Phys 487 Discussion 11 – Time-Dependent Perturbation Theory

As we derived in class, if our Hamiltonian H consists of a time-independent part H_0 with known eigenvalues $E_n^{(0)}$ and eigenstates $\left|n^{(0)}\right\rangle$ plus and a much smaller time-*dependent* part H', then we get the following results to first order in the small perturbation :

Given • $H(t) = H^{(0)} + H'(t)$, • $\left\{ E_n^{(0)}, \left|n^{(0)}\right\rangle \right\}$ = the eigen-* of $H^{(0)}$ • initial state $\left|\psi(t=0)\right\rangle = \left|i^{(0)}\right\rangle$

then $\left|\psi(t)\right\rangle = \sum_n c_n(t)\, e^{-i\omega_n t} \left|n^{(0)}\right\rangle$ with $i\hbar\,\dot{c}_n(t) = \sum_n H'_{fn}\, e^{i\omega_{fn}t} c_n(t)$

• $\omega_{fn} \equiv \left(E_f^{(0)} - E_n^{(0)}\right)/\hbar$

• $H'_{fn} \equiv \left\langle f^{(0)}\right| H' \left|n^{(0)}\right\rangle$

& to <u>1st order</u> in $H' \ll H^{(0)}$, $\quad c_f(t) \approx \delta_{fi} + \frac{1}{i\hbar}\int_0^t H'_{fi}(t')e^{i\omega_{fi}t'}\,dt'$

Some names: ω_{fi} is called the **transition frequency** for going from initial state i to final state f, while $c_f(t)$ is the **transition amplitude** for doing the same thing. The **transition probability** that we are usually trying to calculate is

$$P_{f\to i} = \left| c_f(t) \right|^2$$

Problem 1 : 1D SHO ™ in a decaying electric field

Consider a one-dimensional harmonic oscillator that is in the ground state | 0 > of the unperturbed Hamiltonian at $t = -\infty$. NOTE: since all the states | n > that we are going to talk about are eigenstates of the unperturbed Hamiltonian, we can <u>suppress the $^{(0)}$ subscript</u> without confusion. The following perturbation is applied between t=$-\infty$ and $+\infty$:

$$H'(t) = -qE\,x\,e^{-t^2/\tau^2}$$

where q is the particle's charge and E is a constant electric field. What is the probability that the oscillator is in the state | n > at $t = \infty$?

Problem 2 : 1D SHO ™ again

Show that if the perturbation is $H'(t) = -\dfrac{qEx}{1+\left(t/\tau\right)^2}$ instead, then to first order $P_{0\to 1} = \dfrac{q^2E^2\pi^2\tau^2}{2m\omega\hbar}e^{-2\omega\tau}$.

Problem 3 : Hydrogen Atom with decaying electric field

A hydrogen atom is in the ground state at $t = -\infty$. An electric field $\vec{E}(t) = \hat{z}\,E\,e^{-t^2/\tau^2}$ is applied until $t = +\infty$. Show that the probability that the atom ends up in any of the $n = 2$ states is, to first order,

$$P_{1\to 2} = \left(\frac{qE}{\hbar}\right)^2 \left(\frac{2^{15}a_0^2}{3^{10}}\right) \pi\tau^2 e^{-\omega^2\tau^2/2}$$ where ω is the transition frequency $\omega \equiv \dfrac{E_{2lm} - E_{100}}{\hbar}$.

Does the answer depend on whether or not we incorporate spin in the picture?