Review of Topics

Inertia Terms is << drag (viscosity) term

Sidebar: Fourier Transforms

List of topics we've so-far covered
(12 of them)

- 1. Size vs. strength: L^3 vs. L^2.
- 2. Partition function, Boltzmann distribution: ($Z^{-1}\Sigma e^{-E/k_BT}$)
- 3. Enthalpy ΔH ($\approx\Delta E$), entropy $\Delta S = \ln W_i$, & Free Energy (ΔG): $\Delta G \approx \Delta E - T\Delta S$
- 4. Proteins & amino acids

  a. 20 amino acids: R group: Non-polar, polar, charged; Hydrophobic vs hydrophilic

  b. Bonds & strength of bonds (covalent, ionic, hydrogen, van der Waals)

  c. Primary, secondary, tertiary, quarternary structures
- 5. Enzyme, activation energy
- 6. DNA

  a. Structure and function– 3 parts: base composition (aromatic group (T,A,G,C), sugar (two –OH), phosphate backbone. 3' and 5' end.

  b. Twist

  d. Supercoiling: Twist and Writhe

  e. $N_{buckling}$, $r_{buckling}$

  f. Persistence length

List of topics we've so-far covered

- 6. DNA
  - a. Structure and function
  - b. Base composition
  - c. Twist
  - d. Supercoiling: Twist and Writhe
  - e. $N_{buckling}$: DNA starts to buckle and forms supercoil with slope, $S_{buckling}$
  - f. Persistence length
- 7. RNA
  - a. Structure & Base composition (less -OH and U instead of T)
  - b. Function- more diverse/catalytic; and less good at storage.
  - c. RNA world vs. DNA world. Archea, Prokaryic (Bacteria), Eukaryotic.
- 8. Equipartition theorem: ½ $k_B T$ = degree of freedom which goes like y_i^2.
- 9. Freely Jointed chain (FJC) and worm Like chain (WLC)
- 10. Magnetic trap
- 11. Atomic Force Microscope
- 12. Optical Trap– Bandwidth limits

What is noise in measurement?

The noise in position using equipartition theorem

→ you calculate for noise at all frequencies (infinite bandwidth).

For a typical value of stiffness (k) = 0.1 pN/nm.

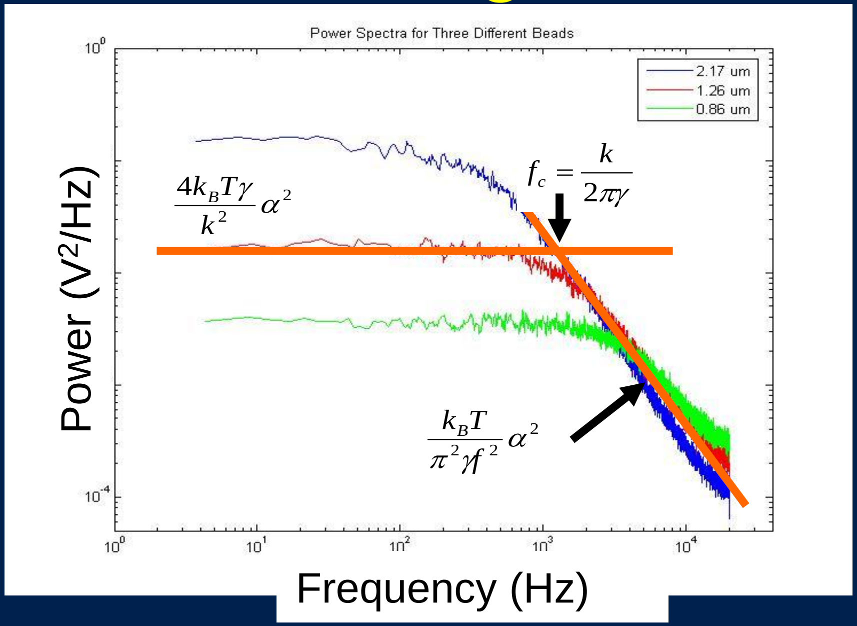
$<x^2>^{1/2} = (k_BT/k)^{1/2} = (4.14/0.1)^{1/2} = (41.4)^{1/2} \sim 6.4$ nm

6.4 nm is a pretty large number.

[Kinesin moves every 8.3 nm; 1 base-pair = 3.4 Å]

How to decrease noise?

Reducing bandwidth reduces noise.



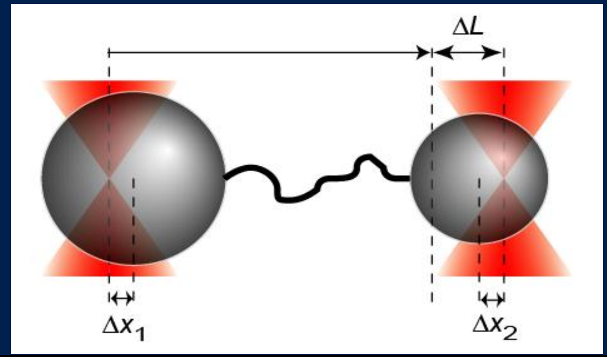
If instead you collect data out to a lower bandwidth BW (100 Hz), you get a much smaller noise.

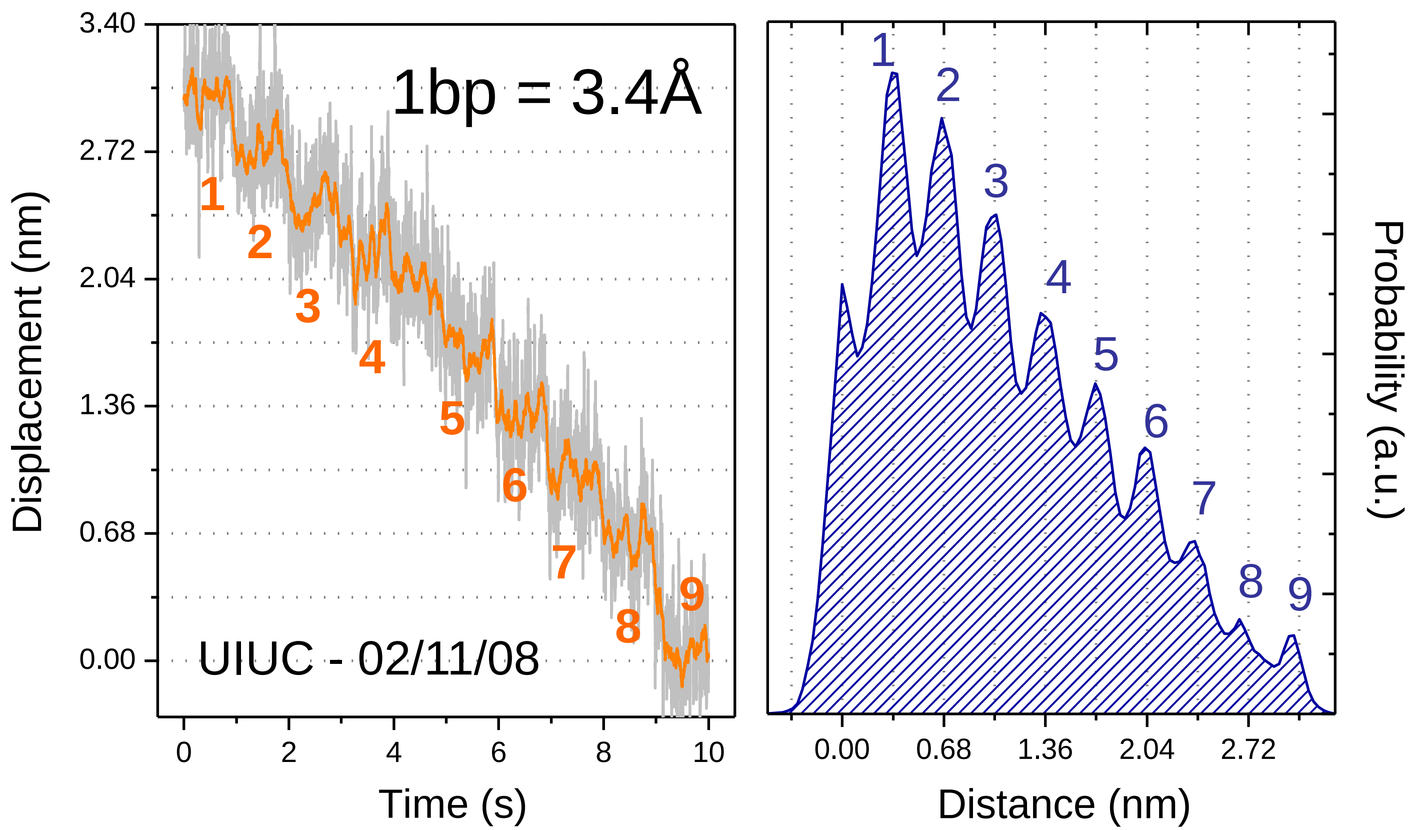
Noise = integrate power spectrum over frequency.
If BW < f_c then it's simple integration because power spectrum is constant, with amplitude = $4k_BT\gamma/k^2$

Let's say BW = 100 Hz: typical value of γ (10^{-6} for ~1 μm bead in water).

But $(\langle x^2\rangle_{BW})^{1/2} = [\int const*(BW)dk]^{1/2} = [(4k_BT\gamma 100)/k]^{1/2} =$
$[4*4.14*10^{-6}*100/0.1]^{1/2}$
~ 0.4 nm = 4 Angstrom!!

Basepair Resolution—Yann Chemla @ UIUC





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3.4 kb DNA

F ~ 20 pN

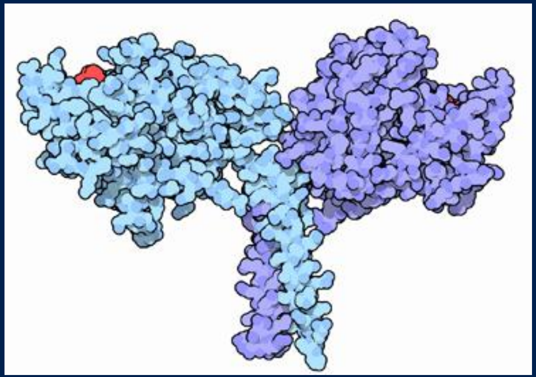
f = 100Hz, 10Hz

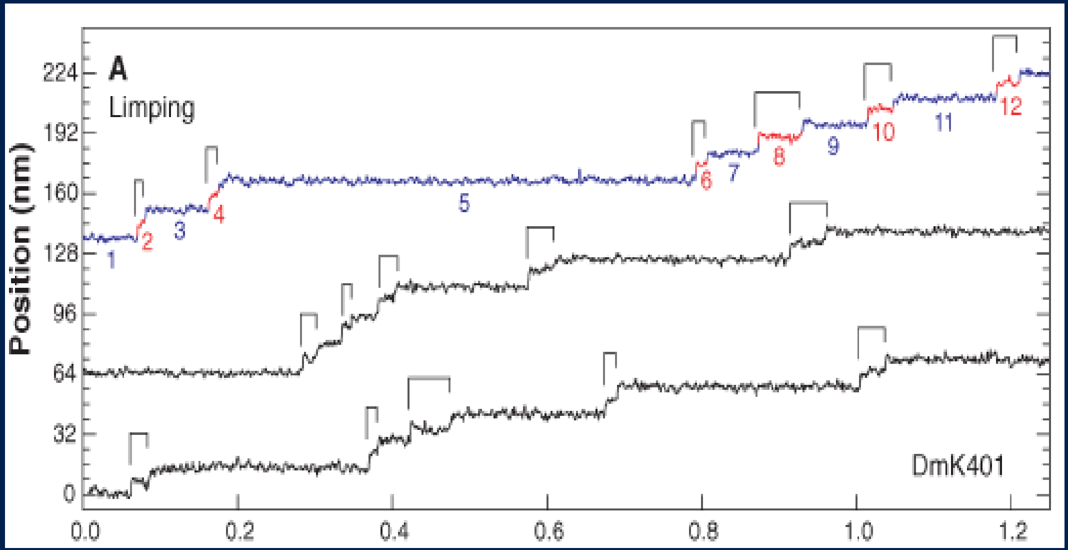
Observing individual steps

Motors move in discrete steps

Detailed statistics on kinetics of stepping & coordination

Kinesin





Step size: 8nm

Asbury, et al. *Science* (2003)

Brownian motion of small particle: ma ≈ 0

k_BT= 4.14pN-nm
k ≈ 0.1 pN-nm

Langevin equation:

$$m\ddot{x} + \gamma\dot{x} + kx = F(t)$$

≈0 (inertia term)

Inertia term (ma)

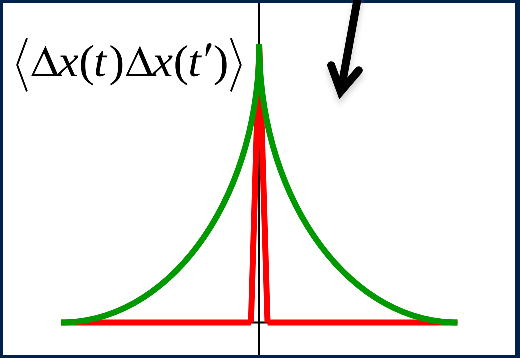
Drag force
$\gamma = 6\pi\eta r$

Trap force

Fluctuating Brownian force

Exponential autocorrelation function

$$\langle \Delta F(t)\Delta F(t')\rangle = \frac{k_B T}{k} e^{-k|t-t'|/\gamma}$$



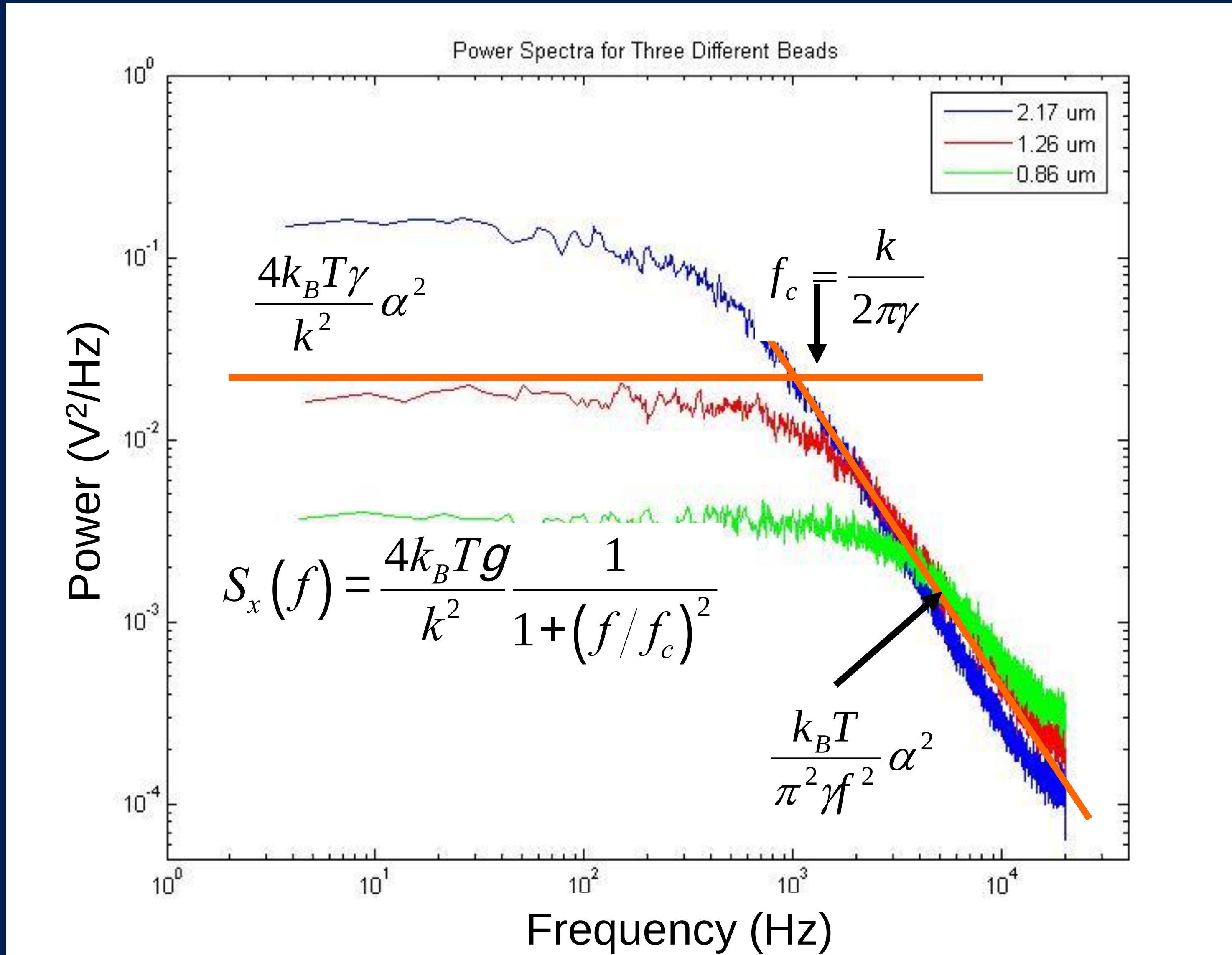
FT → Lorentzian power spectrum

$$S_x(f) = \frac{4k_BT\gamma}{k^2}\frac{1}{1+(f/f_c)^2}$$

Corner frequency
$f_c = k/2\pi\gamma$

Inertia term for μm-sized objects is always small (…for bacteria)

1. Voltages vs. time from detectors.
2. Take FT.
3. Square it to get Power spectrum.
4. Power spectrum = α^2 * $S_x(f)$.



We want to show:

For small objects the inertial force term ≈ 0

$$md^2x/dt^2 << \gamma dx/dt$$

Sidelight into Fourier Transforms

Langevin equation:

$$\underbrace{m\ddot{x}}_{\approx 0} + \gamma\dot{x} + kx = F(t)$$

$$\tilde{x}(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-i\omega t}dt$$

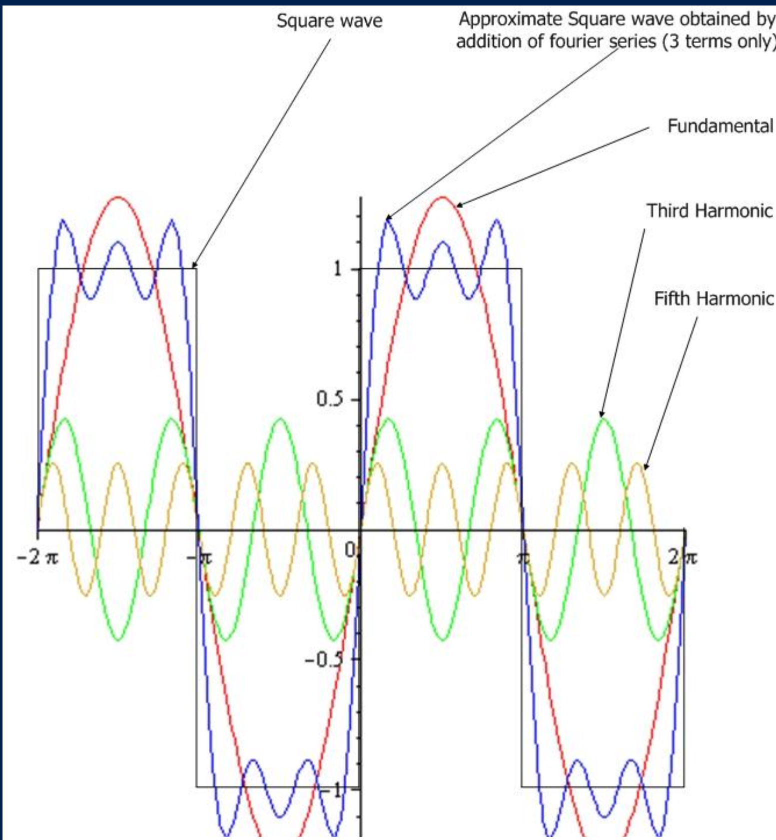
x (ω) = the amplitude of x(t) which has the frequency at ω.

So if you add up all x(ω), you will get back x(t).

To include all possible ω, go from –infinity to + infinity

A differential equation can be looked at as a simple algebraic equation through Fourier Transforms.

Can add up and get all sorts of function



$$v = \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x + \frac{4}{5\pi}\sin 5x + \ldots\ldots$$

Fig-C : Square wave and its Fourier series (only first three terms are taken)

Google: square wave Fourier transform

Square wave

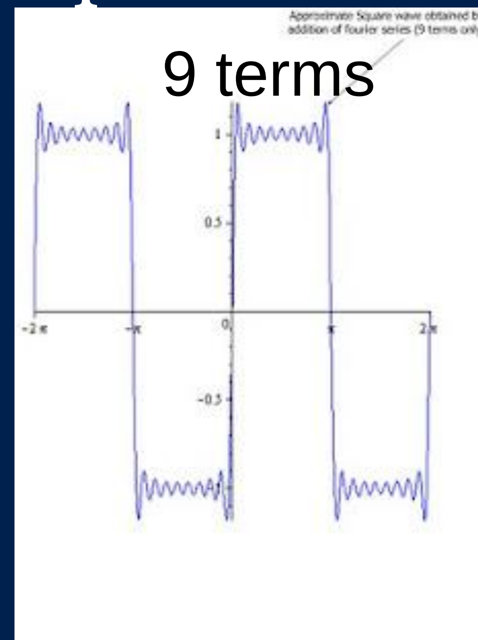


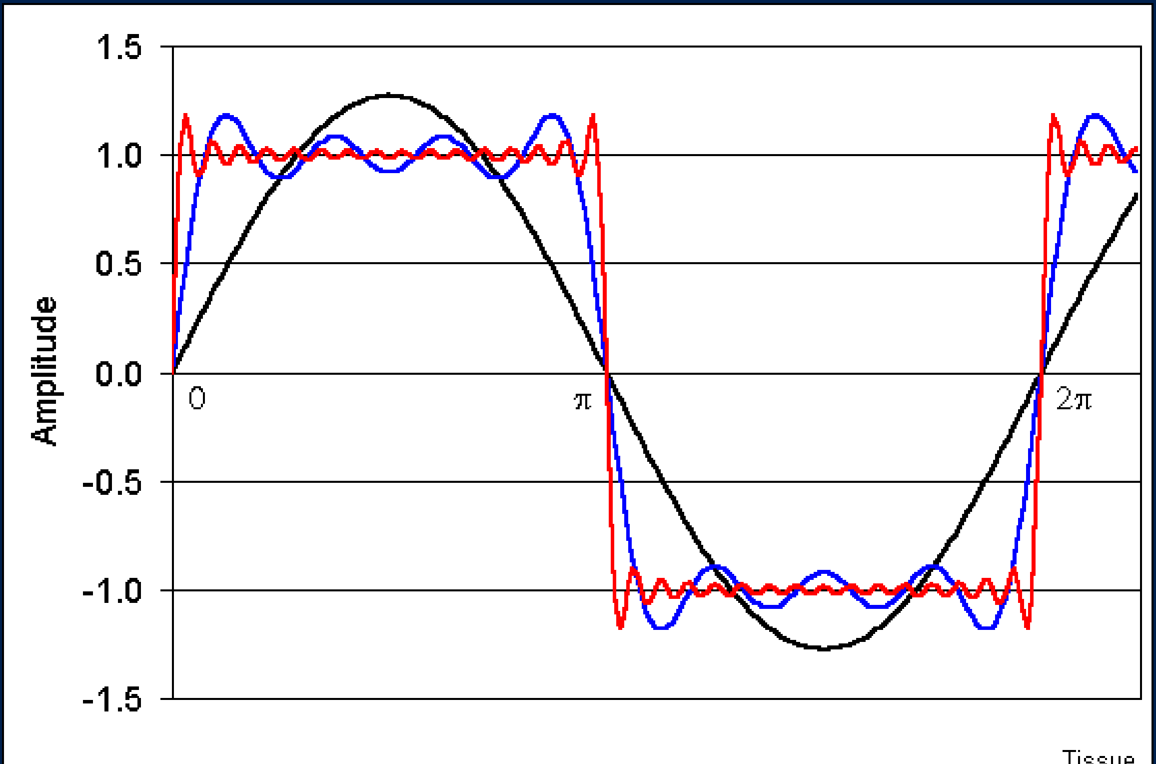
Fig- D: Sum of nine terms of Fourier series of square wave

Since f(x) = f(2L-x), the function is odd, So $b_o = b_n = 0$ where n is even

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$f(x) = \frac{4}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right).$$

Fourier Transform: can view as a f(t), or a f(ω)



The three curves in the plot show the first one term (black line), four terms (blue line), and sixteen terms (red line) in the Fourier expansion. As more terms are added the superposition of sine waves better matches a square wave.

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{-i\omega t}\, d\omega$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{i\omega t}\, dt$$

ω, the angular frequency (2f),

t is time, and:

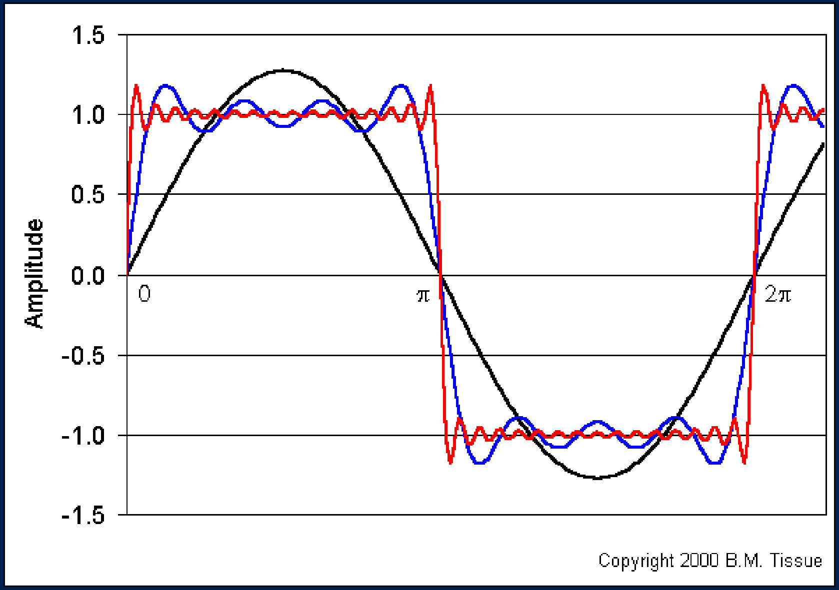
$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

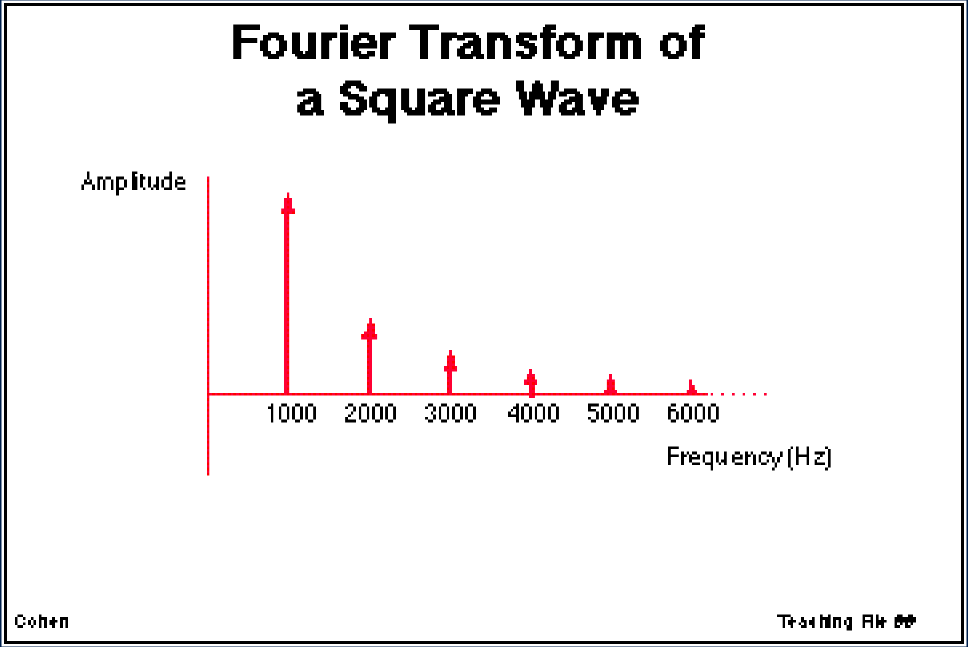
where $i = \sqrt{-1}$

$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n \pi x}{L}\right).$$

http://www.files.chem.vt.edu/chem-ed/data/fourier.html

F(t) and f(ω): Equivalent





$$\tilde{x}(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-i\omega t} dt$$

If the FT of x(t) = x(ω), then
A trick: dx(t)/dt = iω x(ω)
then $d^2x/dt^2 = -\omega^2 x$

[will give as homework]

Langevin equation:

$$m\ddot{x} + \gamma\dot{x} + kx = F(t)$$

Langevin equation:

$$-m\omega^2\tilde{x} + i\omega\gamma\tilde{x} + k\tilde{x} = F(\omega)$$

Some numbers…

$\omega = 2\pi \times 10^4$

$m = 4/3\pi\rho\, r^3$: $r = 0.5\ \mu m$;

$\rho = 1.05\ kg/m^3 = 1.75 \times 10^{-16}$

Inertial term: $m\omega^2 = 2.2 \times 10^{-6}$

$\eta = 10^{-3}$ (viscosity of water)

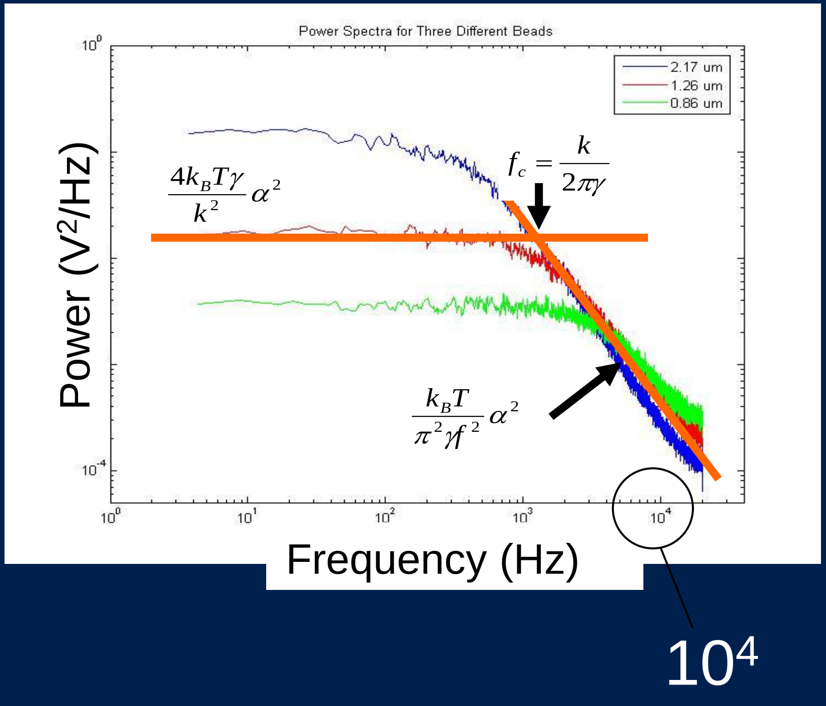
$\gamma = 6.0\, \pi\, \eta\, r = 1.9 \times 10^{-8}$

Viscosity Term: $\gamma\omega = 0.0012$

Harmonic term: $k = 0.1$ pN/nm

Ratio= IT/VT = 0.0018

So Inertial term << Viscous term



Class evaluation

1. What was the most interesting thing you learned in class today?

2. What are you confused about?

3. Related to today's subject, what would you like to know more about?

4. Any helpful comments.

Answer, and turn in at the end of class.