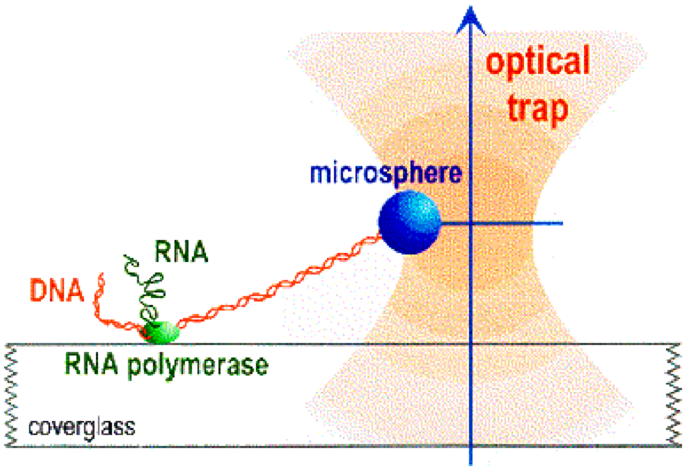
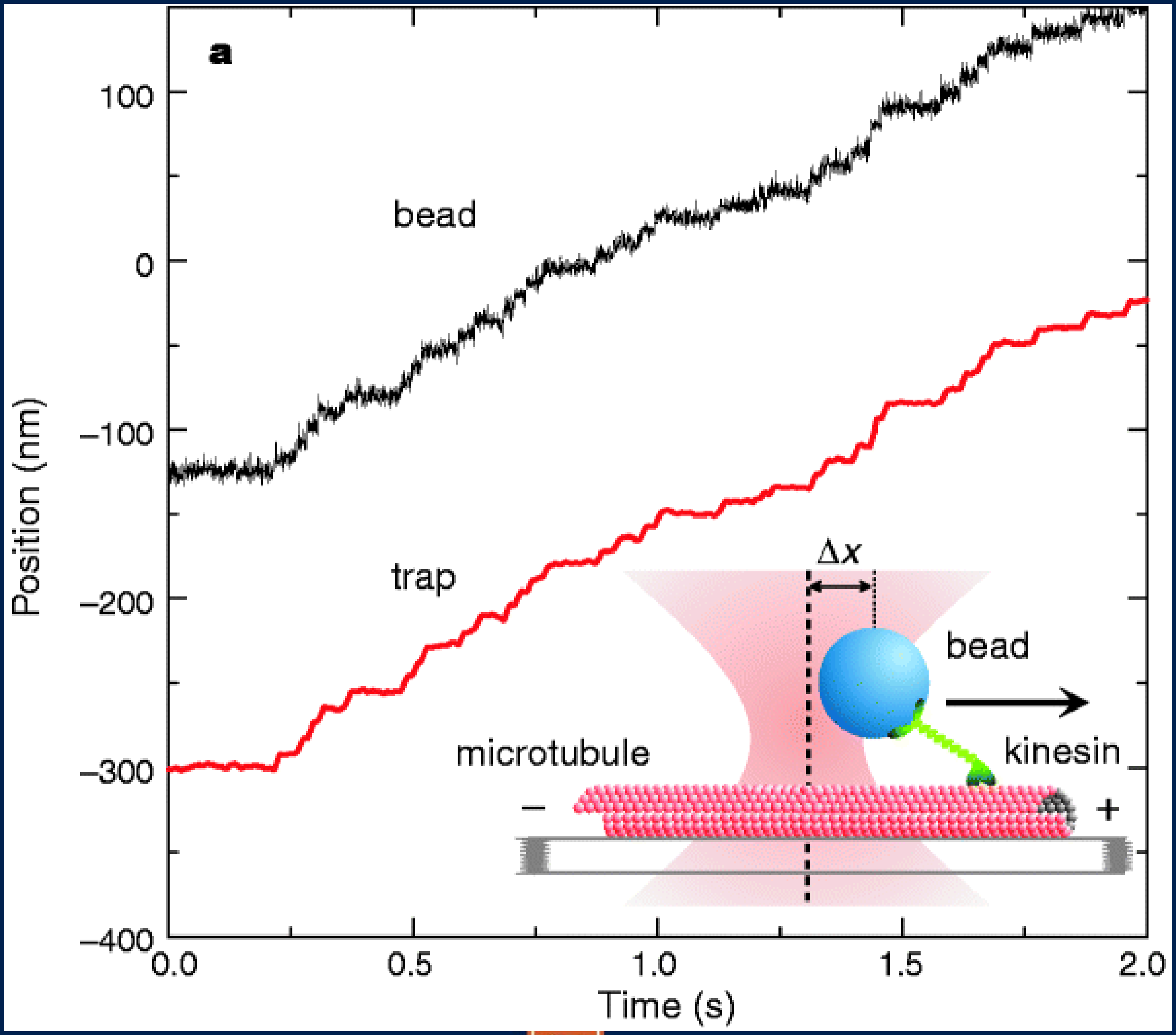

$p = h/\lambda$
6.55×10^{-34} J·s/λ
Slides mostly from Yann Chemla—
blame him if anything is wrong!

Optical Traps

Optical Trap (Nobel Prize, 1997)

Bead is held by "optical force" in trap with effective spring constant k.
Can measure: "stall force" –max force motor can make.
displacement of bead with nm. resolution.




a
bead
trap
Δx
bead
kinesin
microtubule
−
+
Position (nm)
100
0
−100
−200
−300
−400
0.0
0.5
1.0
1.5
2.0
Time (s)

Key points

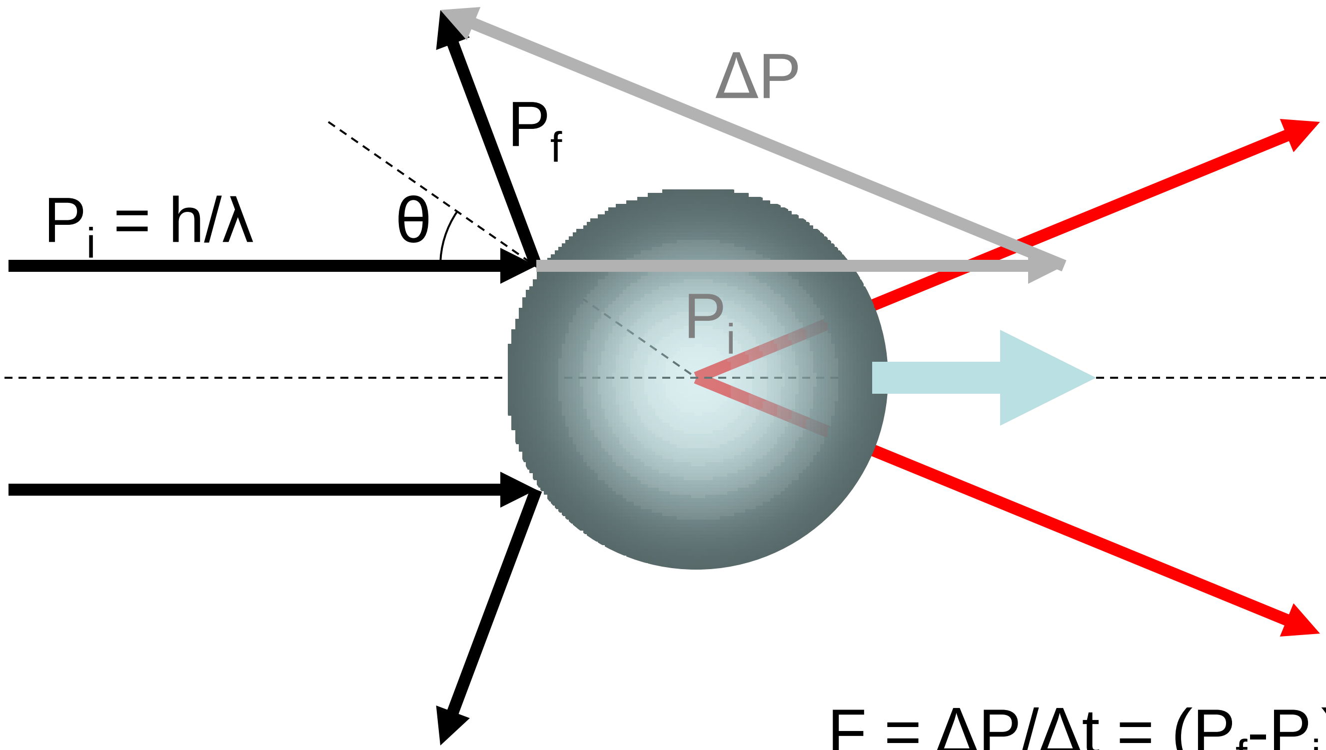
Light generates 2 types of optical forces: *scattering*, *gradient*.

Gradient leas to radiation pressure.

Trap strength depends on light intensity, gradient

Trap is harmonic: k ~ 0.1pN/nm

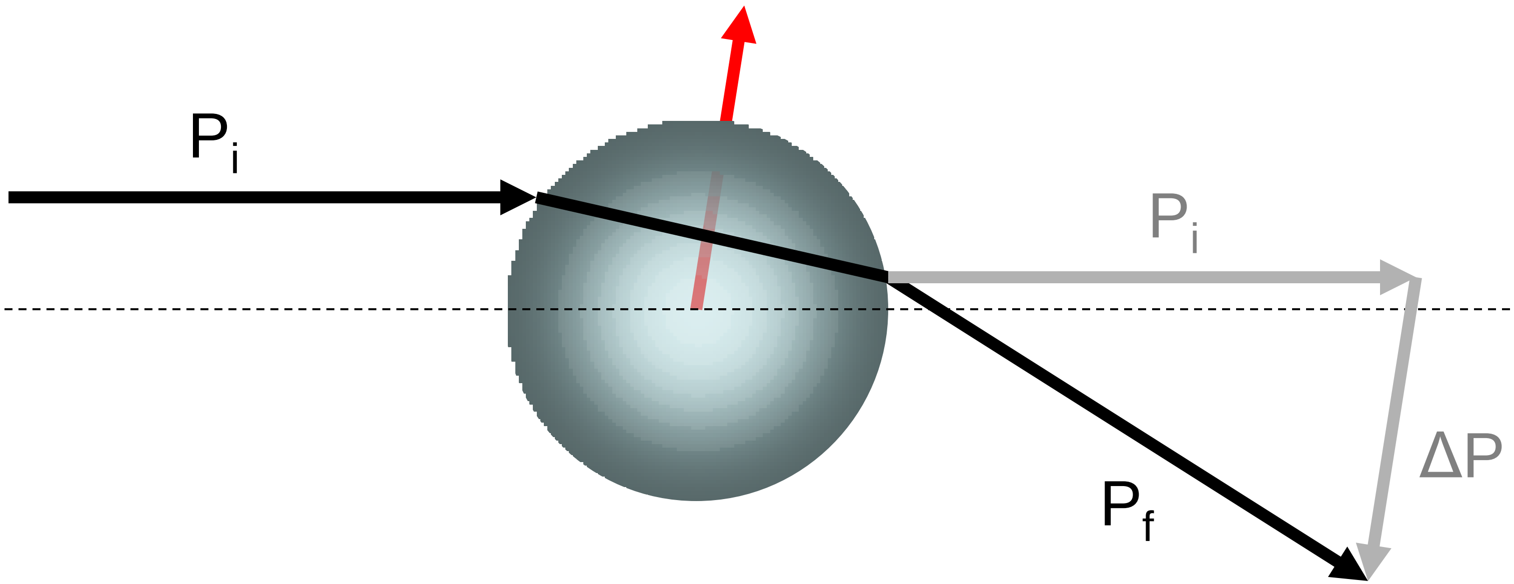
Optical scattering forces – reflection



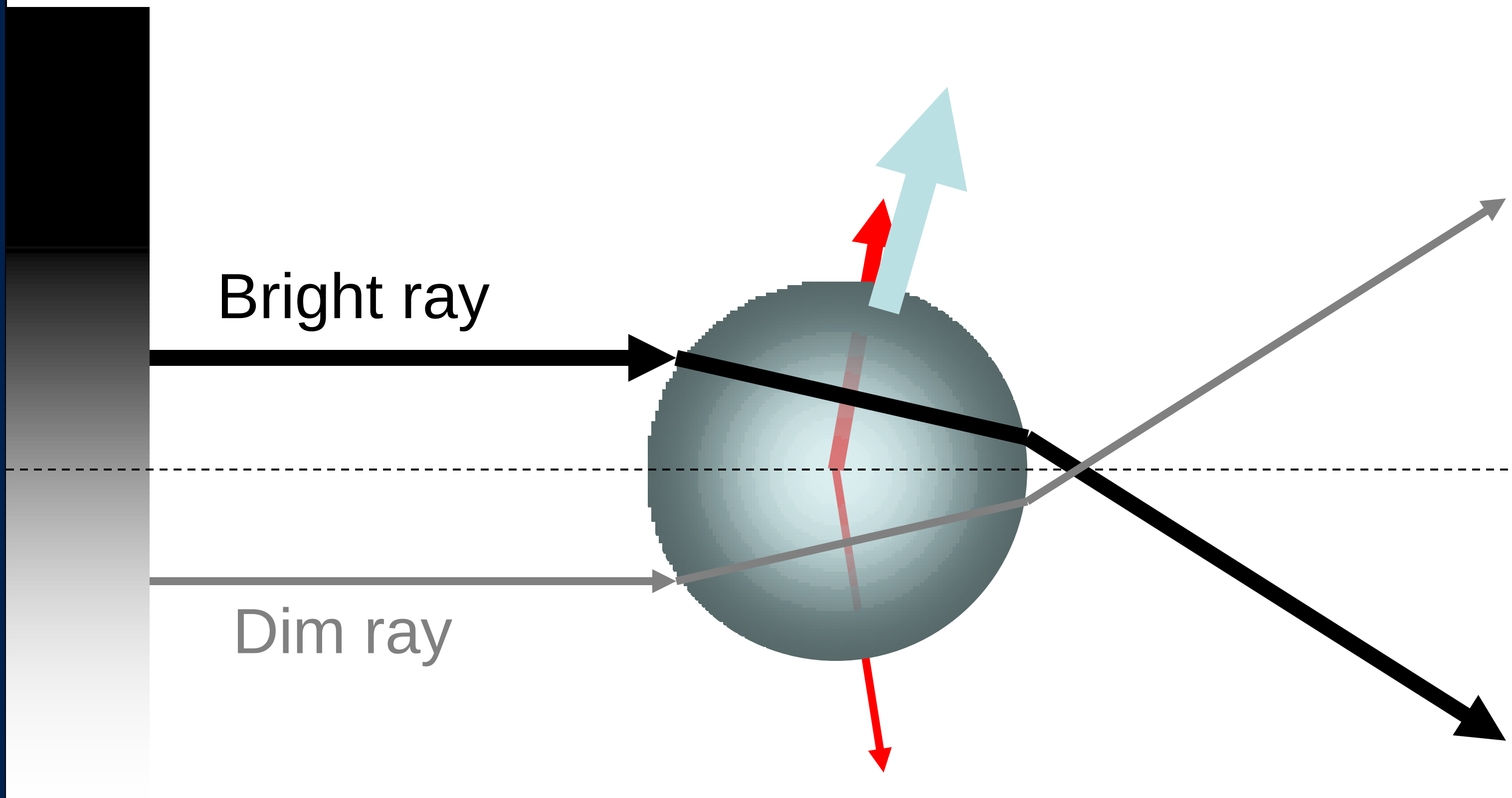
$F = \Delta P/\Delta t = (P_f - P_i)/\Delta t$

Newton's third law – for every action there is an equal and opposite reaction

Optical forces – Refraction

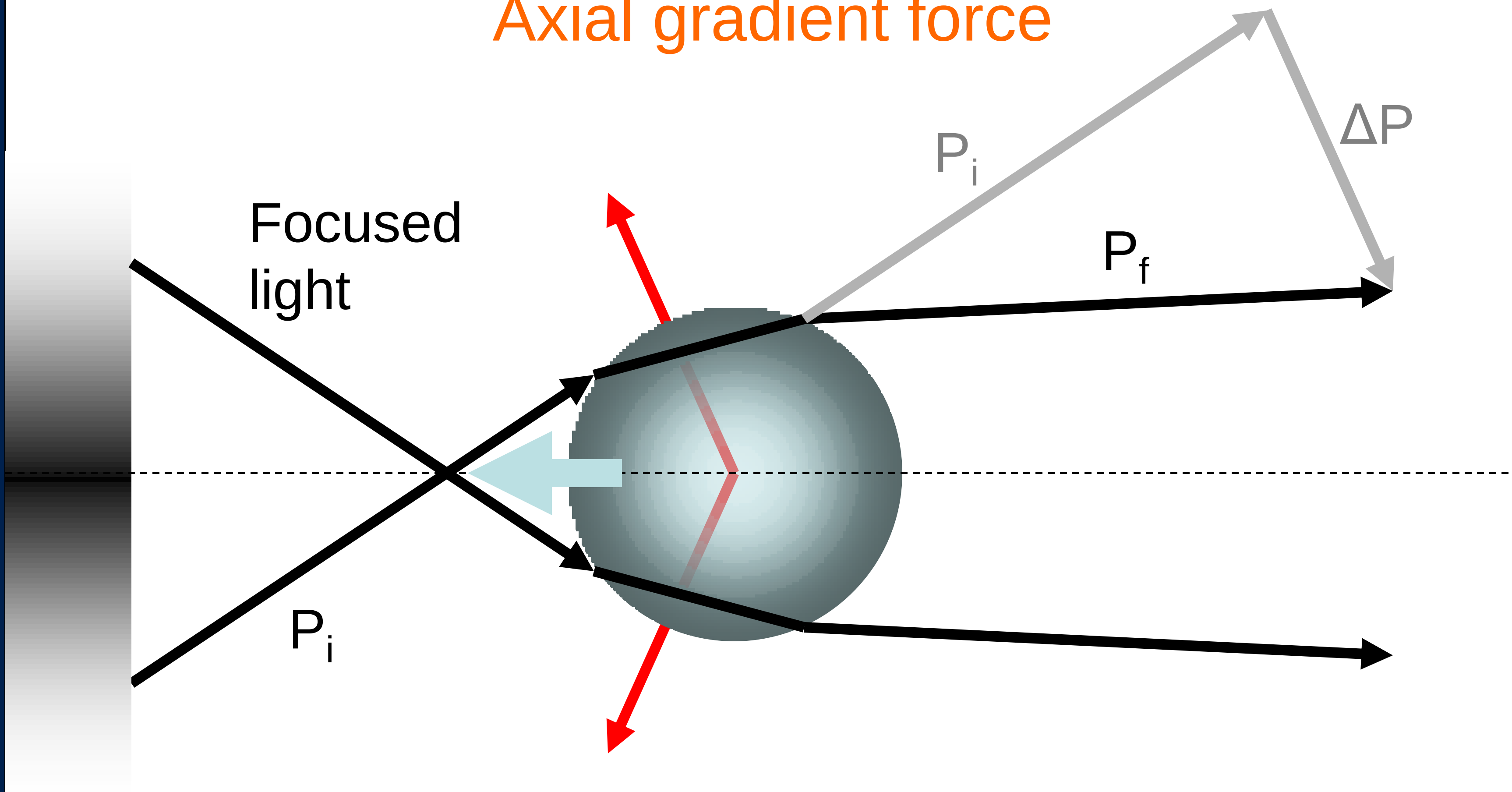


Lateral gradient force



Object feels a force toward brighter light

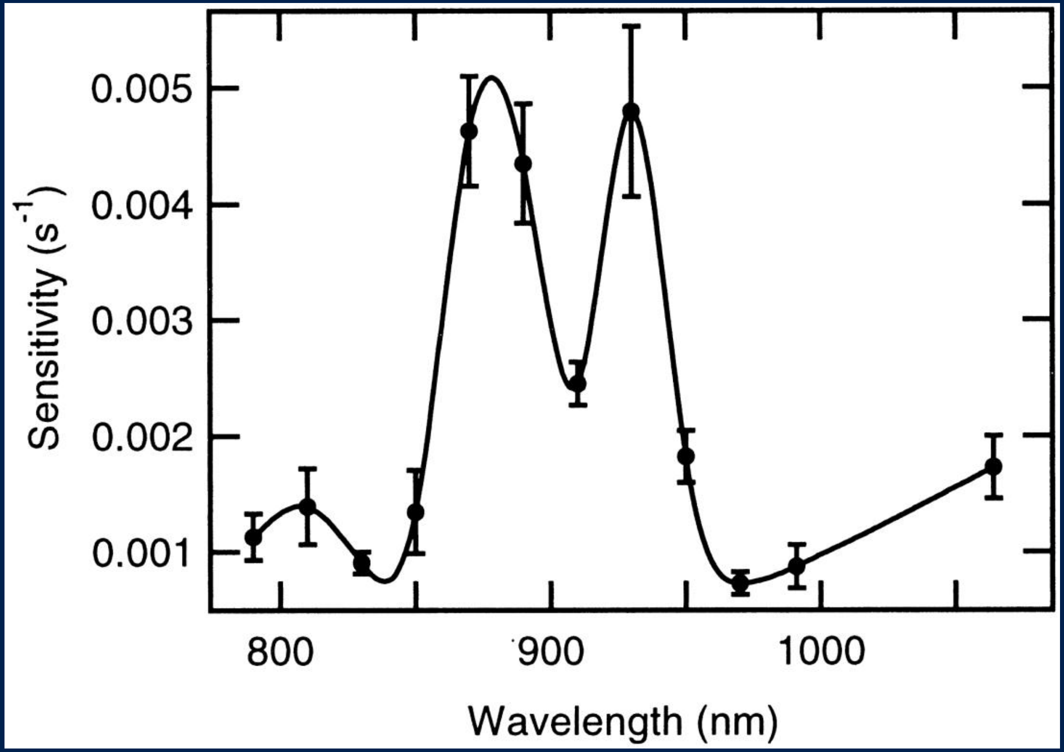
Axial gradient force



Object feels a force toward focus

Force ~ gradient intensity

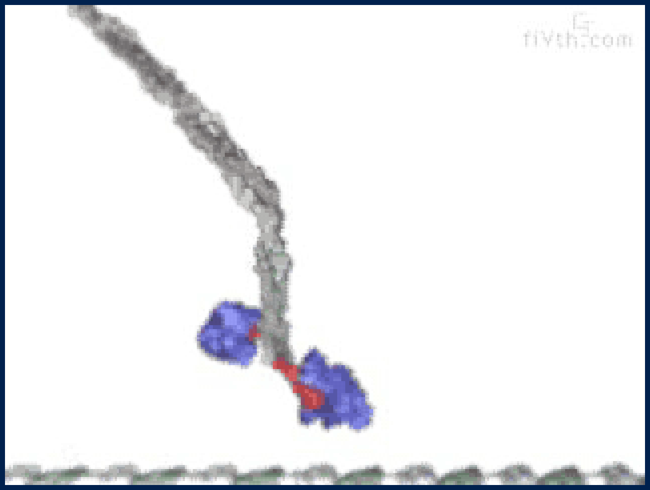
IR traps and biomolecules are compatible



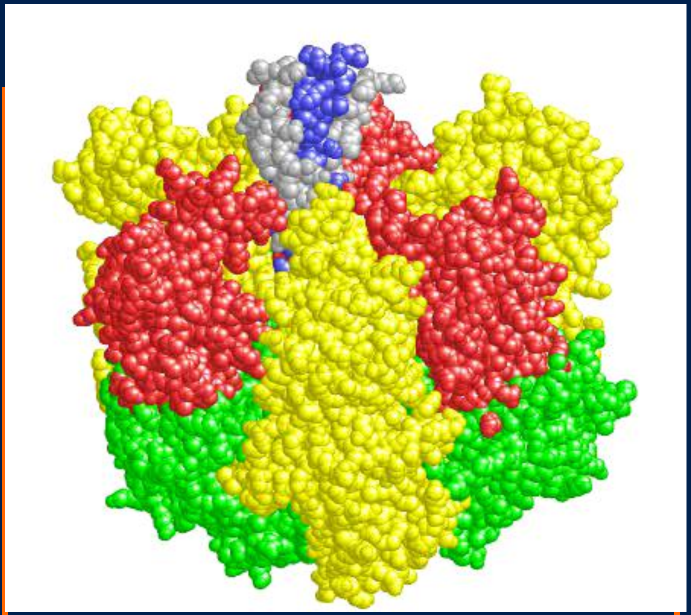
Neuman et al. Biophys J. 1999

Biological scales

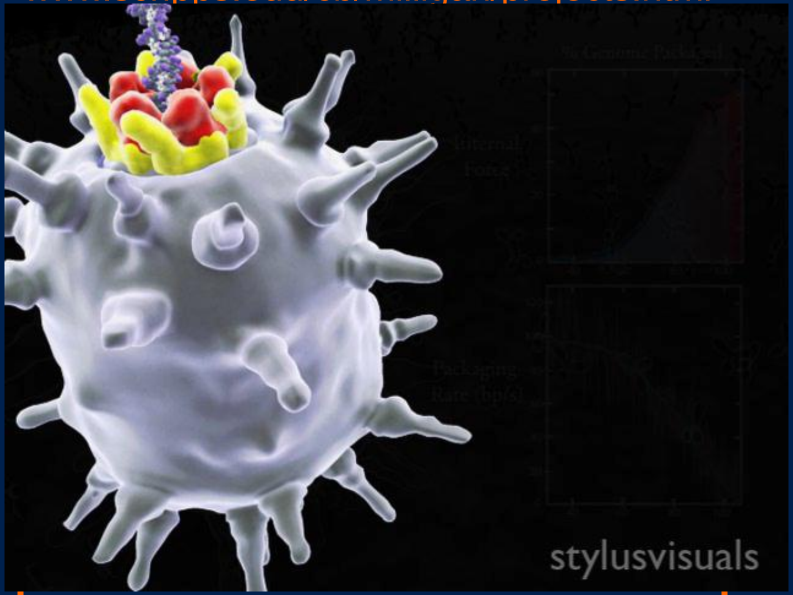
Force: 1-100 picoNewton (pN)
Distance: <1–10 nanometer (nm)



www.scripps.edu/cb/milligan/projects.html



www.cnr.berkeley.edu/~hongwang/Project



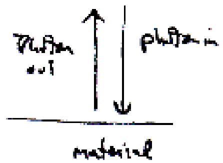
www.alice.berkeley.edu

Range of forces an optical trap can measure.

Estimate size of Trapping force

Force due to scattering of photon(s).

Simple case - reflection (Here we don't have to worry about 2-d + snell's laws + angles)



Force on material $= \frac{\Delta p}{\Delta t} = \frac{2 p_{photon}}{\Delta t}$

($p =$ momentum of photon)

or slightly more general $F \approx \frac{\Delta p}{\Delta t} \sim \frac{Q p}{\Delta t}$

$Q = \leq 1$ (dimensionless) efficiency factor (not perfect reflector / scattered at angle so in general $\Delta p \neq 2p$ but $\Delta p \sim Qp$)

Now we just want to convert momentum/time into something more convenient like Energy/time

= Power of incident light

For light (in vacuum)

$$E = pc$$

For light in material index of refraction n

$$E = pv = pc/n \qquad \text{(energy/photon)}$$

$$\frac{E}{\Delta t} = \frac{pc}{n\Delta t} = \text{Power}$$

$$\frac{P}{\Delta t} = (\text{Power})\, n/c$$ = incident momentum per sec of a ray of Power P in medium of refraction index n

$$F = \frac{Q P}{\Delta t} = \frac{Q(\text{Power})(n)}{c}$$

Q for spherical particle radius $\sim \lambda$

$$Q \sim 0.1$$

For $P = 1\,\text{mW} = 1\,\text{mJ/sec} = 10^{-3}\,\text{N-m/s}$

$$F \sim \frac{(0.1)(10^{-3}\,\text{N-m/s})}{3 \times 10^{8}\,\text{m/sec}} (\sim 1.3)$$

$$\sim \tfrac{1}{2}\,\text{pN}$$

$F \sim 0.5\,\text{pN/mW}$ of laser power

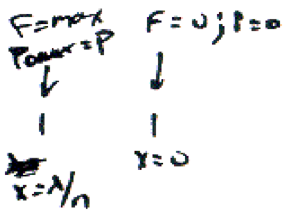
Stiffness (spring constant) of Optical Trap

If power drops from P to zero over λ/n

$$F = Kx$$

@ $x = 0$ $F = 0$

$x = \frac{\lambda}{n}$ $F = \frac{QnP}{c}$



$$\frac{QnP}{c} = \frac{K\lambda}{n}$$

$$\frac{Qn^2P}{\lambda c} \sim K$$

Typical spring constants ~0.01 – 0.1 pN/nm (myosin lever arm 2pN/nm)

for P ~ 100mW on glass/plastic beads ~1μm

Traps roughly linear ~200nm (> this, bead escapes)

Note: Optical trap vs cantilever

Optical traps produce low force (can't look at really strong motors.)

Damping 1μm bead ~10x less than 100μm cantilever

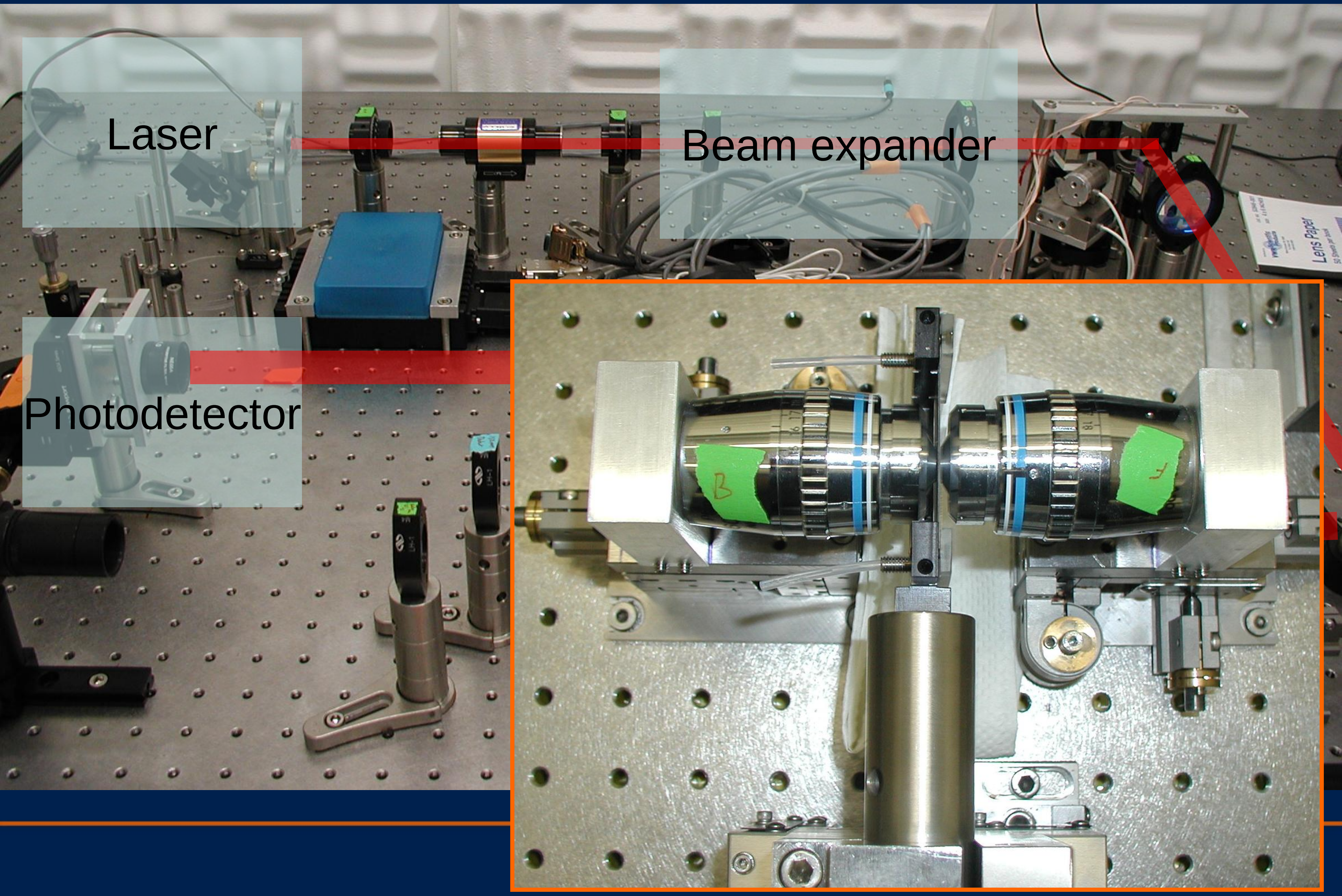
∴ for same force/trap stiffness optical trap has better time resolution ($\tau \sim \gamma/k$)

Requirements for a *quantitative* optical trap:

1) Manipulation – intense light (laser), large gradient (high NA objective), moveable stage (piezo stage) or trap (piezo mirror, AOD, …) [AcoustOptic Device- moveable laser pointer]

2) Measurement – collection and detection optics (BFP interferometry)

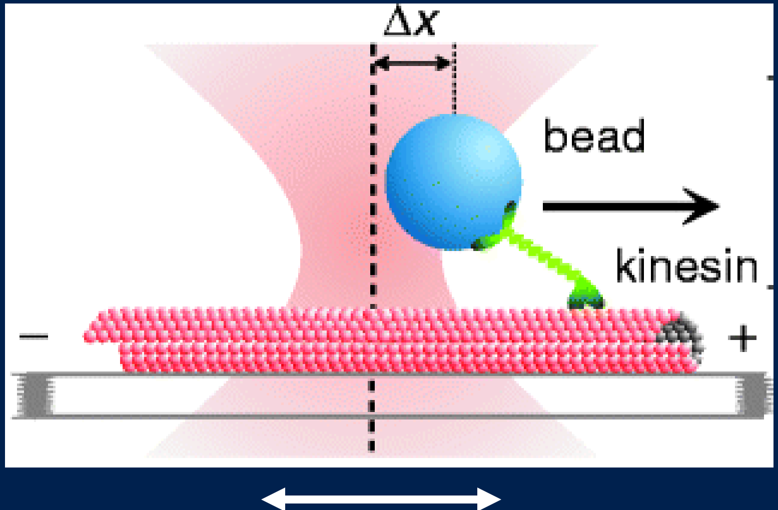
3) Calibration – convert raw data into forces (pN), displacements (nm)

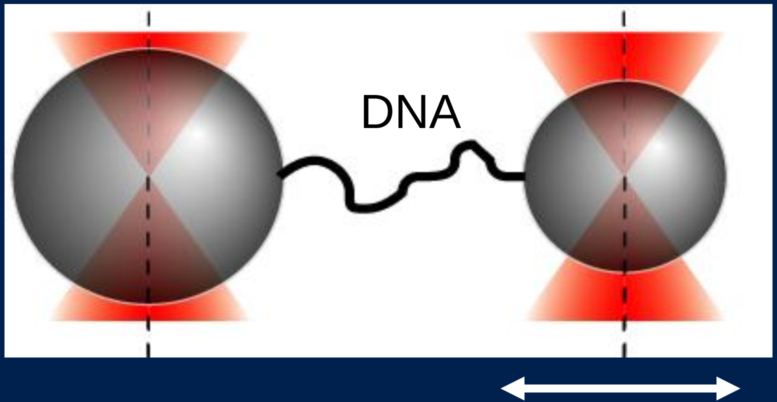

Laser
Beam expander
Photodetector

1) Manipulation

Want to apply forces – need ability to move stage or trap (piezo stage, steerable mirror, AOD…)

(Acouto Optic Device: variable placement of laser)





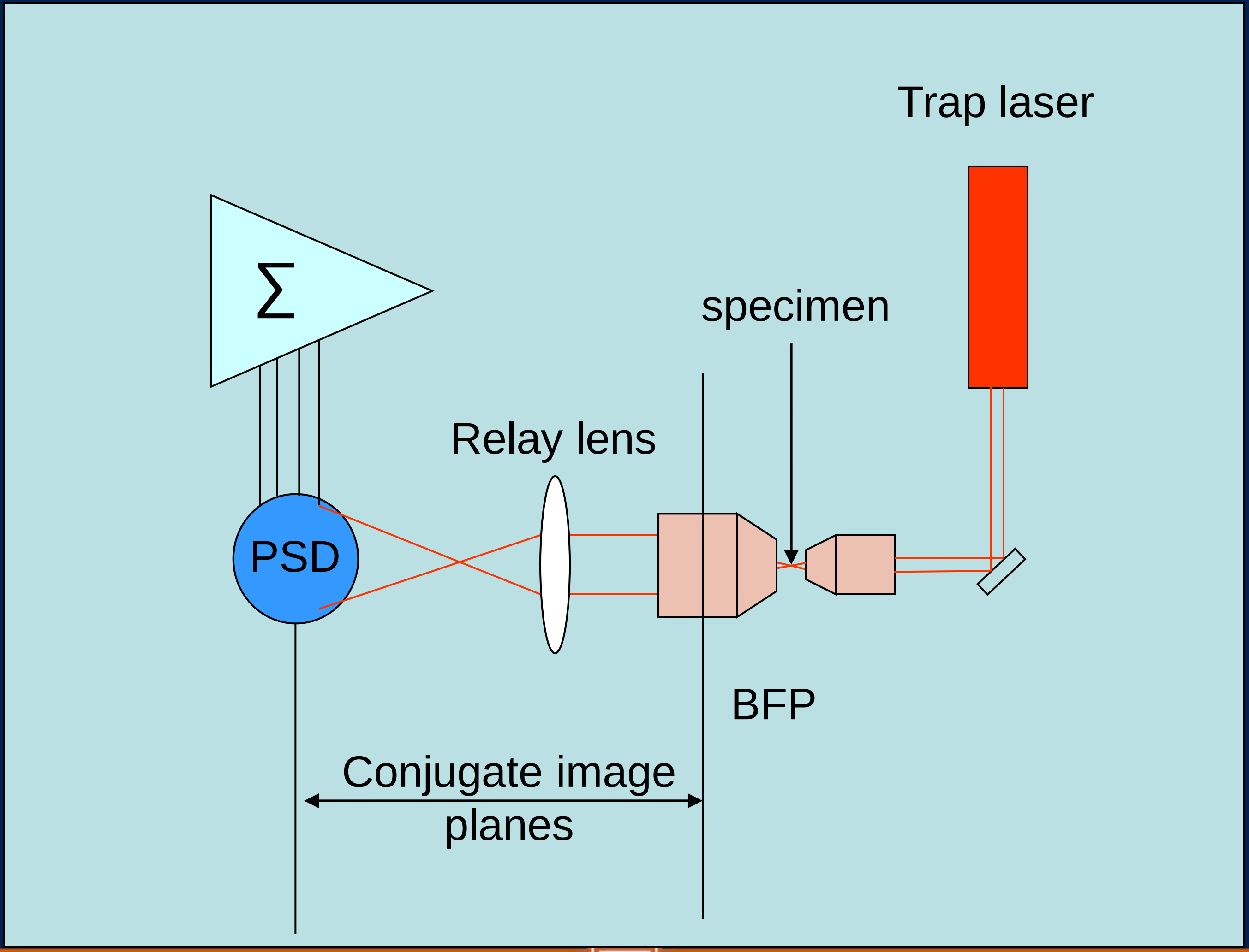
By using two beads, and taking difference, capable of removing floor movement! Get to Angstrom level!

2) Measurement

Want to measure forces, displacements – need to detect deflection of bead from trap center

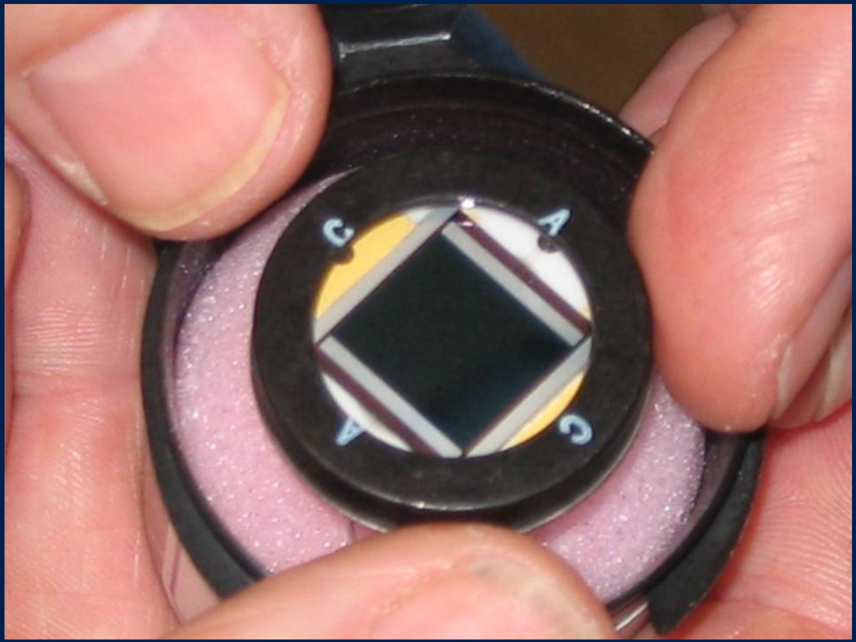
1) Video microscopy
2) Laser-based method – Back-focal plane interferometry

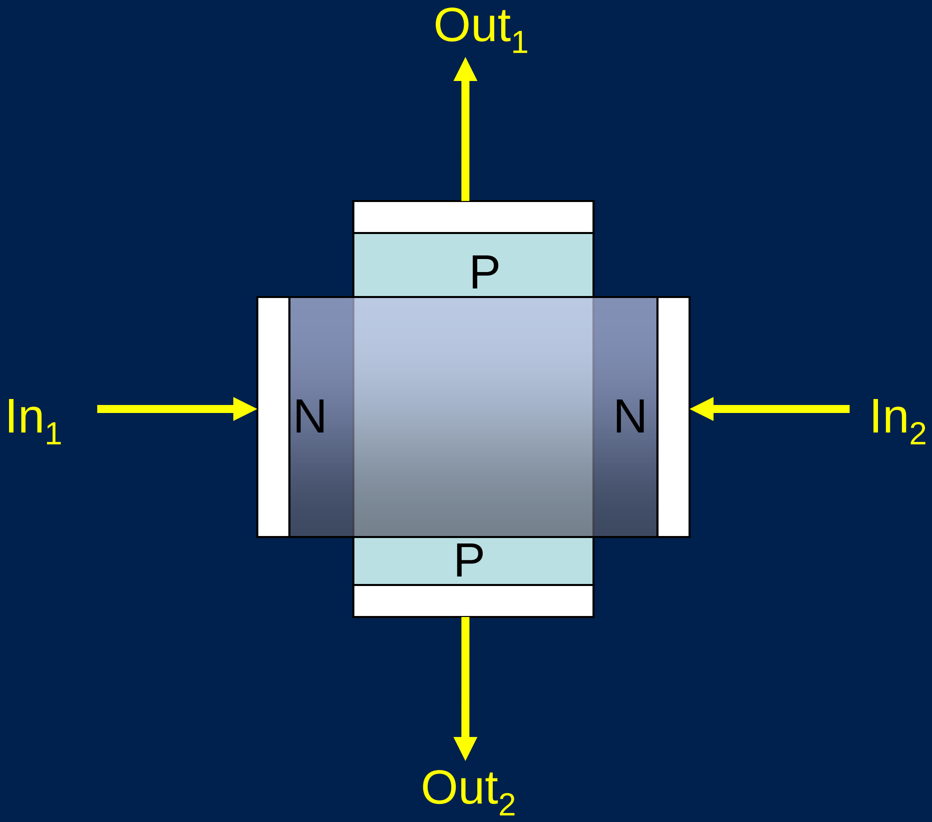
BFP imaged onto detector



Position sensitive detector (PSD)

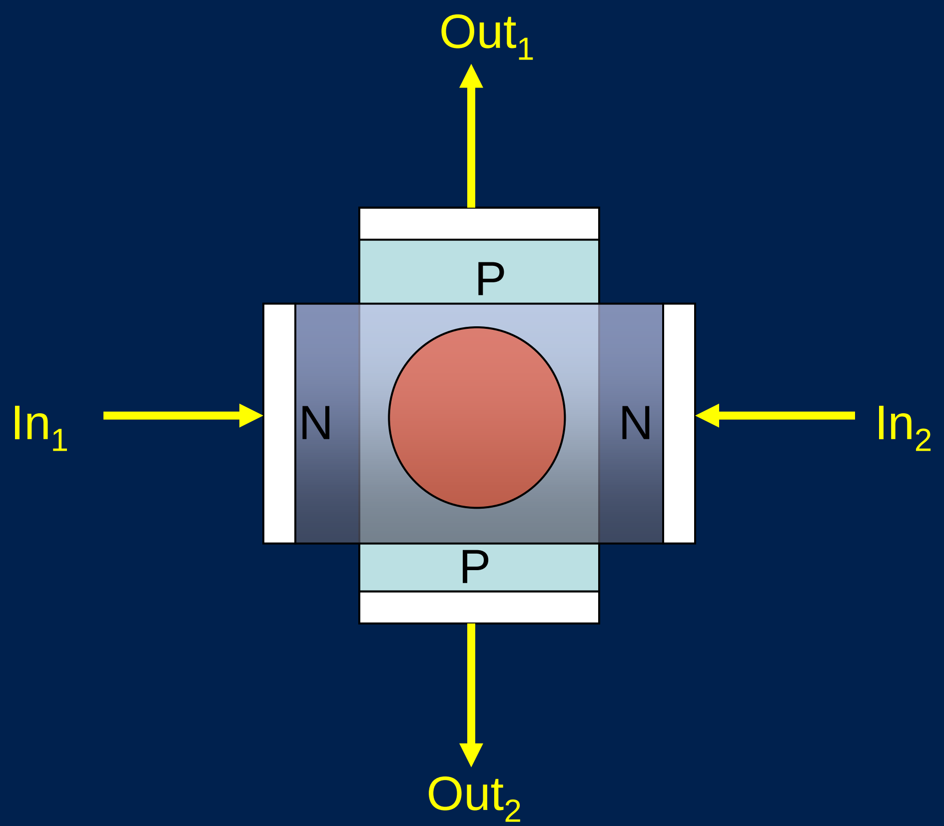
Plate resistors separated by reverse-biased PIN photodiode

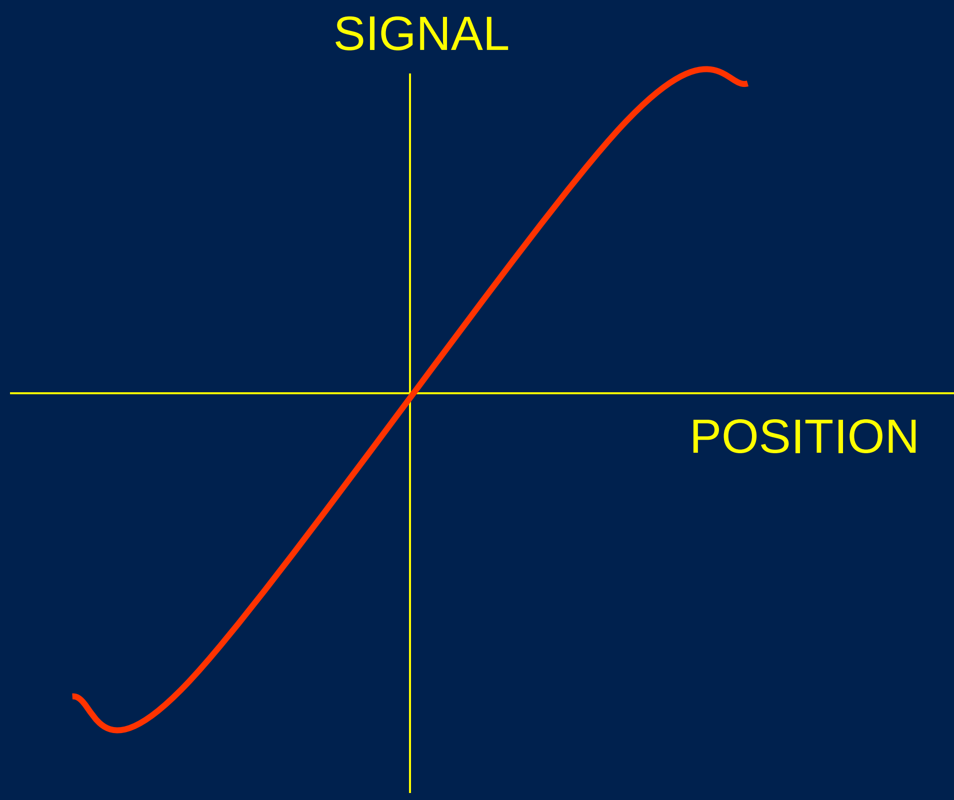




Opposite electrodes at same potential – no conduction with no light

Multiple rays add their currents linearly to the electrodes, where each ray's power adds W_i current to the total sum.





$\Delta X \sim (In_1 - In_2) / (In_1 + In_2)$

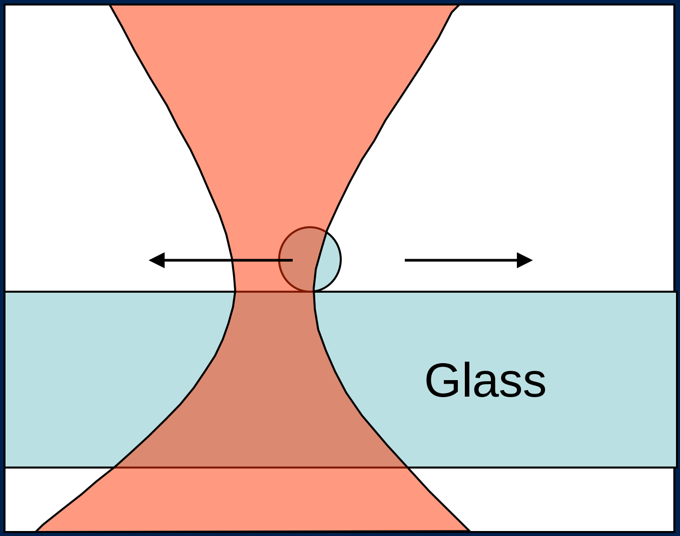
$\Delta Y \sim (Out_1 - Out_2) / (Out_1 + Out_2)$

Calibration

Want to measure forces, displaces – measure voltages from PSD – need calibration

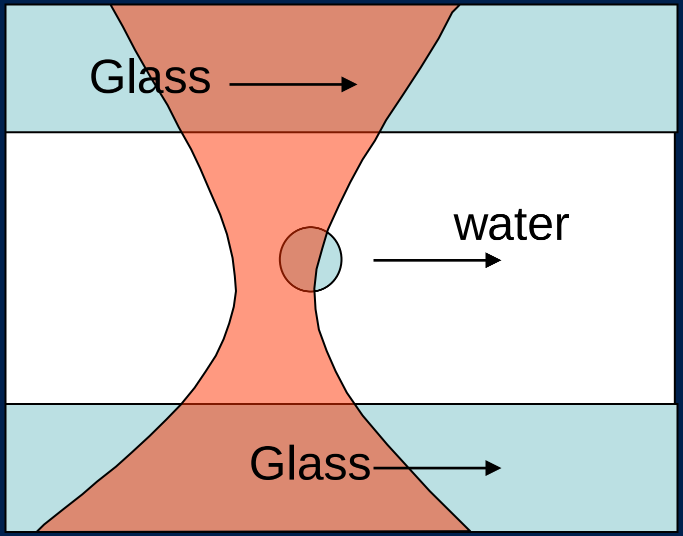
$$\Delta x = \alpha \, \Delta V$$
$$F = k\Delta x = \alpha \, k\Delta V$$

Calibrate with a known displacement



Move bead relative to trap

Calibrate with a known force



Stokes law: $F = \gamma v$

Brownian motion as test force

Langevin equation:

$$\gamma\dot{x} + kx = F(t)$$

Drag force
$\gamma = 3\pi\eta d$

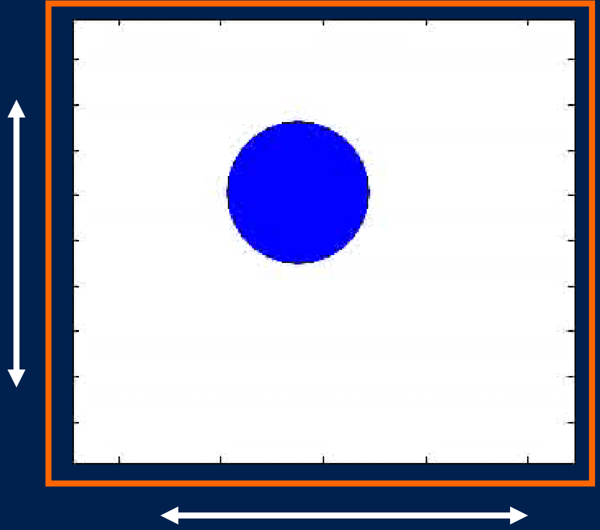
Trap force

Fluctuating Brownian force

$<F(t)> = 0$

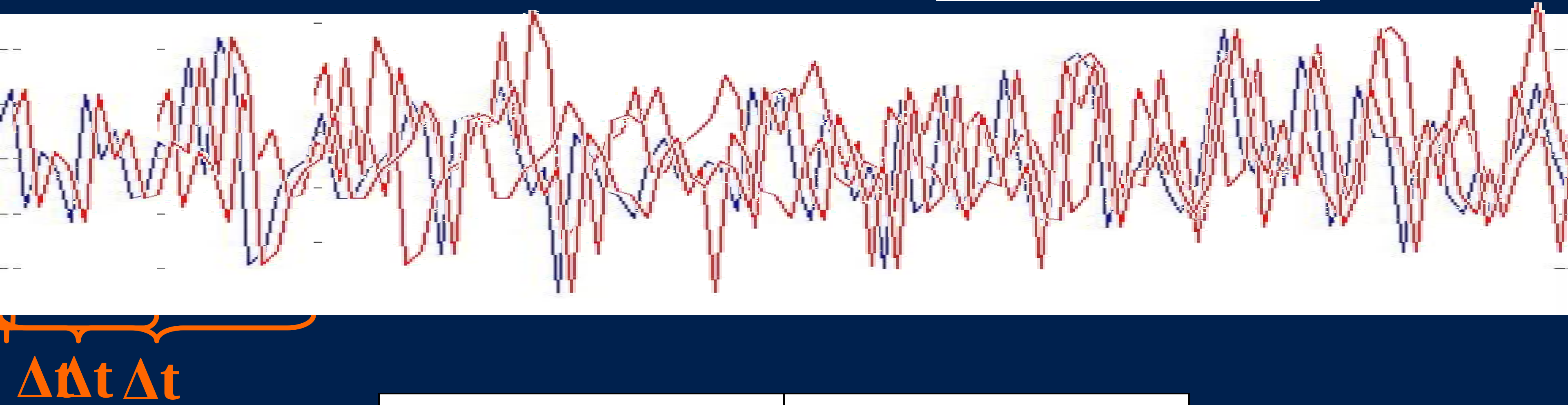
$<F(t)F(t')> = 2k_B T\gamma\delta(t-t')$

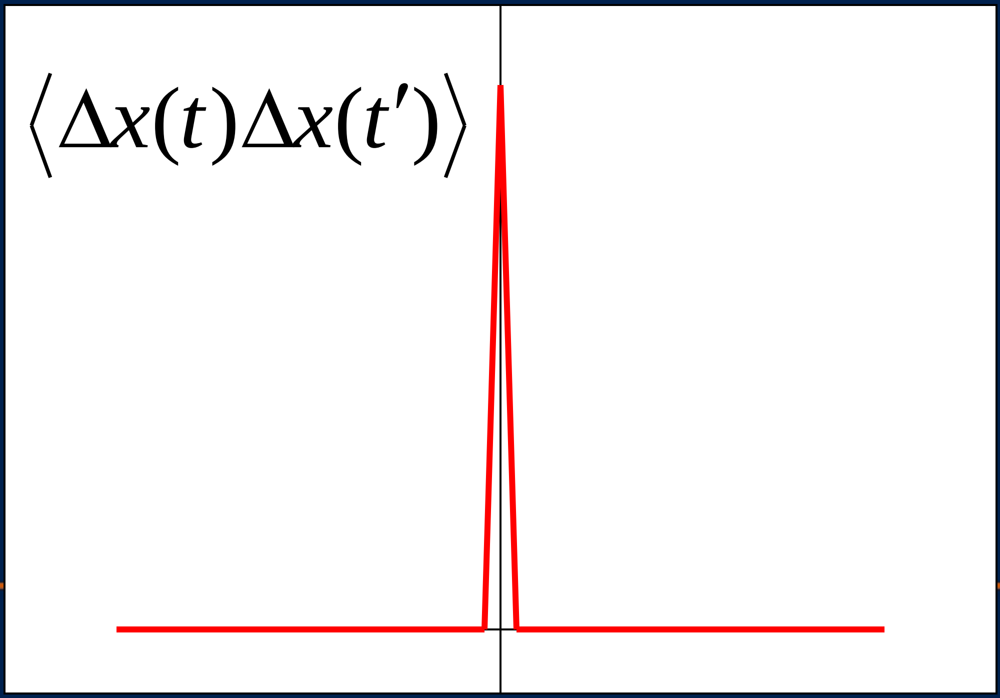
$k_B T$



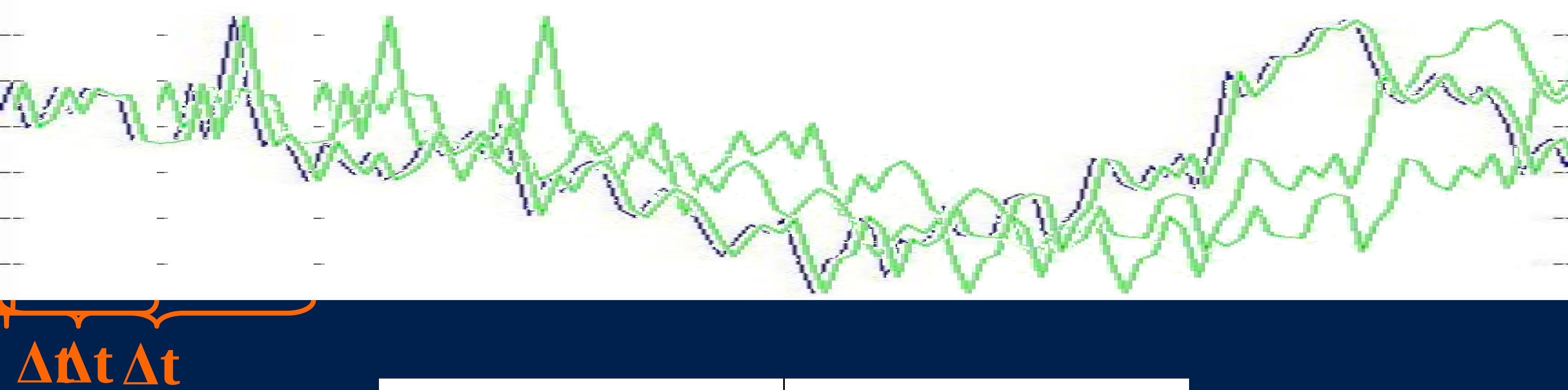
$k_B T$= 4.14pN-nm

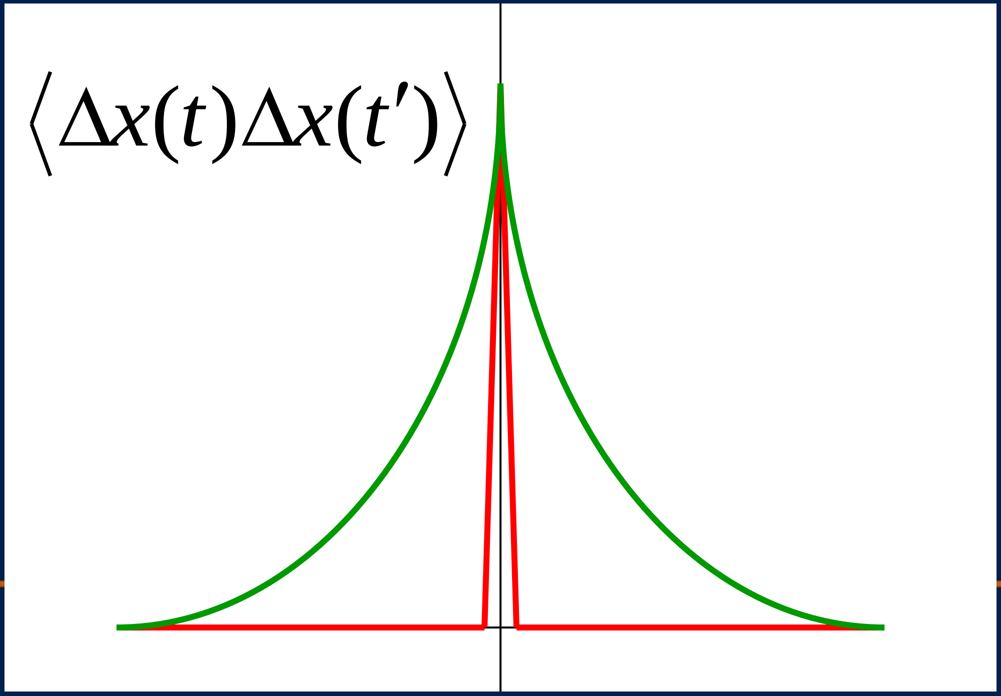
Autocorrelation function $\langle \Delta x(t)\Delta x(t')\rangle$





Autocorrelation function $\langle \Delta x(t) \Delta x(t') \rangle$





Brownian motion as test force

(will continue next time)

Langevin equation:

$$\gamma\dot{x} + kx = F(t)$$

Exponential autocorrelation function

$$\langle \Delta x(t)\Delta x(t')\rangle = \frac{k_B T}{k} e^{-k|t-t'|/\gamma}$$

$$\langle \Delta x^2 \rangle = \frac{k_B T}{k}$$

FT → Lorentzian power spectrum

$$S_x(f) = \frac{4k_B T\gamma}{k^2}\frac{1}{1+(f/f_c)^2}$$

Corner frequency
$f_c = k/2\pi\gamma$

Class evaluation

1. What was the most interesting thing you learned in class today?

2. What are you confused about?

3. Related to today's subject, what would you like to know more about?

4. Any helpful comments.

Answer, and turn in at the end of class.