Bulk Properties: Diffusion

Fig. 2. Effect of temperature on the appearance of mosaic cells within 40 min of cell fusion.

Temperature-dependent diffusion
Why is it moving?  
What is causing the motion?  

How is it moving?  
Do the quantitative details of the motion tell us anything about the cause?
Why is it moving? What is causing the motion?

How is it moving? Do the quantitative details of the motion tell us anything about the cause?

Theoretical Model
“Random Movement in a Fluid Membrane”

Mathematical

Compare Theory to Experiment

Experiment
Measure relevant parameters
Model: “Random Motion in a Fluid”

Brownian Motion (Brown, 1827)

Observed motion of small objects suspended in a fluid

Milk fat suspended in water

http://www.microscopy-uk.org.uk/dww/home/hombrown.htm

Why are the objects moving?

~Proteins/lipids in fluid membrane?
Brownian Motion is due to random collisions with fluid molecules (Einstein 1905, Smoluchowski 1906; Langevin 1908)

\[ E = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT \]

\[ \Rightarrow v_{rms} = \sqrt{\frac{3kT}{m}} \]

Brownian Motion is temperature dependent

Fig. 2. Effect of temperature on the appearance of mosaic cells within 40 min of cell fusion.
1-D Random Walk

Time

\(t = 0, x = 0\)

Flip a coin...

Heads \((p = 0.5)\)

Tails \((q = 0.5)\)

\(\Delta t\)

Flip a coin...

\(\Delta t\)

Flip a coin...

\(\Delta t\)

Flip a coin...

\(\Delta t\)

Total Time \(t = N \cdot \Delta t\)

Total Displacement \(x = (2m - N)L\)

Of course not, it’s random!

Brownian Motion Model

- \(\Delta t \sim\) time between collisions
- \(L \sim\) path taken by particle after collision

Can we predict the displacement as a function of time, \(x(t)\)?
What about a group (ensemble) of random walkers?

We can’t predict individual paths... But we can predict the probability to obtain a given path!
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Probability for each path is:

\[
P_i(m) = p^m q^{N-m}
\]

But there is more than one way to get each \( m \)...

\[
P(m) = \frac{N!}{m!(N-m)!} p^m q^{N-m}
\]

“Binomial Probability Distribution Function”
Now we can find the mean displacement, \( \langle x \rangle \)

\[
x = (2m - N) L \quad \Rightarrow \quad \langle x \rangle = (2\langle m \rangle - N) L
\]

\[
\langle m \rangle = \sum_m m P(m) = \sum_m \frac{N!}{m!(N-m)!} p^m q^{N-m} = p \frac{d}{dp} \sum_m \frac{N!}{m!(N-m)!} p^m q^{N-m} \\
= pN(p+q)^{N-1} = pN
\]

\[
\langle x \rangle = (2\langle m \rangle - N) L \\
= (2N/2 - N) L
\]

\[
\langle x \rangle = 0
\]

And second moment, \( \langle x^2 \rangle \)

\[
x^2 = (2m - N)^2 L^2 \quad \Rightarrow \quad \langle x^2 \rangle = 4\langle m^2 \rangle L^2 - 4\langle m \rangle N + N^2 L^2
\]

Using the same trick,

\[
\langle x^2 \rangle = NL^2 = 2 \frac{L^2}{2\Delta t} t
\]

\[
\langle x^2 \rangle = 2Dt, \quad D = \frac{L^2}{2\Delta t}
\]