PHY460: Problem Set #6 Due: Thursday 12/14 by 5 pm in Yellow PHY460 homework box between Loomis and MRL on 2nd Floor

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1. *Monatomic linear lattice:* Consider a longitudinal wave

$$u_s = u\cos(\omega t - sKa) \tag{1}$$

which propagates in a monatomic linear lattice of atoms M, spacing a, and nearest neighbor interaction C.

(a) Show that the total energy of the wave is

$$E = \frac{1}{2}M\sum_s \left(\frac{du_s}{dt}\right)^2 + \frac{1}{2}C\sum_s (u_s - u_{s+1})^2 \tag{2}$$

where s runs over all atoms.

(b) By substitution of u_s in this expression, show that the time-average total energy per atom is

$$\frac{1}{4}M\omega^2u^2 + \frac{1}{2}C(1-\cos Ka)u^2 = \frac{1}{2}M\omega^2u^2, \tag{3}$$

where in the last we have used the dispersion relation for this problem.

2. *Basis of two unlike atoms:* For the problem of phonons in crystals with a two-atom basis which we solved in class, find the amplitude ratios $\frac{u}{v}$ for the two branches at $K_{max} = \frac{\pi}{a}$. Show that at this value of K, the two lattices act as if decoupled: one lattice remains at rest while the other lattice moves.

3. *Diatomic chain:* Consider the normal modes of a linear chain in which the force constants between the nearest-neighbor atoms are alternately C and $10C$. Let the masses be equal, and let the nearest-neighbor separation be $\frac{a}{2}$. Find $\omega(K)$ at $K = 0$ and $K = \frac{\pi}{a}$. Sketch the dispersion relation by eye. This problem simulates a crystal of diatomic molecules such at H_2.

4. *Enumeration of Phonon Modes:* In this problem we will construct a 3×3 table where the rows are the spatial dimension and the columns are the number of atoms per unit cell. For spatial dimensions $d = 1, 2, 3$ and for $p = 1, 2, 3$ atoms per unit cell list the amount of transverse, longitudinal, acoustic, and optical phonon modes per wave vector in the first Brillouin zone.

5. *Quantum Wells* Suppose the we make 3-material sandwiches out of materials A and B in the order A-B-A. Consider only the valence band and conduction bands of both materials and describe whether or not the electrons and/or the holes are confined into quantum wells. Draw the shapes of the quantum wells to scale and roughly sketch the direction of the band-bending occurring from charge transfer at the material interfaces for the conduction band states. Note: The 0 eV valence band edge of material A is not the same as the 0 eV valence band edge for material B.

(a) $W_A = 1eV$, $E_{cA} = 1.2eV$, $E_{vA} = 0eV$ $E_{FA} = 0.9eV$ and $W_B = 1.1eV$, $E_{cB} = 0.6eV$, $E_{vB} = 0eV$ $E_{FB} = 0.3eV$

(b) $W_A = 1eV$, $E_{vA} = 0eV$, $E_{GA} = 1.2eV$, $E_{FA} = 0.9eV$ and $W_B = 1.1eV$, $E_{vB} = -1eV$, $E_{GB} = 1.6eV$, $E_{FB} = 0.3eV$

where W_i is the work-function, E_{ci} and E_{vi} are the conduction/valence band edges, E_{Fi} is the Fermi level, E_{Gi} is the energy gap defined by $E_{Gi} = E_{ci} - E_{vi}$, and $i = A, B$ is the index labeling the two different materials.

6. *Degeneracy in Landau Levels* Suppose we confine a 3D free electron in an infinite square quantum well that confines the electrons along the z-direction so that the energy is

$$E_j(\mathbf{k}) = \frac{\hbar^2\left(k_x^2 + k_y^2\right)}{2m^*} + \frac{\hbar^2\pi^2}{2m^*d^2}j^2 \tag{4}$$

where $j = 1, 2, 3, \ldots$ and d is the width of the quantum well. Suppose that $d = 50$nm, $L_x = L_y = 1$ mm, and $m^* = 0.1m_0$ where m_0 is the free-electron mass in vacuum. Assume that electrons can only fill the lowest subband $j = 1$.

(a) What is the size of the magnetic length ($\ell_B^2 = \hbar/(eB)$)for $B = 0, 0.1, 1, 10$ and 100 Tesla?

(b) What is the cyclotron energy gap ($\omega_c = eB/m^*$) (spacing between Landau levels) for $B = 0, 0.1, 1, 10$ and 100 Tesla?

(c) How many states can fit in the lowest Landau Level for $B = 0.1, 1, 10$ and 100 Tesla?

(d) What density of electrons in cm^{-2} will exactly fill one Landau level for $B = 0.1, 1, 10$ and 100 Tesla?

(e) Each quantum well subband has its own set of Landau levels what magnetic field strength will cause the second-lowest Landau level of $j = 1$ to be higher energy than the lowest Landau level of $j = 2$?

7. *Density of States of Quantum Wires* Suppose we have a free electron confined in two directions with energy

$$E_{q,r}(\mathbf{k}) = \frac{\hbar^2 k_x^2}{2m^*} + E_{q,r}^0 \tag{5}$$

where $q, r = 1, 2, 3, \ldots$ and $E_{q,r}^0$ does not depend on k. Show that the Density of states of the q, r-th quantum wire subband is proportional to the inverse-velocity as a function of energy.

8. *London penetration depth:*

(a) Take the time derivative of the London equation to show that

$$\frac{\partial \mathbf{j}}{\partial t} = \frac{1}{\mu_0 \lambda_L^2} \mathbf{E} \tag{6}$$

(b) If $m d\mathbf{v}/dt = q\mathbf{E}$, as for free carriers of mass m and charge q, show that $\lambda_L^2 = m/\mu_0 n q^2$.