Light transfers momentum to matter

$p = \frac{h}{\lambda}$

$6.55 \times 10^{-34} \text{ J} \cdot \text{s/} \lambda$

Slides mostly from Yann Chemla—blame him if anything is wrong!
Optical Trap (Nobel Prize, 1997)

Bead is held by “optical force” in trap with effective spring constant k.
Can measure: “stall force” – max force motor can make.
    displacement of bead with nm. resolution.
Key points

Light generates 2 types of optical forces: *scattering, gradient*.

Gradient leads to radiation pressure.

Trap strength depends on light intensity, gradient.

Trap is harmonic: $k \sim 0.1\text{pN/nm}$
Optical scattering forces – reflection

\[ P_i = \frac{h}{\lambda} \]

Newton’s third law – for every action there is an equal and opposite reaction

\[ F = \frac{\Delta P}{\Delta t} = \frac{(P_f - P_i)}{\Delta t} \]
Optical forces – Refraction

\[ P_i, P_f, \Delta P \]
Lateral gradient force

Object feels a force toward brighter light
Axial gradient force

Focused light

Object feels a force toward focus

Force $\sim$ gradient intensity
IR traps and biomolecules are compatible

Neuman et al. Biophys J. 1999
Biological scales

Force: 1-100 picoNewton (pN)
Distance: <1–10 nanometer (nm)
Range of forces an optical trap can measure.

Estimate size of Trapping Force

Force due to scattering of photon(s).

Simple case - reflection (Here we don’t have to carry about 2-d + Snell’s law + angles)

\[ \text{Force on material} = \frac{dp}{dt} = \frac{2 \text{Photon}}{dt} \]

\( p = \text{momentum of photon} \)

For slightly more general \( F = \frac{dp}{dt} - \frac{Qp}{dt} \)

\( Q = \text{(dimensionless) efficiency factor} \)

\( \text{not perfect reflector/scattered at angle so in general } \alpha + 2\alpha \text{ but } \alpha - \alpha \)

Now we just want to convert momentun/time into something more convenient like Energy/Time

= Power of incident light
For light (in vacuum)

\[ E = pc \]

For light in material index of refraction \( n \)

\[ E = pv = \frac{pc}{n} \quad \text{(energy/photon)} \]

\[ \frac{E}{dt} = \frac{pc}{nat} = \text{Power} \]

\[ \frac{P}{dt} = \frac{(\text{Power})n}{c} = \text{incident momentum per sec of a ray of power } P \text{ in medium of refractive index } n \]

\[ F = \frac{Q(P)}{dt} = \frac{Q(\text{Power})(n)}{c} \]

\[ Q \text{ for spherical particle radius } - \lambda \]

\[ Q = 0.1 \]

For \( P = 1 \text{ mW} = 1 \text{ mJ/sec} = 10^{-3} \text{ N-m/s} \)

\[ F = \frac{(0.1)(10^{-3} \text{ N-m/s})}{3 \times 10^8 \text{ m/sec}} \]

\[ \approx \frac{1}{2} \text{ pN} \]

\[ F \approx 0.5 \text{ pN/mW of laser power} \]
Stiffness (spring constant) of Optical Trap

If power drops from P to zero over $\lambda/n$

$$F = \frac{Kx}{\lambda/n}$$

If $x = 0$, $F = 0$

$$\frac{Q_0 P}{c} = \frac{Kx}{\lambda/n}$$

$$\frac{Q_0 P}{xc} \approx K$$

Typical spring constants $\approx 0.01 - 0.1 \text{ pN/nm}$

For $P \approx 100 \text{ mW}$ on glass/plastic beads $\approx 1 \mu m$

Traps roughly linear $\approx 200 \mu m$ (the bead escapes)

Note: Optical trap vs cantilever

Optical traps produce less force (can’t look at really strong motors.)
Damping $1 \mu m$ bead $\approx 10 \times$ less than $100 \mu m$ cantilever

For same force/trap stiffness, optical trap has better force resolution ($F \approx K/k$)
Requirements for a \textit{quantitative} optical trap:

1) Manipulation – intense light (laser), large gradient (high NA objective), moveable stage (piezo stage) or trap (piezo mirror, AOD, …) [AcoustOptic Device- moveable laser pointer]

2) Measurement – collection and detection optics (BFP interferometry)

3) Calibration – convert raw data into forces (pN), displacements (nm)
1) Manipulation

Want to apply forces – need ability to move stage or trap (piezo stage, steerable mirror, AOD…)

(Acouto Optic Device: variable placement of laser)

By using two beads, and taking difference, capable of removing floor movement! Get to Angstrom level!
2) Measurement

Want to measure forces, displacements – need to detect deflection of bead from trap center

1) Video microscopy
2) Laser-based method – Back-focal plane interferometry
BFP imaged onto detector

- **BFP**
- **imaged onto detector**
- **Trap laser**
- **specimen**
- **Relay lens**
- **PSD**
- **∑**
- **Conjugate image planes**

Diagram:
- Trap laser
- Specimen
- Relay lens
- BFP
- Conjugate image planes
- PSD
- ∑
Position sensitive detector (PSD)

Plate resistors separated by reverse-biased PIN photodiode

Opposite electrodes at same potential – no conduction with no light
Multiple rays add their currents linearly to the electrodes, where each ray’s power adds $W_i$ current to the total sum.

$\Delta X \sim \frac{(\text{In}_1 - \text{In}_2)}{(\text{In}_1 + \text{In}_2)}$

$\Delta Y \sim \frac{(\text{Out}_1 - \text{Out}_2)}{(\text{Out}_1 + \text{Out}_2)}$
Calibration

Want to measure forces, displacements – measure voltages from PSD – need calibration

\[ \Delta x = \alpha \Delta V \]
\[ F = k \Delta x = \alpha k \Delta V \]
Calibrate with a known displacement

Move bead relative to trap

Calibrate with a known force

Stokes law: $F = \gamma v$
Brownian motion as test force

Langevin equation:

\[ \gamma \dot{x} + kx = F(t) \]

Drag force
\[ \gamma = 3\pi \eta d \]

Trap force

Fluctuating Brownian force
\[ \langle F(t) \rangle = 0 \]
\[ \langle F(t)F(t') \rangle = 2k_B T \gamma \delta(t-t') \]

\[ k_B T = 4.14 \text{pN-nm} \]
Autocorrelation function \( \langle \Delta x(t) \Delta x(t') \rangle \)
Autocorrelation function \( \langle \Delta x(t) \Delta x(t') \rangle \)
Brownian motion as test force

(Will continue next time)

Langevin equation:

\[ \gamma \dot{x} + k x = F(t) \]

Exponential autocorrelation function

\[
\langle \Delta x(t) \Delta x(t') \rangle = \frac{k_B T}{k} e^{-k|t-t'|/\gamma}
\]

\[
\langle \Delta x^2 \rangle = \frac{k_B T}{k}
\]

FT \rightarrow Lorentzian power spectrum

\[
S_x(f) = \frac{4k_B T \gamma}{k^2} \frac{1}{1 + \left(\frac{f}{f_c}\right)^2}
\]

Corner frequency

\[ f_c = \frac{k}{2\pi \gamma} \]
Class evaluation

1. What was the most interesting thing you learned in class today?

2. What are you confused about?

3. Related to today’s subject, what would you like to know more about?

4. Any helpful comments.

Answer, and turn in at the end of class.