Announcements

Homework #8 assigned today, due Wed 4/18.

Today:

Diffusion

Why x^2= #Dt (from Equipartition Function)

When directed motion (v ≈ constant, x = vt)
is better/worse than diffusion (v not constant)
x (= dv/dt) = # $t^{1/2}$

Biological examples

Bacterial vs. Eukaryotic Cells
Oxygen transport: how close cells need to be to
Oxygen in blood in Lungs
Stopping time of Bacteria.

Diffusion

For "small" things, diffusion is a great way to get around.

For somewhat larger things, need directed motors.

How fast are small molecules moving in a cell?

How often do things come in contact?

Are chemical reactions rates limited by availability of food (ATP)?

Movement by random motion: diffusion.
    Limits to cell size based on oxygen diffusion/availability.

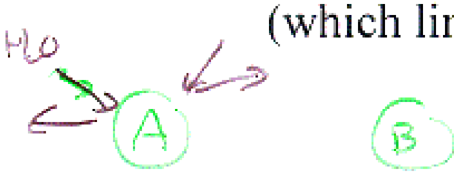
What limits how fast a cell can reproduce?
(<1 hrs for bacteria; ~day for humans)

Inertia does not matter for bacteria or anything that is small / microscopic levels.

Reminder

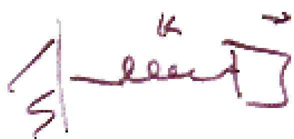
Translation & Equipartition Theorem

For two things to react, need to come in contact.
What is average speed (and distance between) molecules in cell?
Time between collisions?
How long oxygen to take to go across cell (which limits cell size)?



m_A, T

Thermal Energy $= \frac{3}{2} kT$



P.E. $= \frac{1}{2} k x^2$

free

K.E. $= \frac{1}{2} m v^2$

Equipartition Theorem

For each degree of freedom ~~that dep~~ where energy depends on (deg. of freed)^2 $=$ $\frac{1}{2} kT$ of energy

$\frac{1}{2} kT$ P.E. $\frac{1}{2} k x^2$

$\frac{1}{2} kT$ K.E. $\frac{1}{2} m v^2$

$K_B = 1.4 \times 10^{-23}\ J/^{\circ}K$

$T = 300\ ^{\circ}K$

$K_B T = 4.2 \times 10^{-21}\ J$

$(1\ J = 1\ N\text{-}m)$

$1\ N \sim \frac{1}{4}\ lb$

$1\ N \approx \frac{1}{5}\ lb)$

What is velocity of water molecule at room temperature?

$\tau_{collision} = \frac{d}{v}$ → mean free path
→ dist. H_2O mol. will go before hitting another H_2O molecule.

Density of water = 1g/cm³ (55 M H_2O)

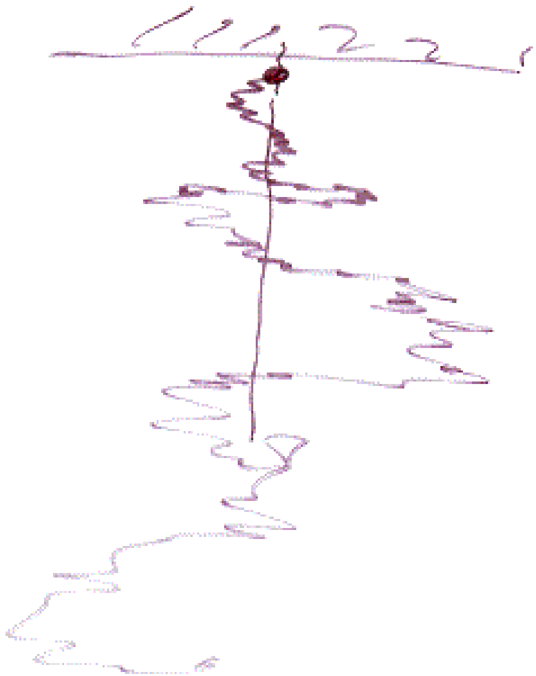
water molecules / cm³

av. dist = $(\;)^{1/3}$ → cm

$\tau_{collision}$ = ??

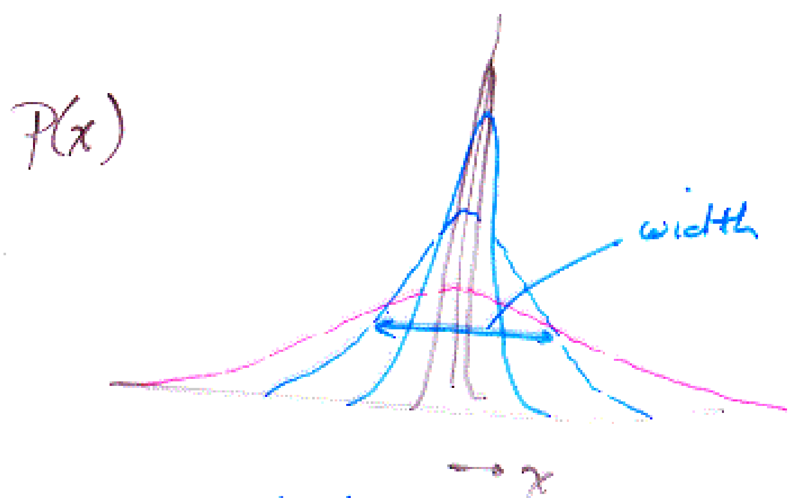
Longer Distances (>> mean free path)

Random Walk: Diffusion



$t=0 \quad P(x=0)=1$

$\qquad P(x\neq 0)=0$



$\langle x \rangle = 0$

average pos. of dye = 0 → as likely to ~~drift~~ randomly move left (−) as right (+)

$$\langle |x| \rangle = \langle \sqrt{x^2} \rangle$$

Can do but a little awkward…

$\langle x^2 \rangle$ at $t = 0$ $= 0$

$\langle x^2 \rangle$ at later time $t \neq 0$

$=$ pos. #.

$\langle x^2 \rangle$ is a measure of width of distribution

$\langle x^2 \rangle$ gets bigger in time.

if linear.

$$x = v t$$

constant speed with no bump.

go!

$x = \text{linear}(t)$

Diffusion: x^2 = # Dt

Diffusion as a Random Walk

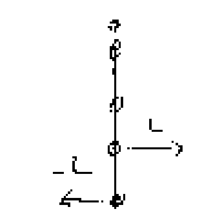
1-D case (first)

Particle at $x=0$ $t=0$.

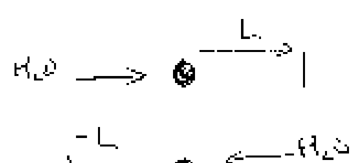
1) Assume equally likely to step to right as step to left.

2) Take steps of length L every τ seconds.

moving with velocity $\pm v_n$ between collisions ($L = \pm v\tau$)
R steps/sec ; total of N steps
(For now take v, τ as constants – they actually depend on size of particle, nature of fluid, Temp)



$x = x_0$
$= 0$
@ $t=0$



Of course in reality, distribution of step sizes but this model works amazingly well.

Thermal Motion: Move ±L

How far do particles move due to thermal motion

Derivation of $\langle x^2 \rangle = 6Dt$

We cannot predict motion of individual molecules, but can make statistical (probabilistic) arguments about average/**mean** properties, as well as distribution (**standard deviation**) of these properties.

Position after N steps = x_N

" " N+1 " = x_{N+1}

$x_{N+1} = x_N \pm L$

$\langle x_N \rangle = x_0 = 0$ for convenience we'll call starting position $x_0 = 0$

$\langle\ \rangle$ means average if we looked at many molecules (or 1 molecule many times, ~~after~~ each time, after N steps)

$\langle x_N \rangle =$ 0 by symmetry– equally likely to step left as right

$$\langle x_N^2 \rangle = F(\langle x_{N-1} \rangle, L)?$$

$$\langle x_N^2 \rangle = \langle (x_{N-1} \pm L)^2 \rangle$$
$$\langle x_N^2 \rangle = \langle x^2_{N-1} \rangle \pm 2L \cancel{\langle x_{N-1} \rangle} + L^2$$

$$\langle x_N^2 \rangle = \langle x^2_{N-1} \rangle + L^2$$

$$\langle x_1^2 \rangle = \langle x_0^2 \rangle + L^2$$

$$\langle x_2^2 \rangle = \langle x_1^2 \rangle + L^2 = \langle x_0^2 \rangle + 2L^2$$

$$\langle x_3^2 \rangle = \langle x_2^2 \rangle + L^2 = \langle x_0^2 \rangle + 3L^2$$

$$\vdots$$

$$\langle X_N^2 \rangle = \langle x_0^2 \rangle + NL^2$$

$$\sqrt{\Delta x_N^2} = \sqrt{\langle x_N^2 \rangle - \langle x_0^2 \rangle} = NL^2$$

$$\boxed{\Delta X_N^2 = NL^2}$$

The average distance moved $\sim \sqrt{N}\, L$

8

In a given period of time
X is smaller w random diffusion
than if molecule just went
at constant speed

$$X \sim \sqrt{t}$$

$$\langle x^2 \rangle = 2Dt$$ one dim

D = constant, diffusion const.
Property of molecule

depends on dimension

1-D: # = 2

2-D: # = 4

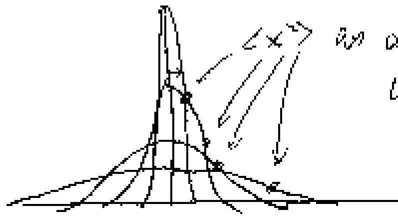
3-D: # = 6

2 = #
t = time

3-D.
$$\langle r^2 \rangle = 6Dt$$

$$|x| \sim \sqrt{2Dt}$$

if in 1 second it's gone a distance x_1
" 2 sec " " " $\sqrt{2}\, x_1$



$\langle x^2 \rangle$ as a function of time
width increases as the sqrt (time)

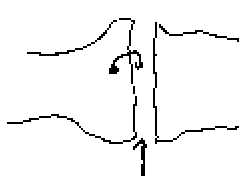
In order for a particle to wander twice as far takes 4 times longer

Plug in some #'s for $D \sim \cancel{300}\ 250\ \mu m^2/sec$
(small molecule in water)

To diffuse ~

$0.1\,\mu m \Rightarrow \quad 0.01\,\mu m^2 = (2)(250\,\mu m^2/sec)\,t$

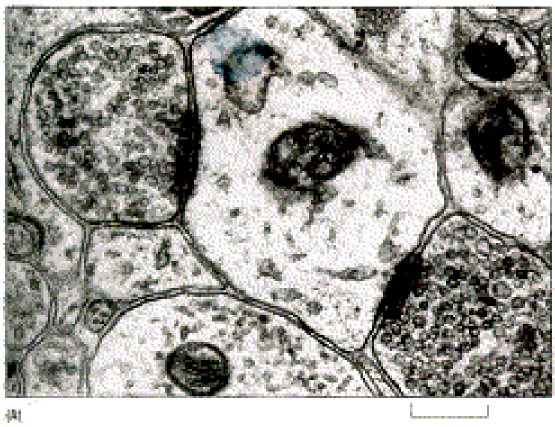
$t \sim 20\,\mu sec$ (fast)



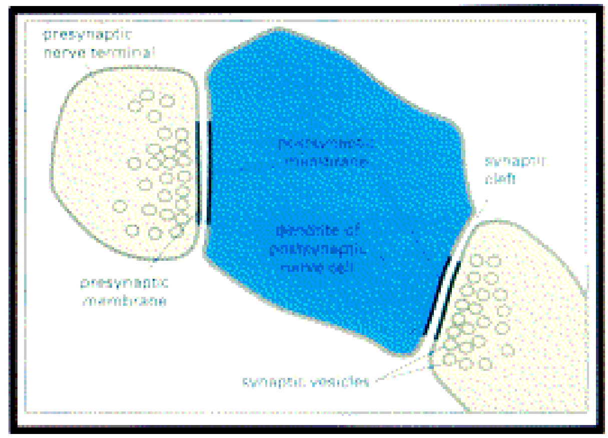
nerve synapse ~0.1μm

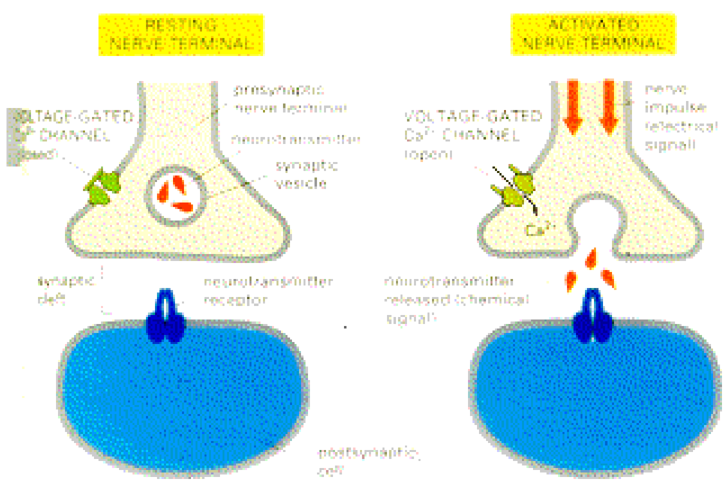
Diffusion across nerve synapse

Cross-sectional slice of nerve synapse



2 μm





How long for neurotransmitter to cross synapse via difffusion?

$D = 250\ \mu m^2/sec$

Nerve synapse: $0.1\ \mu m$ $\quad\quad X^2 = 2Dt$

$0.01\ \mu m^2 = (2)(250\ \mu m^2/sec)t$

$t = 20\ \mu sec$ (fast!)

D - diffusion const. $\langle x^2 \rangle = 6Dt$

if molecule gets bigger

$D \uparrow$ or $D \downarrow$

Object is large $D \downarrow$

because $x \downarrow$ for a given time

ex: How long does it take for O_2 to go from edge of cell to middle?

20 µm cell

$x \sim 10\,\mu m$

$$\langle x^2 \rangle = 6Dt$$

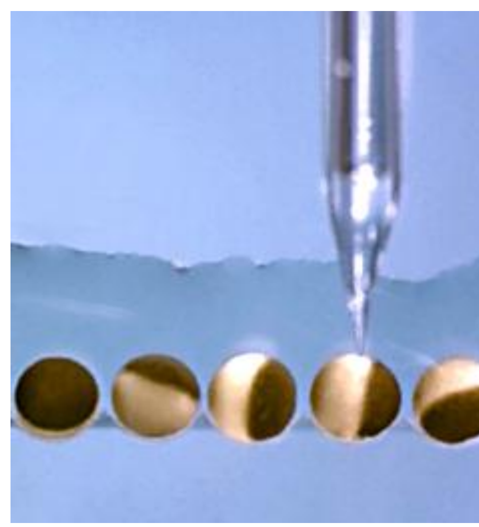
$$t = \frac{\langle x^2 \rangle}{6D} = \frac{(10\,\mu m)^2}{(6)(1000\,\mu m^2/sec)}$$

$$\frac{10^2}{6 \cdot 1000} \sim 0.016\,sec = \underline{16\,msec}$$

Bacterial cell ~1 μm
10x less distance than eukaryotic cell
100x less time

Metabolism of bacterial can (and is) much higher than eukaryotic cell
→ 10-100μm.

bacterial cell ~ 1 x 3 μm
eukaryotic cell → 10-100μm



Oocyte:1-2 mm!

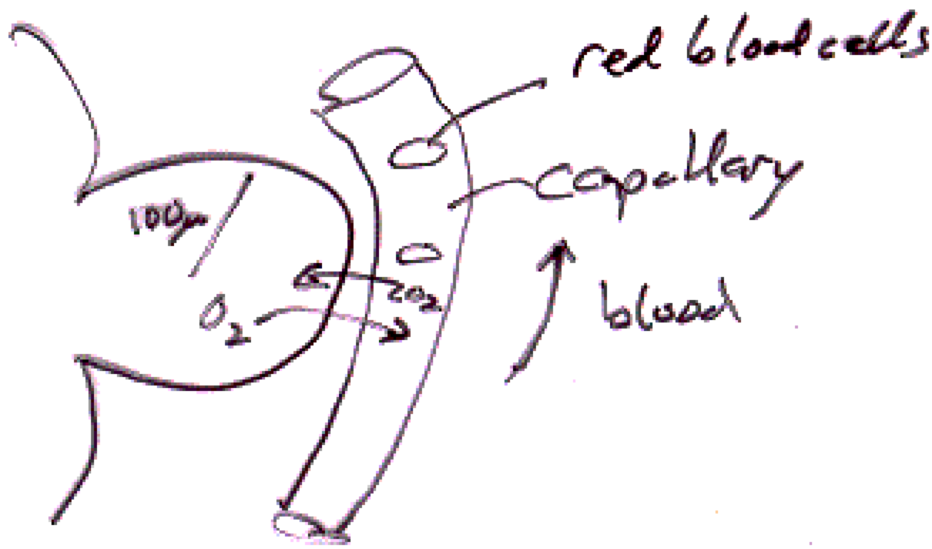
Size of eukaryotes limited by size (diffusion time of O_2). As size gets bigger, everything happens more slowly.

Large cell: frog oocytes– basically everything happens slowly.

Every cell needs to be within 50-100 μm of blood supply!

Lung + Diffusion of O_2/CO_2

- Billions of air sacks (alveoli)



Can diffusion move O_2, CO_2 fast enough?

Efficiency of Diffusion

Diffusion moves things short distances very fast!

$$\langle x^2(t) \rangle = 2Dt$$

$$\sqrt{\langle x^2(t) \rangle} = \sqrt{2Dt}$$

The mean squared displacement increases linearly with time

The mean (average) absolute distance increases as square-root of time.

Distance moved via diffusion $\sim \sqrt{t}$

$$V_{ave} = \frac{d\langle\sqrt{x^2_{ave}}\rangle}{dt}$$

$$\frac{d}{dt}\left(x^2 = 2Dt\right)$$

$$x\frac{dx}{dt} = D$$

$$\frac{dx}{dt} = \frac{D}{x} \approx \sqrt{\frac{D}{t}}$$

$\boxed{\text{very fast spreading for short times}}$

$$t \to 0 \quad v \to \infty!$$

What's wrong? Special Relativity doesn't allow this!

How Bacteria move

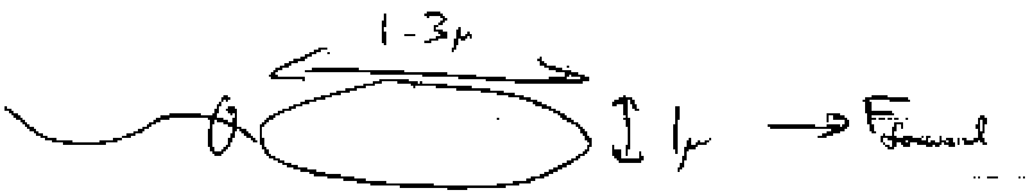
Inertia doesn't matter for microscopic world

Life at low Reynold's number

Why study?

1) Simple Ex of $F = ma$
2) Don't need much bio
3) Results are broadly applicable to bio @ μ level

Bacteria



Go ~ 25 μm/sec ~ $\boxed{\text{10 body lengths/sec}}$

Compare to you: ~~running~~ Walk: 4 miles/hr ~ 6ft/sec

~ $\boxed{\text{1 body length/sec}}$

Swimming: pool ~~length~~ lap ~ (50 yards) ~ min.

~ yard/sec ~ $\frac{1}{2}$ body length/sec

Olympic swimmer ~ body length/sec

$\boxed{\text{Bacteria: good swimmers}}$

If turn off “propeller,” how far Bacteria coast?

$F = ma$

Solve eq'n of motion ~~[illegible]~~

(1) $$\frac{m\,dv}{dt} = -\gamma v$$

$$\int \frac{m\,dv}{v} = \int -\gamma\,dt$$

$$m \ln v = -\frac{\gamma}{m} t$$

$$v = v_0 e^{-\frac{\gamma}{m}t}$$

$$= v_0\, e^{-t/\tau} \qquad \tau = m/\gamma$$

What is mass of bacteria?

$$m \sim \cancel{\tfrac{4}{3}} \; \tfrac{4}{3}\pi r^3 \rho \qquad r \sim 1\mu m \quad \rho = 1 g/cm^3$$

$$\boxed{\sim 4 \times 10^{-15}\,kg = m}$$

$$\gamma = 6\pi\eta r \qquad \eta = 0.001 \quad r = 10^{-6}\,\text{meters}$$

$$\gamma \sim 20 \times 10^{-9} \frac{N\text{-}s}{m} \sim \boxed{20\,\frac{nN\text{-}s}{m} = \gamma}$$

Plugging in #'s

$$m = 4 \times 10^{-15}\ \text{kg}$$

$$\gamma = 20\ \frac{\text{nN-s}}{\text{m}}$$

$$\tau = \frac{m}{\gamma} = 0.2\ \mu\text{sec}$$

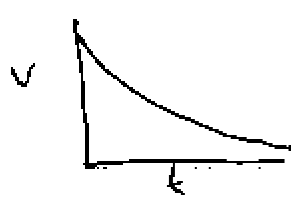
So bacteria stops in 200 nsec! Very fast.

So once forces are turned off, bacteria forgets about history very quickly.

History does not matter to bacteria

How far does bacteria coast in 0.2 μsec?

$$x = \int v\,dt = \int_0^\infty v_0 e^{-t/\tau_0}\,dt$$



$v_0 \sim 25\ \mu\text{m/sec}$

$x = 0.05$ Å ! $<$ diameter of H-atom!

Inertia is irrelevant to bacteria

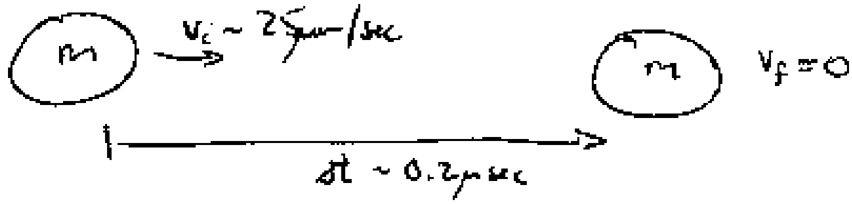
Once force is over, no forward motion!

Person swimming

- a good swimmer coasts ~ 1 body length.

Inertia is much more important to bigger organism

Size of drag force on Bacteria



$$F = \frac{\Delta p}{\Delta t} = \frac{(4\times10^{-15} kg)(25 \mu m/sec)}{0.2 \mu sec}$$

$$= 5\times10^{-13} N$$

$$= 0.5 pN \qquad (pico = 10^{-12})$$

How compare to it's weight?

$$w = mg = (4\times10^{-15} kg)(10 m/s/s) = 0.04 pN$$

So:

$\boxed{\text{Bacteria swim as if dragging } 10\times \text{ their own weight!}}$

Class evaluation

1. What was the most interesting thing you learned in class today?

2. What are you confused about?

3. Related to today's subject, what would you like to know more about?

4. Any helpful comments.

Answer, and turn in at the end of class.