5.20 Switched-on Field (MIT)

Consider a simple harmonic oscillator in one dimension with the usual Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2 \qquad \text{(P.5.20.1)}$$

a) The eigenfunction of the ground state can be written as

$$\psi_0(x) = Ne^{-\alpha^2 x^2/2} \qquad \text{(P.5.20.2)}$$

Determine the constants N and α.

b) What is the eigenvalue of the ground state?
c) At time $t = 0$, an electric field $|\mathbf{E}|$ is switched on, adding a perturbation of the form $H' = e|\mathbf{E}|x$. What is the new ground state energy?
d) Assuming that the field is switched on in a time much faster than $1/\omega$, what is the probability that the particle stays in the ground state?

5.9 One-Dimensional Coulomb Potential (Princeton)

An electron moves in one dimension and is confined to the right half-space $(x > 0)$ where it has a potential energy

$$V(x) = -\frac{e^2}{4x} \qquad \text{(P.5.9.1)}$$

where e is the charge on an electron. This is the image potential of an electron outside a perfect conductor.

a) Find the ground state energy.
b) Find the expectation value $\langle x \rangle$ in the ground state

5.2 Shallow Square Well II (Stony Brook)

A particle of mass m is confined to move in one dimension by a potential $V(x)$ (see Figure P.5.2):

$$V(x) = \begin{cases} \infty & x < 0 \\ -V_0 & 0 < x < a \\ 0 & a < x \end{cases} \qquad \text{(P.5.2.1)}$$

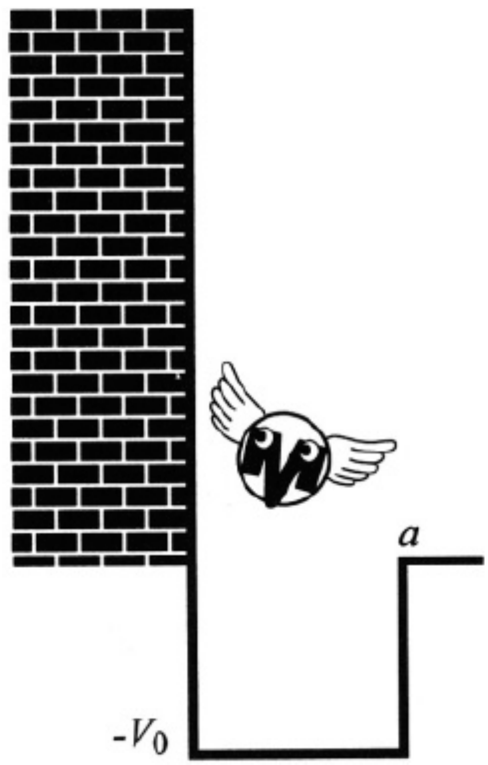

Figure P.5.2

a) Derive the equation for the bound state.
b) From the results of part (a), derive an expression for the minimum value of V_0 which will have a bound state.
c) Give the expression for the eigenfunction of a state with positive energy $E > 0$.
d) Show that the results of (c) define a phase shift for the potential, and derive an expression for the phase shift.