Physics 486 Midterm Exam #1 Spring 2018
Thursday February 22, 9:30 am – 10:50 am

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will not be collected at the end, so anything you write on these question pages will NOT be graded.

You have 80 minutes to work the problems.

At the beginning of the exam:
1) Write your name and netid on your answer booklet(s).
2) Turn your cell phone off.
3) Put away all calculators, phones, computers, notes, and books.

During the exam:
1) Show your work and/or reasoning. Answers with no work or explanation get no points. But ...
2) Don’t write long essays explaining your reasoning. We only need to see enough work to confirm that you understand what you’re doing and are not just guessing. (If you are guessing, explain that, then verify your guess explicitly.) A good annotated sketch is often the best explanation of all!
3) All question parts on this exam are independent: you can get full points on any part even if your answers to all the other parts are incorrect. You should attempt all the question parts! If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. IMPORTANT: If you think you’ve made a mistake but can’t find it, explain what you think is wrong → you may well get partial credit for noticing your error!
5) It is fine to leave answers as radicals or irreducible fractions (e.g. 10\sqrt{3} or 5/7), but you will lose points for not simplifying answers to an irreducible form (e.g. 24(x^2 – y^2) / (√9x – √9y) is unacceptable.)

When you’re done with the exam:

Turn in EVERYTHING: answer booklet and question pages

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.
Problem 1: Always Real

Any complex function $\psi(x)$ can be expressed as the sum of a real part and an imaginary part:

$$\psi(x) = f(x) + ig(x)$$

where $f(x)$ and $g(x)$ are both real. Show that the **expectation value of momentum**, $\langle p \rangle$, for any complex wavefunction $\psi(x) = f(x) + ig(x)$ is a **real number**. Hint: You will need to use one integration by parts.

Problem 2: Bohr Atom

In Bohr’s original model of the atom, electrons of charge $-e$ and mass $m$ move in **circular orbits** of radius $r_n$ around a heavy, stationary nucleus of charge $+Ze$. Since the circular motion of the electrons is periodic in their azimuthal angle $\phi$, the allowed values of the radius $r_n$ can be determined by applying the quantization rule

$$\int_0^{2\pi} p_\phi \, d\phi = nh$$

where generalized momentum $p_\phi = \text{angular momentum } L = mvr$.

Using this quantization rule for angular momentum, and some very elementary classical mechanics expressions, calculate the allowed radii $r_n$ for the electron in terms of $Z, e, m, n$ and physical/numerical constants.

Problem 3: Infinite Well

Consider a particle in this infinite potential well, which is symmetric around $x = 0$:

$$V(x) = \begin{cases} 
0 & \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2} \\
\infty & \text{elsewhere}
\end{cases}$$

Determine the **ground-state wavefunction**, i.e. the wavefunction of lowest energy $\psi_1(x)$. You do **NOT** need to determine the ground-state energy OR the time-dependent part of the wavefunction ... just find $\psi_1(x)$. 

Phys 486 Midterm #1 Spring 2018
**Problem 4 : A Minimalist Wave Packet**

A free particle in a region with $V = 0$ is approximately described by this wavefunction at $t = 0$:

$$\Psi(x,0) = \psi(x) = A(e^{-i11x} + e^{-i12x} + e^{-i10x})$$

where $A$ is a known constant.

Work in units where the particle’s mass $m = 0.5$ and the constant $\hbar = 1$.

(a) Write down the time-dependent wavefunction $\Psi(x,t)$ that describes how the starting wavefunction $\Psi(x,0)$ given above will evolve with time.

**Problem 5 : A Wavefunction to Play With**

At time $t = 0$, a particle is in a state with wavefunction

$$\psi(x) = A e^{-\frac{k|5x|}{2}}$$

where $A$ is a constant of unspecified value.

Hints for dealing with absolute values: It is always safest to deal with the two cases $x > 0$ and $x < 0$ separately. Alternatively, you may sometimes find it efficient to use the “sign function” $\text{sgn}(x)$ to express both cases with one expression; it is defined as $\text{sgn}(x) \equiv +1$ for $x > 0$ and $-1$ for $x < 0$. (It is undefined at $x = 0$.)

(b) Find the mean momentum $\langle p \rangle$ of the particle at time $t = 0$.

(c) What can you conclude about the value of the potential $V(x)$ at $x = 0$? Hint: making a quick sketch of $\psi(x)$ would be extremely helpful!