Phys 487 Discussion 8 – Qual Essentials Requiring Perturbation Theory

Problem 1 : Electron hanging around inside the nucleus *Classique Qual Problem*

To a pretty good approximation, a nucleus of charge Z can be treated as a sphere of radius R_0 with a uniform charge density. The nuclear radius, R_0, is extremely small compared with the Bohr radius, a_0, of the hydrogen atom ($10^4 - 10^5$ times smaller!) , so $R_0 / a_0 << 1$ provides an excellent opportunity for the use of approximation methods like perturbation theory!

(a) Obtain the electrostatic potential energy $V(r)$ between the nucleus and an atomic electron that is valid when the electron is outside ($r > R_0$) or INSIDE ($r < R_0$) the nucleus. (Hint: this is a 212 problem … Gauss' Law …) Defining the potential energy outside a POINT nucleus to be $V_0(r) = -Ze^2 / (4\pi\varepsilon_0 r)$, find the difference $\delta V(r) = V(r) - V_0(r)$ due to the finite size of the nucleus.

(b) A single electron is bound the nucleus in the lowest-energy bound state. What is its wave function when calculated using the potential $V_0(r)$ from a point nucleus of charge Z?

(c) Use first-order perturbation theory to derive an expression for the change in the ground state energy of the electron due to the finite size of the nucleus.

Problem 2 : Hydrogen atom in simultaneous E and B fields *Excellent Qual Problem*

Consider an electron in the n = 2 shell of the hydrogen atom. The 2s and 2p states are initially degenerate (we are ignoring relativistic corrections). Then we impose two simultaneous perturbations that each add a small potential energy term to the Hamiltonian:

- an electric field of constant magnitude E in the $+x$ direction: adds $H'_E = eV(x) = -eEx$
- a magnetic field given by the vector potential $\vec{A} = \frac{B}{2}(-y\hat{x} + x\hat{y}) = \frac{B}{2}s\hat{\phi}$: adds $H'_B \approx -\frac{e}{m}\vec{p}\cdot\vec{A}$

(We are ignoring the magnetic moment of the electron.) Calculate how the four $n = 2$ states are altered by these simultaneous perturbations.

HINT: Same as the Stark effect problem last week, there are a lot of integrals in this problem, but almost all of them are ZERO. So study each one carefully before you do any calculations!