Phys 487 Discussion 9 – The Variational Principle

Problem 1 : Linear Potential *Qual Problem*

A particle of mass m moves in the 1D region $x > 0$ and experiences the following potential energy :

$$V(x) = \begin{cases} \infty & \text{for } x \le 0 \\ F\,x & \text{for } x > 0 \end{cases} \quad \text{where } F \text{ is a real, positive constant.}$$

Use a variational methods to obtain an estimate for the ground state energy. But first, think about these

TIPS for deciding on a trial wavefunction:

- Think about the wavefunction's asymptotic behaviour, i.e. how it must behave / what values it must reach in the limits $x \to \infty$ and $x \to 0$.
- As we mentioned in class, for systems that extend to $\pm\infty$, the easiest trial wavefunctions to work with are almost always Gaussians and/or falling exponentials, i.e. $\exp(-\alpha x^2)$ and $\exp(\mp\alpha x)$. Either form will give you a good answer here, but one will be better than the other. And don't forget …
- The forms in the previous bullet are common choices for taking care of $x \to +\infty$ behaviour … but don't forget about the OTHER boundary condition, which in this case is not $x \to -\infty$ but $x \to 0$! You will have to make a small but significant modification to the previous forms before you can use them as good trial wavefunctions for this problem!

Problem 2 : Connecting Variational Principle & Perturbation Theory *Griffiths 7.5(a)*

Use the variational principle to prove that first-order non-degenerate perturbation theory always overestimates (or more exactly, never underestimates) the ground state energy.

Problem 3 : SHO, 1st excited state *Griffiths 7.4(b)*

Find the best bound on the first excited state of the 1D harmonic oscillator that you can obtain from the trial wavefunction

$$\psi(x) = A\,x\,e^{-bx^2}$$

Problem 4 : δ Function Potential *Griffiths 7.3*

Find the best bound on the ground-state energy E_{gs} for the δ-function potential $V(x) = -\alpha\delta(x)$ using as your trial wavefunction a triangular form that peaks at the origin and falls off linearly on either side to $x = \pm a/2$. (So the total width of the triangle is a … which is your adjustable parameter, and not equal to the Greek α in front of the δ-function potential, which is a given value. a and α look the same in this font, argh!).