PHY460: Problem Set #5 Due: Wednesday 11/29 by 5pm in Yellow PHY460 homework box between Loomis and MRL on 2nd Floor

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1. In a particular semiconductor there are 10^{19} donors/m^3 with an ionization energy (i.e., gap between donor levels and conduction band) E_d of 1 meV and an effective mass $0.01m_e$ (m_e is the vacuum mass of an electron). Estimate the concentration of conduction electrons at 4K.

2. Make a plot of the first two Brillouin zones of a primitive rectangular two-dimensional lattice with axes a, $b = 3a$.

3. A two-dimensional metal has one atom with valence equal to one in a simple rectangular primitive cell $a = 2\AA$; $b = 4\AA$. (a) Draw the first Brillouin zone. Give its dimensions, in m^{-1}. (b) Calculate the radius of the free electron Fermi sphere, in m^{-1}. (c) Draw this sphere to scale on a drawing of the first Brillouin zone.

4. Assume the low-energy part of the conduction band of a metal can be approximated in the effective-mass approximation with an effective mass matrix

$$\left(\frac{1}{m}\right)_{ij} = \frac{1}{m^*}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{1}$$

   Sketch a representative Fermi-surface in the $k_x - k_y$-plane. HINT: Think about what the eigenvalues of the effective mass matrix mean. You can also look in Chapter 17 of Simons for more information on the effective mass matrix.

5. Assume the the bandstructure of a metal is well-described in the tight-binding approximation using a square lattice with a dispersion relation

$$E(\mathbf{q}) = E_0 - 2\gamma\cos(q_x a) - 2\gamma\cos(q_y a). \tag{2}$$

   For simplicity choose $E_0 = 0$, $\gamma = 0.5$ eV and $a = 1\AA$.

   (a)In the $q_x - q_y$-plane draw the boundaries of the Brillouin zone and label the axes with the proper units.

   (b)Sketch the Fermi surfaces at Fermi-energies of $E_F = -0.95$, $E_F = 0.0$, and $E_F = 0.95$. For each Fermi-surface indicate if it would be easier to use an electron or hole description of the electrical carrier conduction.

   (c) For each Fermi-surface mark the points where the line $q_y = q_x$ intersects the Fermi-surface and indicate the direction of the Fermi-velocity at these points.

   (d) Using the same parameters expand E($\mathbf{q}$) around the two points in the first Brillouin zone where an effective-mass approximation (in both the q_x and q_y directions simultaneously) is useful. There are actually four points but I only want you to consider the points where the effective mass matrix is proportional to the identity matrix so that the diagonal elements have the same sign. HINT: One expansion point will have a positive effective mass, and one will have a negative effective mass.

   Do these look similar or different to the same Fermi-energies in part (b)?

   (e) On a separate drawing of the first Brillouin zone in the $q_x - q_y$-plane draw the Fermi-surfaces for (i) $E_F = -0.95$ for the positive effective mass case from part (d) and (ii) $E_F = 0.95$ for the negative effective mass case from part (d).

   HINT: It will be helpful to use Mathematica's contour plot function to determine the shapes of the Fermi-surfaces in part (b) and for comparison in part (e)