn} \equiv \left< f^{(0)} | H' | n^{(0)} \right> \)

& to 1st order in \( H' \ll H^{(0)} \),

\[
c_f(t) = \delta_{fi} + \frac{1}{i\hbar} \int_0^t H'_{fi}(t') e^{i\omega_{ji} t'} dt'
\]

Some names: \( \omega_{fi} \) is called the transition frequency for going from initial state \( i \) to final state \( f \), while \( c_f(t) \) is the transition amplitude for doing the same thing. The transition probability that we are usually trying to calculate is

\[
P_{f\rightarrow i} = |c_f(t)|^2
\]

**Problem 1: 1D SHO™ in a decaying electric field**

Consider a one-dimensional harmonic oscillator that is in the ground state \( |0\rangle \) of the unperturbed Hamiltonian at \( t = -\infty \). NOTE: since all the states \( |n\rangle \) that we are going to talk about are eigenstates of the unperturbed Hamiltonian, we can suppress the \( ^{(0)} \) subscript without confusion. The following perturbation is applied between \( t = -\infty \) and \( +\infty \):

\[
H'(t) = -q E x e^{-t^2/\tau^2}
\]

where \( q \) is the particle’s charge and \( E \) is a constant electric field. What is the probability that the oscillator is in the state \( |n\rangle \) at \( t = \infty \)?

**Problem 2: 1D SHO™ again**

Show that if the perturbation is \( H'(t) = -q E x (1/(t/\tau)^3) \) instead, then to first order \( P_{0\rightarrow 1} = \frac{q^2 E^2 \pi^2 \tau^2}{2m\omega h} e^{-2\omega \tau} \).

**Problem 3: Hydrogen Atom with decaying electric field**

A hydrogen atom is in the ground state at \( t = -\infty \). An electric field \( E(t) = \dot{E} e^{-t^2/\tau^2} \) is applied until \( t = +\infty \). Show that the probability that the atom ends up in any of the \( n = 2 \) states is, to first order,

\[
P_{1\rightarrow 2} = \left( \frac{qE}{\hbar} \right)^2 \left( \frac{2\sqrt{15} a_0^2}{3^{10}} \right) \pi \frac{1}{\tau} e^{-\omega^2 \tau^2/2}
\]

where \( \omega \) is the transition frequency \( \omega \equiv \frac{E_{2m}-E_{100}}{\hbar} \).

Does the answer depend on whether or not we incorporate spin in the picture?